Mon. Wed. Thurs Fri.	(C 16) 2.3.4-.3.5 Electric Potential & Boundary Conditions 2.4.1-.4.2 Work & Energy in Electrostatics T3 Contour Plots 2.4.3-4.4 Work & Energy in Electrostatics	 HW2
Mon. Wed. 	2.5 Conductors Summer Science Research Poster Session: Hedco7pm~9pm 	 HW3

Electric Potential & Boundary Conditions

Electric Potential (Difference)

$$\Delta V \equiv -\int_a^b \vec{E}(\vec{r}) \cdot d\vec{\ell}$$

$$V(P) \equiv -\int_{\mathscr{ref}}^{P} \vec{E} \cdot d\vec{\ell}$$

For a single point charge

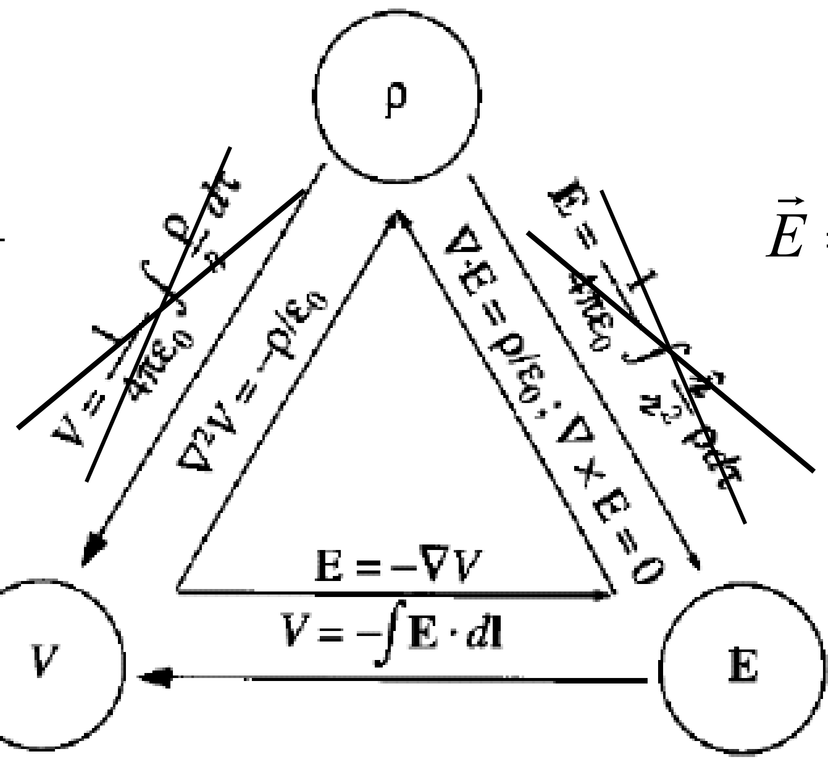
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{\mathscr{r}^2} \hat{\mathscr{r}}$$ So, $$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{\mathscr{r}}$$

For several point charges

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{\mathscr{r}_i^2} \hat{\mathscr{r}}_i$$ So, $$\boxed{V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{\mathscr{r}_i}}$$

