Mon. 9/23 Wed. 9/25 Thurs 9/26 Fri., 9/27	**2.5** Conductors **3.1-.2** Laplace & Images ***Summer Science Research Poster Session****: Hedco7pm~9pm* **Review**	HW3
Mon. 9/30	**Exam 1 (Ch 2)**	

Materials

Announcements

Last Time

Work to build a Continuous Charge Distribution

$$W_{assemble} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{j<i}\frac{q_i q_j}{\mathscr{r}_{ij}} = \frac{1}{2}\left[\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{i\neq j}\frac{q_i q_j}{\mathscr{r}_{ij}}\right] = \frac{1}{2}\sum_{i=1}^{n} q_i\left(\frac{1}{4\pi\varepsilon_0}\sum_{i\neq j}\frac{q_j}{\mathscr{r}_{ij}}\right) = \frac{1}{2}\sum_{i=1}^{n} q_i V(r_i)$$

No Self-Energy.

$$W_{assemble} = \lim_{dq\to 0}\frac{1}{2}\sum_{i=1}^{n} dq_i V(r_i) = \lim_{d\tau\to 0}\frac{1}{2}\sum_{i=1}^{n}\rho\, d\tau V(r_i)$$

$$W_{assemble} = \frac{1}{2}\int \rho V\, d\tau$$

$$W = \frac{\varepsilon_0}{2}\int_{\text{all space}} E^2\, d\tau$$

Almost Self-Energy. $q(r') = \lim_{d\tau\to 0}\rho(r')d\tau' \to 0$

Inapplicable for Point Charges. $q(r') = \lim_{d\tau\to 0}\rho_{point}(r')d\tau' = q \neq 0$

Where's the Energy?

This Time

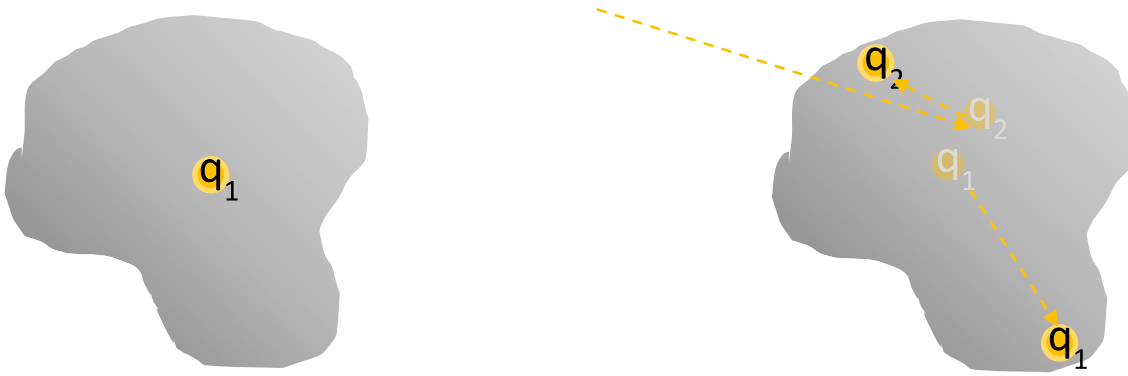
Summary

Speaking of continuous charge distributions. There are two kinds of media over which you might distribute charges.

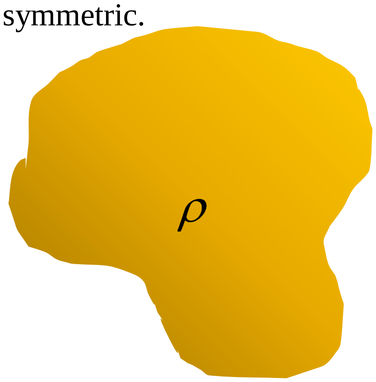
- *insulators*, for which charges stay right where you put them because they get attached to particular atoms.
- *Conductors*, over which the charges are free to (and will) distribute themselves in response to each other and any other charges around.

