Fri. 9/6	*(C 15)* **2.1.4** Continuous Charge Distributions	
Mon. 9/9 Wed. 9/11 Thurs 9/12	*(C 21.1-.5,.8)* **1.2, 2.2.1-.2.2** Gauss & Div, **T2** Numerical Quadrature *(C 21.1-.5,.8)* **2.2.3** Using Gauss	HW1

Equipment

- Load on Vpython
- Bring in ppt's of rod, ring, disc, and sphree

NOTE: Very detailed notes are still online from my teaching of Phys 232 in which I either did or set up the integrals for line, ring, disc, and sphere of charge's E field.

Summary

Electric Fields for Charge Distributions

To calculate electric fields for charge distributions, do not start with Eqs. 2.6-2.8. It is better to start with a diagram and the summation equation (Eq. 2.4) when setting up an integral! Notice that is what is done in Example 2.1.

$$\vec{E}(\vec{r}) = \sum_{i=1}^{n} \vec{E}_i(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^2}\,\hat{\mathscr{r}}_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^3}\,\vec{\mathscr{r}}_i \qquad [2.4]$$

There are three types of continuous charge distributions: λ = charge per unit length, σ = charge per unit area, and ρ = charge per unit volume. They can be constant or depend on position. The charge in a small (differential) length, area, or volume is:

$$q \to \lambda(\text{length}) \to \sigma(\text{area}) \to \rho(\text{volume}).$$

Symmetry

The direction of the electric field can often be determined first, which can save work!

If some operation (mirror inversion or rotation) does not change the distribution of charges, it can't change the electric field (especially its direction).

Example:

Worth doing one in excruciating detail, say field due to rod. Then come down to special case.

1st – Rather quickly ran through field due to a rod powerpoint

Example 2.2 – (*worked in text*) Find the electric field a distance z above the midpoint of a straight line segment of length $2L$ which carries a uniform line charge λ.

Divide it into small pieces of length dx'. Consider one piece at x'. The positions and separation are

$$\vec{r} = z\hat{z} \text{ (field point)},$$

$$\vec{r}' = x'\hat{x} \text{ (source point)},$$

$$\vec{\boldsymbol{\imath}} = \vec{r} - \vec{r}' = -x'\hat{x} + z\hat{z}.$$

By symmetry (inversion about z axis), the electric field can only point in the z direction, so we only need to find that component. The contribution from the one piece is

$$dE_z = \frac{1}{4\pi\varepsilon_0}\frac{q}{\boldsymbol{\imath}^3}\boldsymbol{\imath}_z = \frac{1}{4\pi\varepsilon_0}\frac{(\lambda\, dx')}{\left(x'^2+z^2\right)^{3/2}}\, z\,.$$

Integrate over x' to find the total field:

$$E_z = \frac{\lambda z}{4\pi\varepsilon_0}\int_{-L}^{L}\frac{dx'}{\left(x'^2+z^2\right)^{3/2}}\,.$$

The integral needed (you can use a table of the Wolfram Integrator webpage) is

$$\int\frac{dt}{\left(t^2+a^2\right)^{3/2}} = \frac{t}{a^2\left(t^2+a^2\right)^{1/2}},$$

The total field is

$$E_z = \frac{\lambda z}{4\pi\varepsilon_0}\left[\frac{x'}{z^2\left(x'^2+z^2\right)^{1/2}}\right]_{-L}^{L} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda L}{z\sqrt{L^2+z^2}}.$$

In the limit that the line is long ($L >> z$),

$$E_z \approx \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}.$$

Infinite Sheet with Uniform Surface Charge Density (positive)

Let the sheet lie in the *xy* plane ($z = 0$). Divide it into small rectangles with dimensions dx' and dy'. Consider one piece at x' and y'. Find the electric field at a point along the *z* axis (it will only depend on the distance from the sheet by translational symmetry). The positions and separation are

$$\vec{r} = z\hat{z} \text{ (field point)},$$

$$\vec{r}' = x'\hat{x} + y'\hat{y} \text{ (source point)},$$

$$\vec{\boldsymbol{\mathscr{r}}} = \vec{r} - \vec{r}' = -x'\hat{x} - y'\hat{y} + z\hat{z}.$$

By rotational symmetry, the electric field can only point in the *z* direction, so we only need to find that component. The contribution from the one piece is

$$dE_z = \frac{1}{4\pi\varepsilon_0}\frac{q}{\mathscr{r}^3}\mathscr{r}_z = \frac{1}{4\pi\varepsilon_0}\frac{(\sigma\, dx'dy')}{\left(x'^2 + y'^2 + z^2\right)^{3/2}}\, z.$$

Integrate over x' and y' to find the total field:

$$E_z = \frac{\sigma z}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dx'dy'}{\left(x'^2 + y'^2 + z^2\right)^{3/2}}.$$

The first integral needed is

$$\int\frac{dt}{\left(t^2 + a^2\right)^{3/2}} = \frac{t}{a^2\left(t^2 + a^2\right)^{1/2}},$$

which gives (x' cancels with the square root at the limits)

$$E_z = \frac{\sigma z}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\left[\frac{x'}{\left(y'^2 + z^2\right)\left(x'^2 + y'^2 + z^2\right)^{1/2}}\right]_{-\infty}^{\infty} dy' = \frac{\sigma z}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{2}{\left(y'^2 + z^2\right)}dy'.$$

The second integral needed is

$$\int\frac{dt}{t^2 + a^2} = \frac{1}{|a|}\tan^{-1}\left(\frac{t}{a}\right),$$

so

$$E_z = \frac{\sigma z}{2\pi\varepsilon_0}\left[\frac{1}{|z|}\tan^{-1}\left(\frac{y'}{z}\right)\right]_{-\infty}^{\infty}.$$

Since

$$\tan^{-1}(\pm t)\xrightarrow{t\to\infty}\pm\pi/2,$$

the result (answer does <u>not</u> depend of the distance *z*!) is

$$E_z = \pm\frac{\sigma}{2\varepsilon_0} \text{ (+ above or z > 0, - below or z < 0).}$$

2nd – asked them what geometry they wanted us to do in ‘real time.’ Someone asked for a finite rectangular sheet of charge. So we followed the same basic process as used for the rod to set that up and work through. We took it as far as setting up the integrals and recognizing that, since integrating over a plane is the same as first integrating over a rod, and then summing over parallel rods, the first integral was the same as we’d seen in the powerpoint – so we could adapt and then adopt that result. We merely noted that the second integral could be simplified to the form Int(dx/sqrt(x^2 + 1)) = log(x + sqrt(x^2 + 1)).

They also asked about the differential areas used in polar and spherical coordinates.

Exercises:

Problem 2.5: Find the electric field a distance *z* above the center of a circular loop of radius *r* that carries a uniform charge of linear density λ.

Problem 2.4 Find the electric field a distance *z* above the center of a square loop (side a) carrying a uniform line charge λ.

Preview

For Wednesday, you’ll read about the divergence and curl of the electric field due to static charges.

Should start working on HW #1 – we will finish all of the material for that on Wednesday