Fri., 11/15	**10.2** Continuous Distributions	
Mon., 11/18 Wed.,11/20 Fri., 11/22	**10.3** Point Charges *(C 14)* **4.1** Polarization *(C 14)* **4.2** Field of Polarized Object	HW9

Last Time.

We replace Maxwell's 4 laws with 4 other relations:

$$\vec{\nabla}\times\vec{A}\equiv\vec{B} \qquad\qquad \vec{E}=-\vec{\nabla}V-\frac{\partial\vec{A}}{\partial t}$$

$$\Box^2\vec{A}_L=-\mu_0\vec{J} \qquad \Box^2 V_L=-\frac{\rho}{\varepsilon_0} \quad \text{where } \Box^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

Where we'd made the Gauge choice

$$\vec{\nabla}\cdot\vec{A}_L=-\mu_0\varepsilon_0\frac{\partial V_L}{\partial t}$$

In order to simplify these last two.

We'd then accepted Griffith's proposed Solutions

$$V_L(\vec{r},t)=\frac{1}{4\pi\varepsilon_o}\int\frac{\rho(\vec{r}',t_r)}{\mathscr{r}}d\tau' \qquad \vec{A}_L(\vec{r},t)=\frac{\mu_o}{4\pi}\int\frac{\vec{J}(\vec{r}',t_r)}{\mathscr{r}}d\tau'$$

Where

$$t_r\equiv t-\frac{\mathscr{r}}{c}$$

But set about proving that they were solutions.

We'd gotten thus far:

$$\nabla^2V_L=\vec{\nabla}\cdot\left(\nabla\frac{1}{4\pi\varepsilon_o}\left(\int\frac{\rho(\vec{r}',t_r)}{\mathscr{r}}d\tau'\right)\right)=\nabla\cdot\frac{1}{4\pi\varepsilon_o}\left(\int-\frac{\dot\rho(\vec{r}',t_r)\hat{\mathscr{r}}}{c\mathscr{r}}-\frac{\rho(\vec{r}',t_r)\hat{\mathscr{r}}}{\mathscr{r}^2}d\tau'\right)$$

So now we need to take the divergence of the two terms.

$$\frac{1}{4\pi\varepsilon_o}\left(\int-\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}}\frac{\nabla\dot\rho(\vec{r}',t_r)}{c}+\frac{\dot\rho(\vec{r}',t_r)}{c}\nabla\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}}\right)\right)-\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\nabla\rho(\vec{r}',t_r)+\rho(\vec{r}',t_r)\nabla\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)\right)d\tau'\right)$$

Now, just as we'd found that

$$\nabla\left[\rho(\vec{r}',t_r)\right]=-\frac{\dot\rho(\vec{r}',t_r)}{c}\hat{\mathscr{r}}, \qquad \nabla\left[\dot\rho(\vec{r}',t_r)\right]=-\frac{\ddot\rho(\vec{r}',t_r)}{c}\hat{\mathscr{r}}$$

And, from Ch 1, $\nabla\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}}\right)=\frac{1}{\mathscr{r}^2}$ and $\nabla\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)=4\pi\delta^3(\vec{\mathscr{r}})$

So, the integral simplifies to

$$\frac{1}{4\pi\varepsilon_o}\left(\int -\left(-\frac{\ddot{\rho}(\vec{r}',t_r)}{\mathscr{r}c^2}+\frac{\dot{\rho}(\vec{r}',t_r)}{c\mathscr{r}^2}\right)-\left(-\frac{\dot{\rho}(\vec{r}',t_r)}{\mathscr{r}^2 c}+\rho(\vec{r}',t_r)4\pi\delta^3(\vec{\mathscr{r}})\right)d\tau'\right)$$

$$\frac{1}{4\pi\varepsilon_o}\int\frac{\ddot{\rho}(\vec{r}',t_r)}{\mathscr{r}c^2}d\tau'-\frac{\rho(r,t)}{\varepsilon_o}$$

(the delta function killed the integral everywhere but *r'=r*, of course there $t_r = t$.)