This means conductors require a bit of thought to figure out just how charges *will* distribute themselves and just what fields they will produce.

Let's conceptually build up a mental picture of a conductor. As a thought experiment, imagine an object of some material that's 'perfectly slippery' for charges. You place one charge on it, and with no other charges to push it around, it stays put. Then you bring in a second charge, and while you're doing so, *it* pushes the first charge to the far end of the material. When you release the second charge, it's pushed by the *first* charge to the opposite end of the material.



If you proceeded to load charges on this thing, they would continue to distribute themselves all over the object, as far from each other as they can be so you end up with a fairly uniform charge distribution over the whole thing. (note: the symmetry / asymmetry of the charge density reflects the geometric symmetry / asymmetry of the object itself – only on a sphere would the distribution be perfectly symmetric.



The difference between what we imagined happening in this scenario and what happens with a real conductor is that a conductor *already has* mobile charges on it – the conduction electrons. When the object is neutral, they're distributed just like the atoms of the object – so there's no charge build up anywhere. But the moment you bring another charge toward the object, these electrons start redistributing themselves in response to it.

Properties of Conductors – ask students to list and explain/justify them!

(i) *E* = 0 inside a conductor (eventually)

If there were an electric field inside, the free charges would move until there was no longer one.

Suppose conductor is placed in an external electric field. The *induced charges* on the metal produce an electric field inside the conductor that is same size but the opposite direction as the external electric field.

(ii) ρ =0 inside a conductor

This can be explained with Gauss's law, $\vec{\nabla}\cdot\vec{E}=\rho/\varepsilon_0$. If E = 0 everywhere inside, then ρ = 0.

(iii) Any net charge resides on the surface(s) of a conductor

There is nowhere else that it can be, if not inside.

(iv) *V* is constant throughout a conductor

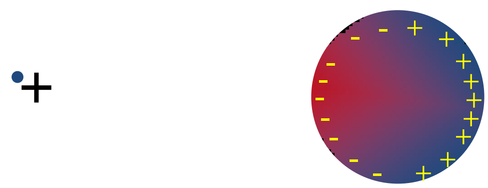
Since E = 0 everywhere inside, the potential difference between any two point inside the conductor is $V(b)-V(a)=-\int_a^b \vec{E}\cdot d\vec{\ell}=0$. This is more obvious if you use a path that is completely inside the conductor.

(v) $\vec{E}$ is perpendicular to the surface, just outside a conductor

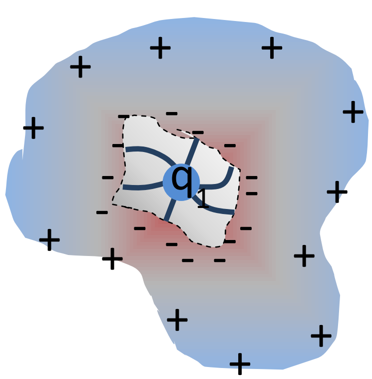
If there were a tangential component to the electric field, it would cause the surface charge to move along the surface *until* there were no tangential component.

Induced Charges

If an external charge is near a conductor, there will be induced surface charges on the conductor. Unless charge is transferred, the net charge of the conductor will not change (charge conservation). A simple example is Fig 2.44 – inside E = 0.