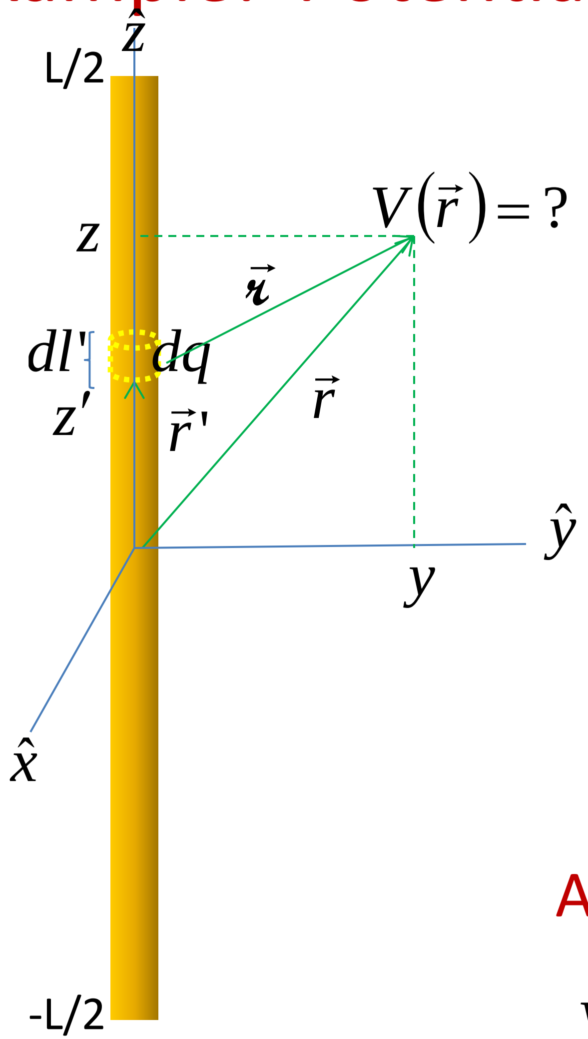
Electro-static Relations

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{\boldsymbol{\mathscr{r}}_i}$$



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{\boldsymbol{\mathscr{r}}_i^{\,2}}\,\hat{\boldsymbol{\mathscr{r}}}_i$$

Example: Potential due to a rod of uniform charge



$$dV(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\mathscr{r}}$$

where, $\mathscr{r} = \left(y^2 + (z - z')^2\right)^{\frac{1}{2}}$

$$\frac{dq}{dl'} = \frac{Q}{L} = \lambda$$

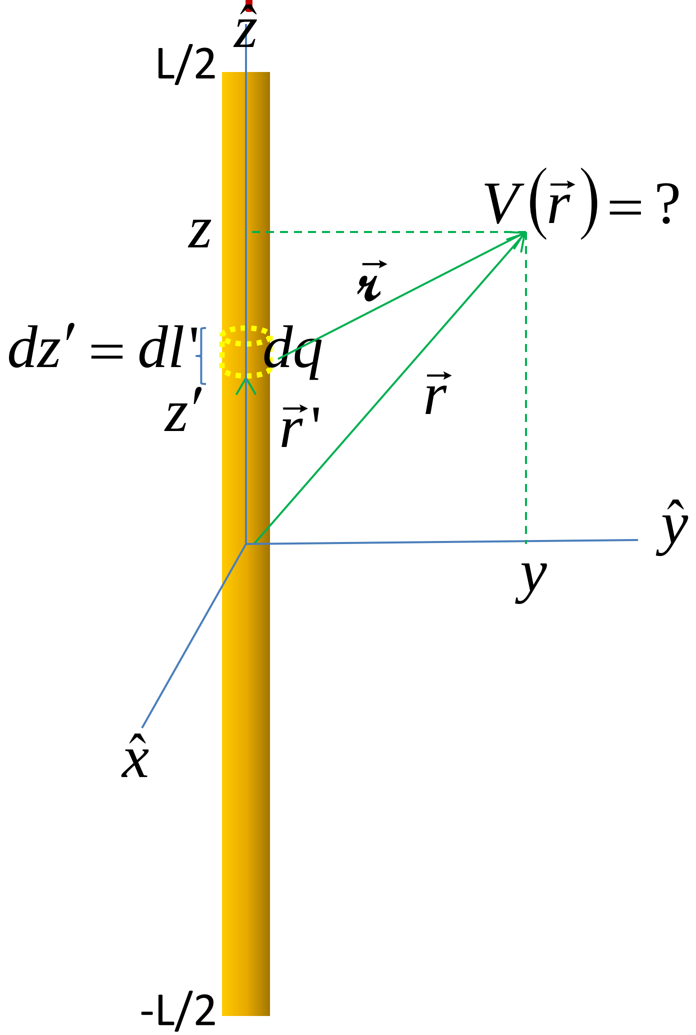
So,

$$dV(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dz'}{\left(y^2 + (z - z')^2\right)^{\frac{1}{2}}}$$

And *total* potential is

$$V(\vec{r}) = \int dV(\vec{r})$$

Example: Potential due to a rod of uniform charge



$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_{z'=-\frac{L}{2}}^{z'=\frac{L}{2}} \frac{\lambda dz'}{\left(y^2 + (z - z')^2\right)^{\frac{1}{2}}}$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_{z'=-\frac{L}{2}}^{z'=\frac{L}{2}} \frac{\lambda dz'}{y\left(1 + \left(\frac{z - z'}{y}\right)^2\right)^{\frac{1}{2}}}$$

