Wed.	(C 21.1-.5,.8) 2.2.3 Using Gauss, (T2 Numerical Quadrature, 1.3 Integral Calc)	
Thurs		HW1
Fri.	(C21.1-.5,.8) 2.2.3-.2.4 Using Gauss	

Using Gauss's Law

Last Time

Not just $$\vec{E}_{net} = \int_{charge} \frac{1}{4\pi\varepsilon_o} \frac{\hat{\boldsymbol{\mathscr{r}}}}{\boldsymbol{\mathscr{r}}^2} dq$$

But also $$\oint_S \vec{E} \cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{\text{enc}}$$

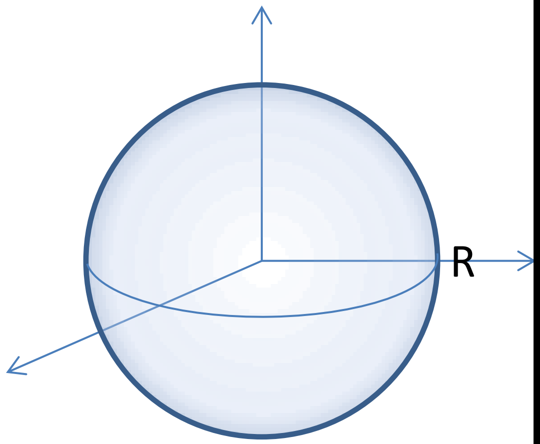
Or $$\lim_{Vol \to 0} \frac{\oint_S \vec{E} \cdot d\vec{a}}{Vol} = \lim_{Vol \to 0} \frac{1}{\varepsilon_0} \frac{Q_{\text{enc}}}{Vol}$$

$$Div\vec{E} = \frac{1}{\varepsilon_0} \rho$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_0} \rho$$

Example Hollow, spherical shell or radius R with Q uniformly distributed over the surface.

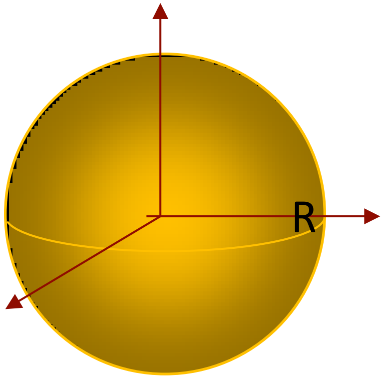
Field inside and out.



[redacted]

Example Solid sphere of radius R with varying charge density: $\rho(r') = \rho_o \frac{r'^3}{R^3}$

Field inside and out.



[redacted]

Exercise

Uniformly-Charged Rod

A thin rod of length *L* has a positive charge *Q* distributed uniformly along its length.

1. **Field Geometry**. Use a *symmetry* argument to determine the *direction* of the electric field near the center of the rod.
2. **Choosing the Gaussian bubble.** What shape of Gaussian surface can you draw so that each part is either perpendicular or parallel to the electric field?
3. **Doing the Math.** Use Gauss's law to find the magnitude of the electric field at a radial distance *r* from the rod near its center.

Exercise

Solid Uniformly-Charged Sphere (inside and out)

A solid sphere of radius *R* has a positive charge *Q* distributed uniformly throughout its volume.

1. **Field Geometry.** Use a symmetry argument to determine the direction of the electric field.
2. **Choosing a Gaussian Bubble.** What shape of Gaussian surface can you draw so that each part is either perpendicular or parallel to the electric field?
3. **Doing the Math.** Use Gauss's law to find the magnitude of the electric field at a distance *r* from the center of the sphere for:

[redacted]

Curl of E

In Spherical Coordinates: $\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{\mathscr{r}^2}\hat{\mathscr{r}} = E_r\hat{\mathscr{r}}$

$$\vec{\nabla}\times\vec{E} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\,\cancel{E_\phi}\right) - \frac{\partial}{\partial\phi}\cancel{E_\theta}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\left(E_r\right) - \frac{\partial}{\partial r}\left(r\cancel{E_\phi}\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\cancel{E_\theta}\right) - \frac{\partial}{\partial\theta}\left(rE_r\right)\right]\hat{\phi}$$

$$\vec{\nabla}\times\vec{E} = \frac{1}{r}\left[\frac{1}{\sin\theta}\cancel{\frac{\partial}{\partial\phi}}\left(E_r\right)\right]\hat{\theta} - \frac{1}{r}\left[\cancel{\frac{\partial}{\partial\theta}}\left(rE_r\right)\right]\hat{\phi} = 0$$

A little *too* easy

In Cartesian Coordinates:

$$\vec{E}_1(\vec{r}) = \frac{q}{4\pi\varepsilon_0}\frac{\vec{\mathscr{r}}}{\mathscr{r}^3} = \frac{q}{4\pi\varepsilon_0}\frac{\left[(x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}\right]}{\left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{3/2}}$$

$$\vec{\nabla}\times\vec{E}_1 = \left(\frac{q}{4\pi\varepsilon_0}\right)\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \dfrac{(x-x')}{\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{3/2}} & \dfrac{(y-y')}{\left[\;\right]^{3/2}} & \dfrac{(z-z')}{\left[\;\right]^{3/2}} \end{vmatrix}$$

$$\frac{\partial}{\partial y}\left\{\frac{(z-z')}{[\;]^{3/2}}\right\} - \frac{\partial}{\partial z}\left\{\frac{(y-y')}{[\;]^{3/2}}\right\} = (z-z')\frac{\left(-\frac{3}{2}\right)2(y-y')}{[\;]^{5/2}} - (y-y')\frac{\left(-\frac{3}{2}\right)2(z-z')}{[\;]^{5/2}} = 0$$

Etc.