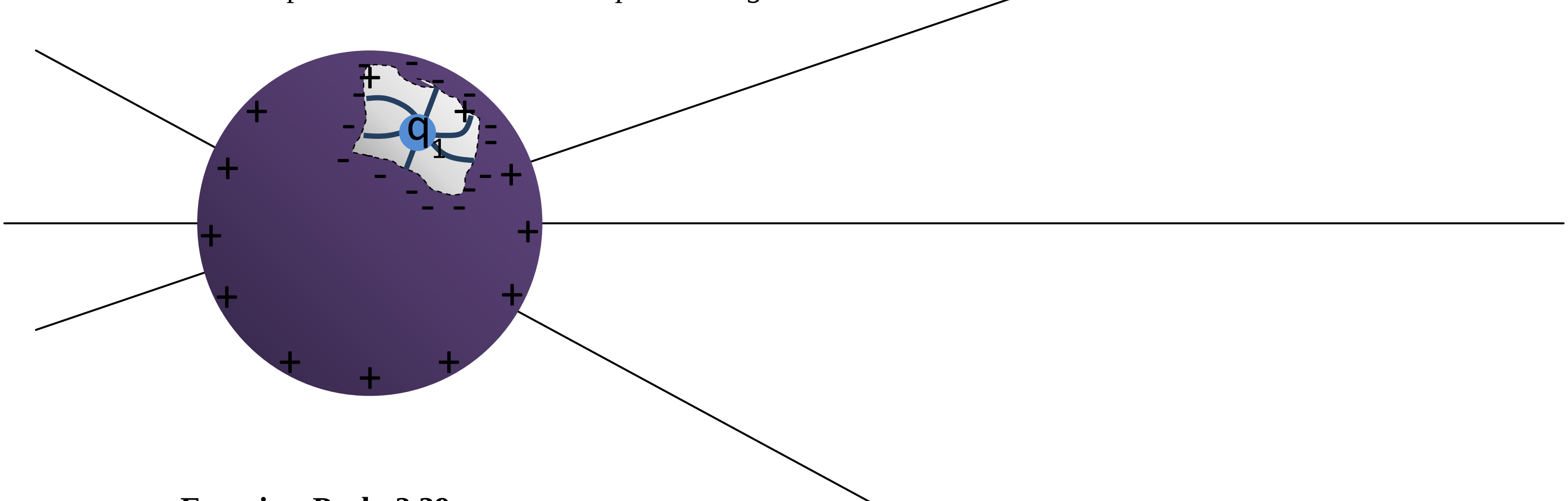


Two more interesting examples are Fig. 2.45 and **2.46**. Suppose that there is no net charge on the conductor. In both cases, the total surface charge on the cavity's wall is –q, which can be seen using a Gaussian surface just inside the conductor (taking it as a given that there's no field in the body of the conductor.) This doesn't tell us how the charge is distributed if the cavity has an irregular shape. The total surface charge on the outside of the conductors is +q because they are neutral. In the first case, that is all we can tell. On the sphere, the charge is uniformly distributed because of its regular shape.



Griffiths makes the interesting argument that, from the outside, the field emanates from the outside of the sphere as if there were a point charge at its center, *not* where the actual extra charge is. The argument is that the actual point charge already gathers to it surface of charge that cancels its field throughout the conductor. What is left is that the, now unpaired charges in the conductor distribute themselves optimally with respect to each other – uniformly over the surface.

Sphere with off-center Cavity. This point is particularly driven home by example 2.10. The charge in the cavity draws charges to line the cavity and cancel it's field everywhere. Of course, that leaves behind unbalanced charges in the conductor who push each other to the surface *oblivious to the charges around the cavity.* They distribute themselves uniformly and thus generate the same field (outside the surface) as would a point charge at the center of the sphere *not where the extra point charge is.*



Exercise: Prob. 2.39

Two spherical cavities, of radii *a* and *b*, are hollowed out from the interior of a (neutral) conducting sphere of radius *R*. At the center of each cavity a point charge is placed – call these charges q_a and q_b.

(a) Find the surface charge densities σ_a, σ_b, and σ_R.

$-q_a/(4\pi a^2)$ $-q_a/(4\pi b^2)$ $(q_a+q_b)/(4\pi R^2)$

(b) What is the field outside the conductor?

$(q_a+q_b)/(4\pi\varepsilon_o r^2)$

(c) What is the field within each cavity?

$-q_a/(4\pi\varepsilon_o a^2)$ $-q_a/(4\pi\varepsilon_o b^2)$

(d) What are the forces on the two charges?