So,

$$\nabla^2 V_L = \frac{1}{4\pi\varepsilon_o}\int\frac{\ddot{\rho}(\vec{r}',t_r)}{\mathscr{r}c^2}d\tau'-\frac{\rho(r,t)}{\varepsilon_o}$$

Pulling the time derivative outside the integral, we have

$$\nabla^2 V_L = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\frac{1}{4\pi\varepsilon_o}\int\frac{\rho(\vec{r}',t_r)}{\mathscr{r}}d\tau'-\frac{\rho(r,t)}{\varepsilon_o}$$

$$\nabla^2 V_L = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}V_L-\frac{\rho(r,t)}{\varepsilon_o}$$

$$\Box^2 V_L = -\frac{\rho}{\varepsilon_0}$$

Tada.

Similarly, the same thing can be done for each component of a A, proving that both

$$V_L(\vec{r},t)=\frac{1}{4\pi\varepsilon_o}\int\frac{\rho(\vec{r}',t_r)}{\mathscr{r}}d\tau' \qquad \vec{A}_L(\vec{r},t)=\frac{\mu_o}{4\pi}\int\frac{\vec{J}(\vec{r}',t_r)}{\mathscr{r}}d\tau'$$

Are correct Solutions.

Examples

Now that we've got them, let's use them.

Pr. 10.9 a) Suppose an infinite straight wire carries a linearly increasing current $I(t)=kt$ for t>0. Find the electric and magnetic fields generated.

We'll only go yeahy far on this one, because there's a valuable point to make in the set-up of the problem.

It's worth emphasizing that this current is piecewise defined

$$I(t)=\begin{cases}0 & for \quad t<0\\ kt & for \quad t>0\end{cases}$$

While that may just look like a *time* issue, we'll see that it's also a *position* issue.

Now, $\vec{\nabla} \times \vec{A} \equiv \vec{B}$ and $\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$

Where

$$V_L(\vec{r},t) = \frac{1}{4\pi\varepsilon_o} \int \frac{\rho(\vec{r}\,',t_r)}{\mathscr{r}} d\tau' = 0, \quad \vec{A}_L(\vec{r},t) = \frac{\mu_o}{4\pi} \int \frac{\vec{J}(\vec{r}\,',t_r)}{\mathscr{r}} d\tau' = \frac{\mu_o}{4\pi} \int \frac{I(\vec{r}\,',t_r)}{\mathscr{r}} dz'\hat{z}$$

We can say

$$\vec{A}_L(\vec{r},t) = \frac{\mu_o}{4\pi} \int_{-\infty}^{\infty} \frac{I(\vec{r}\,',t_r)}{\mathscr{r}} dz'\hat{z}$$

But that's a little deceptive since, our observer won't see distant segments of the current turned on yet. We can see that a little better if we note

$$I(t_r) = \begin{cases} 0 & for \quad t_r < 0 \\ kt_r & for \quad t_r > 0 \end{cases}$$

Where $t_r = t - \frac{c}{\mathscr{r}}$ rephrasing this back in terms of plain old *t*,

$$kt_r = k\left(t - \frac{\mathscr{r}}{c}\right)$$

and the condition for turning this on is

$$t - \frac{\mathscr{r}}{c} > 0$$

$$t > \frac{\mathscr{r}}{c}$$

$$tc > \mathscr{r}$$

Of course, $\mathscr{r} = \sqrt{s^2 + z'^2}$ so, putting this threshold in terms of the integration variable,

$$tc > \sqrt{s^2 + z'^2}$$

$$(tc)^2 - s^2 > z'^2$$

$$z'^2 > \left(\frac{c}{t}\right)^2 - s^2$$

$$\sqrt{(tc)^2 - s^2} > |z'|$$

So at a given time *t*, we'll see current only in the segment of wire this far down the line (either way). Now, we can rephrase our piece-wise defined current in terms of our variable of integration

$$
I(t,z') = \begin{cases} 0 \quad for \quad t < \dfrac{c}{s}, \quad |z'| < \sqrt{(ct)^2 - s^2} \\ k\left(t - \dfrac{\sqrt{s^2 + z'^2}}{c}\right) \quad for \quad t > \dfrac{c}{s}, \quad |z'| > \sqrt{(ct)^2 - s^2} \end{cases}
$$

The time threshold is simply a) saying that before a certain time, not even a message from the nearest point will reach the observer and b) we don't want imaginary terms in the spatial limits!