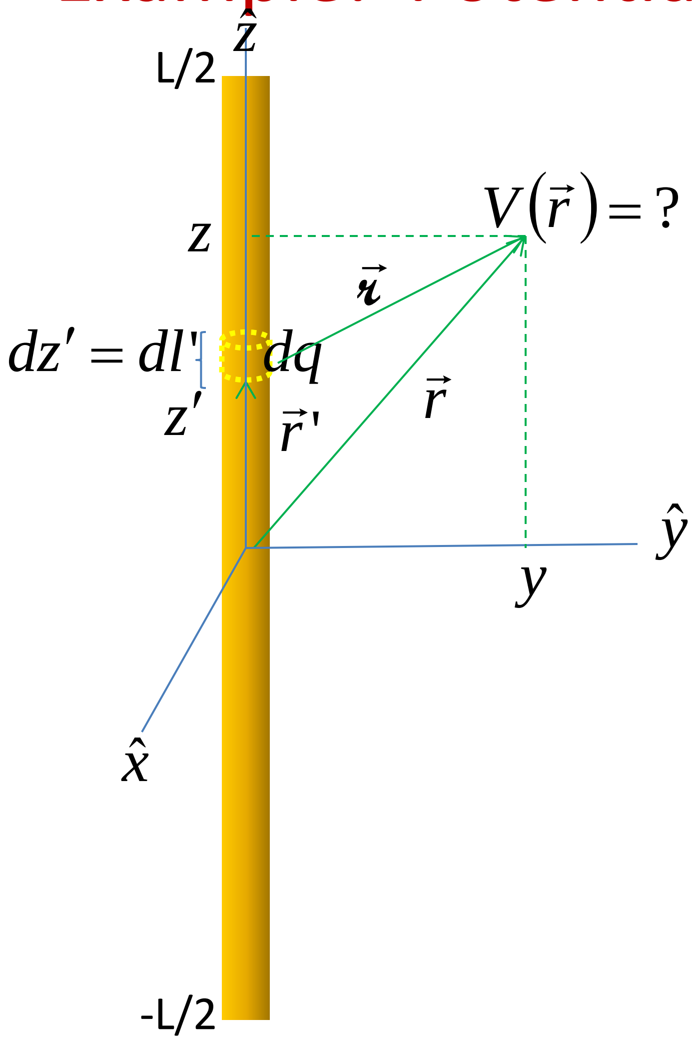
Change of Variables

$$\xi \equiv \left(\frac{z - z'}{y}\right)$$

$$\frac{d\xi}{dz'} = -\frac{1}{y} \Rightarrow dz' = -y d\xi$$

$$V(\vec{r}) = -\frac{\lambda}{4\pi\varepsilon_0} \int_{\xi=\left(\frac{z+\frac{L}{2}}{y}\right)}^{\xi=\left(\frac{z-\frac{L}{2}}{y}\right)} \frac{d\xi}{\left(1 + \xi^2\right)^{\frac{1}{2}}} = -\frac{\lambda}{4\pi\varepsilon_0} \ln\left(\xi + \left(1 + \xi^2\right)^{\frac{1}{2}}\right)\Bigg|_{\xi=\left(\frac{z+\frac{L}{2}}{y}\right)}^{\xi=\left(\frac{z-\frac{L}{2}}{y}\right)}$$

Example: Potential due to a rod of uniform charge



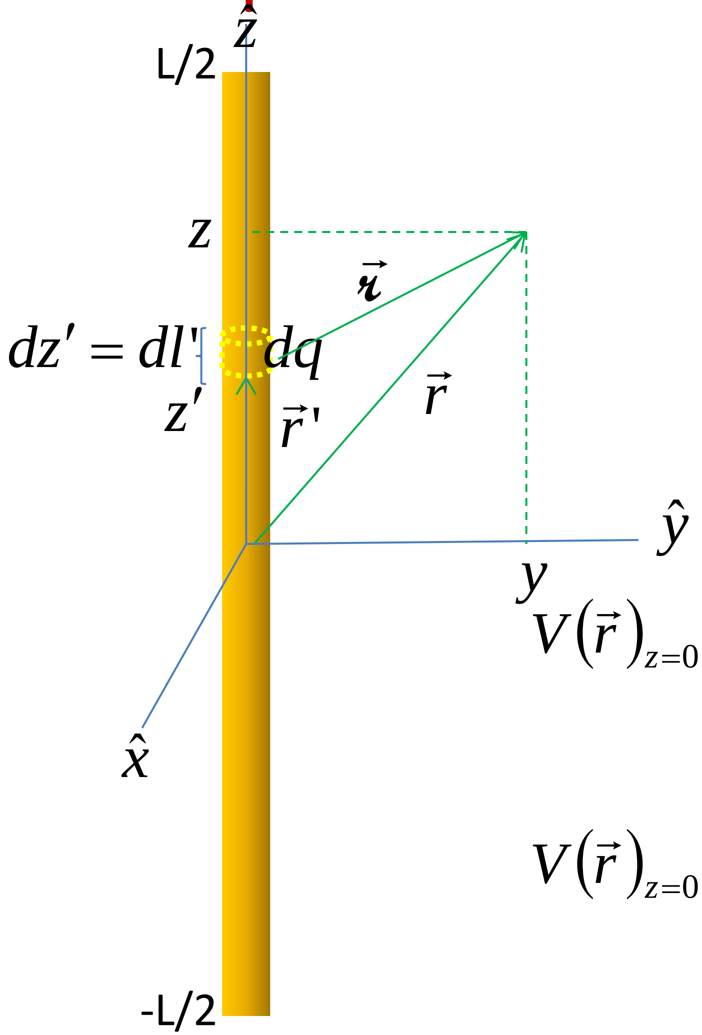
$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_{z'=-\frac{L}{2}}^{z'=\frac{L}{2}} \frac{\lambda\, dz'}{\left(y^2 + (z - z')^2\right)^{\frac{1}{2}}}$$