0; the fields of the surface charges surrounding them cancel (note: wouldn't be the case if the cavities weren't spherical)

Surface Charge and the force on a conductor

I'll do the special case of a solid conductor. The book does the more general case of a simple sheet.

Say you expose a conductor to an external field. Whether the conductor had any surface charge distribution before, it certainly does now.

The external force applied to a patch of the conductor is

$$\vec{F}_{ext} = \left(\sigma A_{patch}\right)\vec{E}_{ext}$$

Now, what *is* the "external field" in terms of the *total* field right there? You should have done a problem on this back in Phys 232. The *total field* is that due to the patch in question + that due to *all* other charges; that includes the field due to the rest of the surface and the field due to any external sources

$$E_{ext} = E_{tot} - E_{patch}$$

Now, we know that if we only had the patch of surface charge, then it would radiate equal and opposite field both up (out of the conductor) and down (into the conductor) to the tune of

$E_{patch} = \frac{\sigma}{2\varepsilon_o}$ (thank you Gauss's Law)

Look inside Capacitor

$\vec{E}_{patch} = -\frac{\sigma}{2\varepsilon_o}\hat{n}$ That the total field inside is 0 tells us

$$E_{ext} = 0 - \vec{E}_{patch} = \frac{\sigma}{2\varepsilon_o}\hat{n}$$

Extrapolate to above as well

Now, if that's the external field just *below* the surface of the conductor, then that's also the external field just *above* the surface of the conductor – that's the external field *at* the surface of the conductor.

$$\vec{F}_{ext} = \left(\sigma A_{patch}\right)\vec{E}_{ext}$$

$$\vec{F}_{ext} = \left(\sigma A_{patch}\right)\left(\frac{\sigma}{2\varepsilon_o}\hat{n}\right)$$

$$\vec{F}_{ext} = \left(A_{patch}\right)\left(\frac{\sigma^2}{2\varepsilon_o}\hat{n}\right)$$

$$P = \frac{\vec{F}_{ext}}{A_{patch}} = \frac{\sigma^2}{2\varepsilon_o} = \varepsilon_o\frac{E_{net}^2}{2}$$

This pressure makes the conductor want to move into the field.

Note: even if there is *no* external source, then this pressure *still* exists - all the charges on the conductor repel each other and so would make the conductor want to stretch away from itself

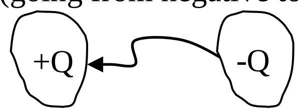
(naturally, all those extra surface charges have gotten *as far away from each other as possible*, but they're still pushing to get further.)

Example: A "very long" metal cylinder of radius *s* and length *L* carries a total charge Q. What is the outward electrostatic pressure due to all the charges repelling each other.

Capacitors

We're used to thinking of capacitors as two, parallel conducting plates. The "capacitance" is the ratio of the charge that's been separated across them to the change in voltage between them. However, anytime you have a charge separation, you can speak of the capacitance; for that matter, any time you have two conductors (charged or not), you can speak of their capacitance (what the ratio of charge to voltage *would* be).

Suppose there are two conductors and we put a charge +*Q* on one and –*Q* on the other. The potential difference between the two (going from negative to positive charge) is



$$\Delta V = V_+ - V_- = -\int_{(-)}^{(+)} \vec{E}\cdot d\vec{\ell}\ .$$

The *capacitance* is defined as

$$C = \frac{Q}{\Delta V}\ .$$

While this was calculate imagining a specific charge and a specific voltage, it's actually independent of those: if the charges are double, the size of the electric field everywhere and the potential difference double, so it's really a property of the conductors and their separations rather than of specific charges placed upon them.

Units: Capacitance is measured farads (F) are coulombs per volt. A farad is a very large capacitance! Common values are microfarads (10^{-6} F) and picofarads (10^{-12} F).