Now we've done all the new-stuff thinking, from here out, it's old hat.

$$
\vec{A}_L(\vec{r},t) = \frac{\mu_o}{4\pi}\int_{-\infty}^{\infty}\frac{I(\vec{r}\,',t_r)}{\mathscr{r}}dz'\hat{z} = \frac{\mu_o}{4\pi}\int_{z'_{\min}=-\sqrt{(ct)^2-s^2}}^{z'_{\max}=+\sqrt{(ct)^2-s^2}}\frac{k\left(t-\dfrac{\sqrt{s^2+z'^2}}{c}\right)}{\sqrt{s^2+z'^2}}dz'\hat{z}
$$

$$
\vec{A}_L(\vec{r},t) = \frac{\mu_o k}{4\pi}\left(t\int_{z'_{\min}}^{z'_{\max}}\frac{1}{\sqrt{s^2+z'^2}}dz' - \frac{1}{c}\int_{z'_{\min}}^{z'_{\max}}dz'\right)\hat{z} = \frac{\mu_o k}{4\pi}\left(t\ln\left(\sqrt{s^2+z'^2}+z'\right) - \frac{z'}{c}\right)\Bigg|_{z'_{\min}}^{z'_{\max}}\hat{z}
$$

$$
\vec{A}_L(\vec{r},t) = \frac{\mu_o k}{4\pi}\left(t\ln\left(\frac{\sqrt{s^2+z'^{2}_{\max}}+z'_{\max}}{\sqrt{s^2+z'^{2}_{\min}}+z'_{\min}}\right) - \frac{z'_{\max}-z'_{\min}}{c}\right)\hat{z}
$$

$$
\vec{A}_L(\vec{r},t) = \frac{\mu_o k}{4\pi}\left(t\ln\left(\frac{\sqrt{s^2+\sqrt{(ct)^2-s^2}^{\,2}}+\sqrt{(ct)^2-s^2}}{\sqrt{s^2+\sqrt{(ct)^2-s^2}^{\,2}}-\sqrt{(ct)^2-s^2}}\right) - \frac{\sqrt{(ct)^2-s^2}--\sqrt{(ct)^2-s^2}}{c}\right)\hat{z}
$$

$$
\vec{A}_L(\vec{r},t) = \frac{\mu_o k}{4\pi}\left(t\ln\left(\frac{tc+\sqrt{(ct)^2-s^2}}{tc-\sqrt{(ct)^2-s^2}}\right) - \frac{2\sqrt{(ct)^2-s^2}}{c}\right)\hat{z}
$$

$$
\vec{A}_L(\vec{r},t) = \frac{\mu_o k}{4\pi}t\left(\ln\left(\frac{1+\sqrt{1-\left(\dfrac{s}{tc}\right)^2}}{1-\sqrt{1-\left(\dfrac{s}{tc}\right)^2}}\right) - 2\sqrt{1-\left(\dfrac{s}{tc}\right)^2}\right)\hat{z}
$$

$$
\vec{B} = \vec{\nabla}\times\vec{A}
$$

Since this expression only has an s-dependent z-component, the curl is simply

$$\vec{B} = \vec{\nabla} \times \vec{A} = -\frac{\partial A_z}{\partial s}\hat{\phi} = -\frac{\mu_o k}{4\pi} t\left(\left(\frac{1}{1+\sqrt{1-\left(\frac{s}{tc}\right)^2}} + \frac{1}{1-\sqrt{1-\left(\frac{s}{tc}\right)^2}}\right)\frac{-2s}{2(tc)^2\sqrt{1-\left(\frac{s}{tc}\right)^2}} - 2\frac{-2s}{2(tc)^2\sqrt{1-\left(\frac{s}{tc}\right)^2}}\right)\hat{\phi}$$

$$\vec{B} = \frac{\mu_o k}{2\pi c}\left(\sqrt{\left(\frac{ct}{s}\right)^2 - 1}\right)\hat{\phi}$$

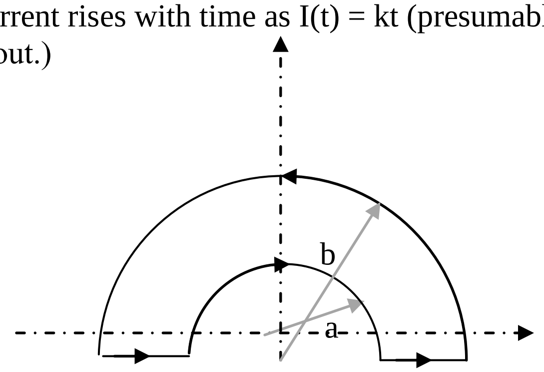
As for the electric field,

$\vec{E} = -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t}$ but with no V, that's just