$$V(\vec{r}) = -\frac{\lambda}{4\pi\varepsilon_0} \ln\left(\xi + \left(1 + \xi^2\right)^{\frac{1}{2}}\right)\Bigg|_{\xi=\left(\frac{z+\frac{L}{2}}{y}\right)}^{\xi=\left(\frac{z-\frac{L}{2}}{y}\right)}$$

$$V(\vec{r}) = -\frac{\lambda}{4\pi\varepsilon_0} \left\{ \ln\left(\frac{z-\frac{L}{2}}{y} + \left(1 + \left(\frac{z-\frac{L}{2}}{y}\right)^2\right)^{\frac{1}{2}} \right) - \ln\left(\frac{z+\frac{L}{2}}{y} + \left(1 + \left(\left(\frac{z+\frac{L}{2}}{y}\right)\right)^2\right)^{\frac{1}{2}} \right) \right\}$$

$$V(\vec{r}) = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{z + \frac{L}{2} + \left(y^2 + \left(z + \frac{L}{2}\right)^2\right)^{\frac{1}{2}}}{z - \frac{L}{2} + \left(y^2 + \left(z - \frac{L}{2}\right)^2\right)^{\frac{1}{2}}} \right)$$

Example: Potential due to a rod of uniform charge



$$V(\vec{r}) = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{z + \frac{L}{2} + \left(y^2 + \left(z + \frac{L}{2}\right)^2\right)^{\frac{1}{2}}}{z - \frac{L}{2} + \left(y^2 + \left(z - \frac{L}{2}\right)^2\right)^{\frac{1}{2}}} \right)$$

Limits

Z = 0, y>>L

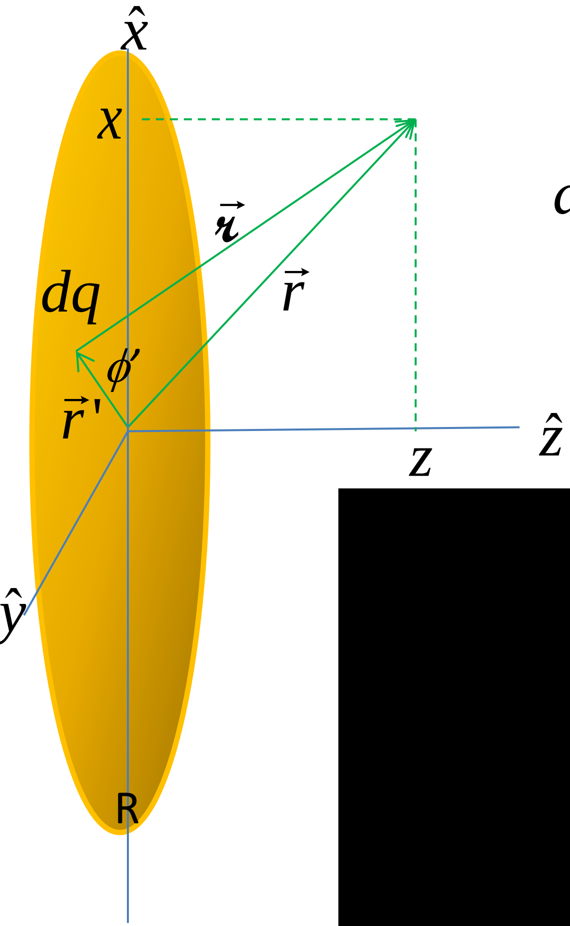
$$V(\vec{r})_{z=0} = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{+\frac{L}{2} + \left(y^2 + \left(\frac{L}{2}\right)^2\right)^{\frac{1}{2}}}{-\frac{L}{2} + \left(y^2 + \left(\frac{L}{2}\right)^2\right)^{\frac{1}{2}}} \right)$$