Results for a large parallel plate capacitor: (see Ex. 2.5 for *E*)

$$\left|\vec{E}\right| = \sigma/\varepsilon_0 = Q/A\varepsilon_0 \text{ (between the plates)}$$

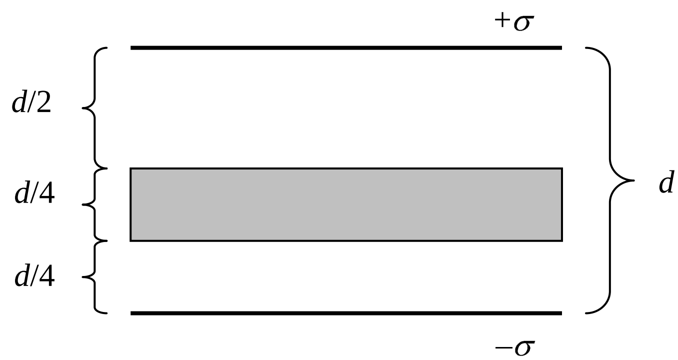
$$V = \left|\vec{E}\right| d = Qd/A\varepsilon_0$$

$$C = \frac{Q}{V} = \frac{A\varepsilon_0}{d}$$

Exercises:

Say you have two concentric cylinders of radii b>a and charge +Q, -Q. What is their capacitance?

Metal Plate inside a Capacitor – induced charges



What are the induced charge densities on the metal plate inserted inside the capacitor as shown above?

Must be equal and opposite to the charge densities of the plates they face in order to cancel the field within.

The "external" field due to the capacitor is $\left|\vec{E}\right| = \sigma/\varepsilon_0$ (inside) which is downward. There must be surface charges on the metal that produce a field inside it that cancels the external field. There must be positive charge on the bottom and negative on the top. Since it also acts like a capacitor, the field doesn't depend on the distance and the surface charge must be the same as on the capacitor plates. Note that the field outside of the metal is unchanged, but inside the metal $E = 0$.

What is the change in voltage across the whole gap?

Integrating the field along the path length, now get field only for ¾ of the separation.

What is the capacitance of the two plates, assume area A?

This concludes Ch. 2 and Exam 1's coverage.

Preview

On Wednesday, we'll talk about properties of solutions to Laplace's equation, $\nabla^2 V = 0$ and the image method.

"I became a little conceptually confused when Griffiths stated that all charges naturally accumulate on the surface of a conductor (the part that made sense), and then turns around and later on pg. 99, states "but [energy of a sphere] is greater when"

Rachael Hach Hide response Post a response Flag as inappropriate

Admin

...the charge is uniformly distributed through the volume." So can you uniformly distribute a charge say, in a solid sphere?

Rachael Hach

"Could you get an electric field in a cavity of a conductor if there is another conductor inside the cavity and the whole system is in an electric field?"

Spencer Post a response Flag as inappropriate

Admin

"Can we talk more about the brief argument made about the faraday cage using Ampere's law - I didn't quite understand that. Also, what happened to the negative sign when calculating V for the //-plate capacitor?"

Casey McGrath Post a response Flag as inappropriate

Admin

"I'm having a little trouble getting how charges fall on the surface of conductors and cavities. Field must be zero in a conductor but not always in a cavity, yet neither can hold charge and both carry charges on the surface. How does this work?"

Anton Post a response Flag as inappropriate

Admin

"In Ex 2.10, how do we know that the charge surrounding the cavity is -q and not +q?"

Jessica Post a response Flag as inappropriate

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"Can we talk a little bit about what is meant by "electrostatic pressure" and what Griffiths means when he says that it tends to draw the conductor into the field."

Ben Kid Post a response Flag as inappropriate

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"Can we go over the argument on p. 101 about how the sum of the three fields being zero is due to the sum of only the first being zero and the third being zero on its own? Also can we have an explanation of the concept of 'electrostatic pressure?'"

Sam Post a response

Admin