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} = -\frac{\mu_o k}{4\pi}\left(\ln\left(\frac{1+\sqrt{1-\left(\frac{s}{tc}\right)^2}}{1-\sqrt{1-\left(\frac{s}{tc}\right)^2}}\right) + 2\sqrt{1-\left(\frac{s}{tc}\right)^2}\right)\hat{z}$$

10.10 (assigned)

A current loop made of two concentric arcs. The current rises with time as I(t) = kt (presumably just since t=0, but we'll assume we're long enough out.)



Griffiths asks why you can't find B from this expression for A. Because we've specialized it too much for being at the origin, so we can't see the true dependence on the observation location, thus we can't take the curl.

10.2.2 Jefimenko's Equations

So, we've developed our expressions for V_L and A_L, they have suggestively causal forms; however, it's E and B that are physically significant, so the real question is what do *they* look like. In fact, Griffith's had noted that V_{coul} does *not* depend upon retarded time values, so we really can't put too much stock in the mathematical forms of V and A.

Now,

$$\vec{E} = -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t} \text{ and } \vec{B} = \vec{\nabla}\times\vec{A}$$

where

$$V_L(\vec{r},t) = \frac{1}{4\pi\varepsilon_o}\int\frac{\rho(\vec{r}\,',t_r)}{\mathscr{r}}d\tau' \qquad \vec{A}_L(\vec{r},t) = \frac{\mu_o}{4\pi}\int\frac{\vec{J}(\vec{r}\,',t_r)}{\mathscr{r}}d\tau'$$

In the process of proving that this expression for V satisfies the d'Alembertian, we already took the gradient of V, in fact, that's right where we picked up today.

$$\nabla V_L = \nabla\frac{1}{4\pi\varepsilon_o}\left(\int\frac{\rho(\vec{r}\,',t_r)}{\mathscr{r}}d\tau'\right) = -\frac{1}{4\pi\varepsilon_o}\left(\int\frac{\dot{\rho}(\vec{r}\,',t_r)\hat{\mathscr{r}}}{c\mathscr{r}} + \frac{\rho(\vec{r}\,',t_r)\hat{\mathscr{r}}}{\mathscr{r}^2}d\tau'\right)$$

Meanwhile, $\frac{\partial\vec{A}_L}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\mu_o}{4\pi}\int\frac{\vec{J}(\vec{r}\,',t_r)}{\mathscr{r}}d\tau'\right) = \frac{\mu_o}{4\pi}\int\frac{\dot{\vec{J}}(\vec{r}\,',t_r)}{\mathscr{r}}d\tau' = \frac{1}{4\pi\varepsilon_o}\int\frac{\dot{\vec{J}}(\vec{r}\,',t_r)}{c^2\mathscr{r}}d\tau'$

The last little step used $\frac{1}{\varepsilon_o\mu_o} = c^2$ just to get the same factor out front.

Putting these two together gives

$$\vec{E} = -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t} = \frac{1}{4\pi\varepsilon_o}\int\left(\frac{\dot{\rho}(\vec{r}\,',t_r)\hat{\mathscr{r}}}{c\mathscr{r}} + \frac{\rho(\vec{r}\,',t_r)\hat{\mathscr{r}}}{\mathscr{r}^2} - \frac{\dot{\vec{J}}(\vec{r}\,',t_r)}{c^2\mathscr{r}}\right)d\tau'$$

Note that the dependence on retarded times persists. This has a very causal form. As far as we know, this is it, this is what makes an electric field, i.e., what pushes/pulls stationary charges – stationary charges, time varying charge densities, and time-varying current densities.