$$V(\vec{r})_{z=0} = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{1 + \frac{L}{2} \Big/ \left(y^2 + \left(\frac{L}{2}\right)^2\right)^{\frac{1}{2}}}{1 - \frac{L}{2} \Big/ \left(y^2 + \left(\frac{L}{2}\right)^2\right)^{\frac{1}{2}}} \right) \approx \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{1 + \frac{L}{2}/y}{1 - \frac{L}{2}/y} \right)$$

$$V(\vec{r})_{z=0} \approx \frac{\lambda}{4\pi\varepsilon_0} \left\{ \frac{L}{2y} + \frac{L}{2y} \right\}$$

$$V(\vec{r})_{z=0} \approx \frac{\lambda}{4\pi\varepsilon_0} \left\{ \frac{L}{y} \right\} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda L}{y} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\mathscr{r}}$$

Exercise: Potential due to a disc of uniform charge (just set up the integral)



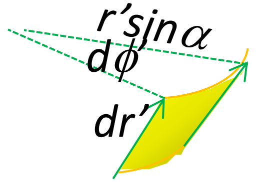
$$dV(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\mathscr{r}}$$

$$dq = \sigma da \qquad da = ?$$

$$\mathscr{r} = \left((x-x')^2 + (y-y')^2 + (z-0)^2\right)^{\frac{1}{2}}$$

$$x' = ? \qquad y' = ?$$

Example: Potential due to a cone of uniform charge (build integral)



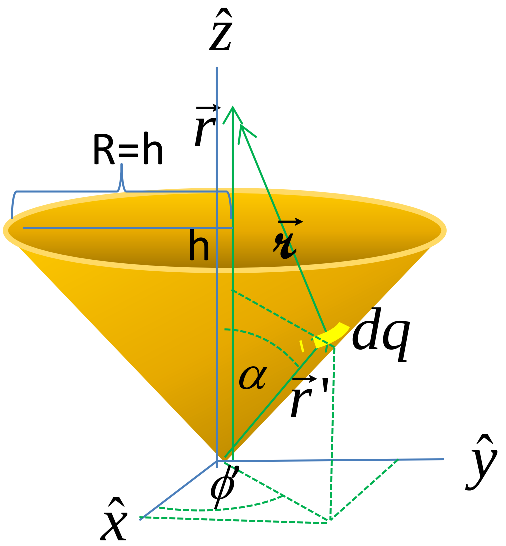
$$dV(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\mathscr{r}}$$

$$dq = \sigma da$$

$$da = dr' \cdot (r'\sin\alpha)d\varphi$$

$$\mathscr{r} = \left((x')^2 + (y')^2 + (z - z')^2\right)^{\frac{1}{2}}$$

$$x' = \ldots \qquad y' = \ldots \qquad z' = \ldots$$



Or:

$$\mathscr{r} = \left(\vec{\mathscr{r}} \cdot \vec{\mathscr{r}}\right)^{\frac{1}{2}} \qquad \textit{where} \qquad \vec{\mathscr{r}} = \vec{r} - \vec{r}\,'$$

so $\vec{\mathscr{r}} \cdot \vec{\mathscr{r}} = (\vec{r} - \vec{r}\,') \cdot (\vec{r} - \vec{r}\,') = r^2 + r'^2 - 2\vec{r} \cdot \vec{r}\,'$

$$\vec{\mathscr{r}} \cdot \vec{\mathscr{r}} = r^2 + r'^2 - 2rr'\cos\alpha$$

so $\mathscr{r} = \left(\vec{\mathscr{r}} \cdot \vec{\mathscr{r}}\right)^{\frac{1}{2}} = \left(z^2 + r'^2 - 2zr'\cos\alpha\right)^{\frac{1}{2}}$

Putting it together

$$dV(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\sigma da}{\mathscr{r}} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(\sin\alpha)r'dr'd\phi'}{\left(z^2 + r'^2 - 2zr'\cos\alpha\right)^{\frac{1}{2}}}$$

$$V(\vec{r}) = \frac{\sigma(\sin\alpha)}{4\pi\varepsilon_0}\int_{r'=0}^{r'_{max}}\int_{\phi'=0}^{2\pi}\frac{r'dr'd\phi'}{\left(z^2 + r'^2 - 2zr'\cos\alpha\right)^{\frac{1}{2}}}$$

Clean up: can express $\sin\alpha$, $\cos\alpha$, and r_{max} in terms of R and h

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Example: Potential due to a rod of uniform charge

Limits Z ,y>>L

$$V(\vec{r}) \approx \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{z+\frac{L}{2}+\left(y^2+z^2\left(1+2\frac{L}{2z}\right)\right)^{\frac{1}{2}}}{z-\frac{L}{2}+\left(y^2+z^2\left(1-2\frac{L}{2z}\right)\right)^{\frac{1}{2}}}\right) \approx \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{z+\frac{L}{2}+\left(y^2+z^2+Lz\right)^{\frac{1}{2}}}{z-\frac{L}{2}+\left(y^2+z^2-Lz\right)^{\frac{1}{2}}}\right)$$

$$\approx \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{z+\frac{L}{2}+\left(y^2+z^2\right)^{\frac{1}{2}}\left(1+\frac{Lz}{y^2+z^2}\right)^{\frac{1}{2}}}{z-\frac{L}{2}+\left(y^2+z^2\right)^{\frac{1}{2}}\left(1-\frac{Lz}{y^2+z^2}\right)^{\frac{1}{2}}}\right) \approx \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{z+\frac{L}{2}+\left(y^2+z^2\right)^{\frac{1}{2}}\left(1+\frac{1}{2}\frac{Lz}{y^2+z^2}\right)}{z-\frac{L}{2}+\left(y^2+z^2\right)^{\frac{1}{2}}\left(1-\frac{1}{2}\frac{Lz}{y^2+z^2}\right)}\right)$$

$$\approx \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{z+\frac{L}{2}+\left(y^2+z^2\right)^{\frac{1}{2}}+\frac{1}{2}\frac{Lz}{\left(y^2+z^2\right)^{\frac{1}{2}}}}{z-\frac{L}{2}+\left(y^2+z^2\right)^{\frac{1}{2}}-\frac{1}{2}\frac{Lz}{\left(y^2+z^2\right)^{\frac{1}{2}}}}\right) \approx \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{z+\left(y^2+z^2\right)^{\frac{1}{2}}+\frac{L}{2}\left(1+\frac{z}{\left(y^2+z^2\right)^{\frac{1}{2}}}\right)}{z+\left(y^2+z^2\right)^{\frac{1}{2}}-\frac{L}{2}\left(1+\frac{z}{\left(y^2+z^2\right)^{\frac{1}{2}}}\right)}\right)$$

$$\approx \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{1+\frac{L}{2\left(z+\left(y^2+z^2\right)^{\frac{1}{2}}\right)}\left(1+\frac{z}{\left(y^2+z^2\right)^{\frac{1}{2}}}\right)}{1-\frac{L}{2\left(z+\left(y^2+z^2\right)^{\frac{1}{2}}\right)}\left(1+\frac{z}{\left(y^2+z^2\right)^{\frac{1}{2}}}\right)}\right) \approx \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{L}{2\left(z+\left(y^2+z^2\right)^{\frac{1}{2}}\right)}\left(1+\frac{z}{\left(y^2+z^2\right)^{\frac{1}{2}}}\right)+\frac{L}{2\left(z+\left(y^2+z^2\right)^{\frac{1}{2}}\right)}\left(1+\frac{z}{\left(y^2+z^2\right)^{\frac{1}{2}}}\right)\right)$$

$$\approx \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{L}{\left(z+\left(y^2+z^2\right)^{\frac{1}{2}}\right)}\left(1+\frac{z}{\left(y^2+z^2\right)^{\frac{1}{2}}}\right)\right) = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{L}{\left(z+\left(y^2+z^2\right)^{\frac{1}{2}}\right)}\left(\frac{\left(y^2+z^2\right)^{\frac{1}{2}}+z}{\left(y^2+z^2\right)^{\frac{1}{2}}}\right)\right) \approx \frac{1}{4\pi\varepsilon_0}\left(\frac{\lambda L}{\left(y^2+z^2\right)^{\frac{1}{2}}}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r}\right)$$