Now for the Magnetic Field.

$$\vec{B} = \vec{\nabla}\times\vec{A} = \vec{\nabla}\times\frac{\mu_o}{4\pi}\int\frac{\vec{J}(\vec{r}\,',t_r)}{\mathscr{r}}d\tau' = \frac{\mu_o}{4\pi}\int\vec{\nabla}\times\left(\frac{\vec{J}(\vec{r}\,',t_r)}{\mathscr{r}}\right)d\tau'$$

Vector Identity 7 tells us how to take the cross product of a vector times a scalar

$$\frac{\mu_o}{4\pi}\int\left(\frac{\vec{\nabla}\times\vec{J}(\vec{r}\,',t_r)}{\mathscr{r}} - \vec{J}(\vec{r}\,',t_r)\times\vec{\nabla}\left(\frac{1}{\mathscr{r}}\right)\right)d\tau' = \frac{\mu_o}{4\pi}\int\left(\frac{\vec{\nabla}\times\vec{J}(\vec{r}\,',t_r)}{\mathscr{r}} - \vec{J}(\vec{r}\,',t_r)\times\left(-\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)\right)d\tau'$$

As for that curl of J with respect to *r* (which J depends upon only through t_r), Griffiths suggests that we take it one component at a time.

$$\vec{\nabla}\times\vec{J}(\vec{r}\,',t_r) = \left(\frac{\partial J_z(\vec{r}\,',t_r)}{\partial y} - \frac{\partial J_y(\vec{r}\,',t_r)}{\partial z}\right)\hat{x} + \left(\frac{\partial J_x(\vec{r}\,',t_r)}{\partial z} - \frac{\partial J_z(\vec{r}\,',t_r)}{\partial x}\right)\hat{y} + \left(\frac{\partial J_x(\vec{r}\,',t_r)}{\partial y} - \frac{\partial J_y(\vec{r}\,',t_r)}{\partial x}\right)\hat{z}$$

But each of these derivatives really needs to be chain-ruled through; for example,

$$\frac{\partial J_z(\vec{r}',t_r)}{\partial y}=\frac{\partial J_z(\vec{r}',t_r)}{\partial t_r}\frac{\partial t_r}{\partial \mathscr{r}}\frac{\partial \mathscr{r}}{\partial y}=\dot{J}_z(\vec{r}',t_r)\left(\frac{-1}{c}\right)\frac{y}{\mathscr{r}}$$

Similarly for all the terms

$$\vec{\nabla}\times\vec{J}(\vec{r}',t_r)=-\frac{1}{c\mathscr{r}}\left[\left(\dot{J}_z(\vec{r}',t_r)y-\dot{J}_y(\vec{r}',t_r)z\right)\hat{x}+\left(\dot{J}_x(\vec{r}',t_r)z-\dot{J}_z(\vec{r}',t_r)x\right)\hat{y}+\left(\dot{J}_x(\vec{r}',t_r)y-\dot{J}_y(\vec{r}',t_r)x\right)\hat{z}\right]=\frac{\dot{\vec{J}}(\vec{r}',t_r)\times\vec{\mathscr{r}}}{c\mathscr{r}}$$

$$\vec{\nabla}\times\vec{J}(\vec{r}',t_r)=\frac{\dot{\vec{J}}(\vec{r}',t_r)\times\hat{\mathscr{r}}}{c}$$

(drop the negative sign because all terms are backwards from what they would be for J×r, so it's actually r×J=-J×r.)

$$\vec{B}=\vec{\nabla}\times\vec{A}=\frac{\mu_o}{4\pi}\int\left(\frac{\dot{\vec{J}}(\vec{r}',t_r)\times\hat{\mathscr{r}}}{c\mathscr{r}}+\frac{\vec{J}(\vec{r}',t_r)\times\hat{\mathscr{r}}}{\mathscr{r}^2}\right)d\tau'$$

So,

$$\vec{B}(\vec{r},t)=\frac{\mu_o}{4\pi}\int\left(\frac{\dot{\vec{J}}(\vec{r}',t_r)}{c\mathscr{r}}+\frac{\vec{J}(\vec{r}',t_r)}{\mathscr{r}^2}\right)\times\hat{\mathscr{r}}\,d\tau'$$

There we have it, the magnetic field. Again, this is appealingly causal – the field in here and now in terms of the sources there and then. As you can see, what causes a magnetic field, i.e., that which exerts a force on a moving charge is a current and a changing current density.

When arguing for the Gauge freedoms of the potentials I noted that it's the fields that we can detect, not the potentials, and, at the end of the day, you'd get the same fields regardless of what gauge you used for your calculations – it's just a matter of making a choice that makes the work easy for you. Presumably, had we chosen to work in the Coulomb gauge we'd have arrived at these same field expressions.

Now, what we *really* detect is the influence of the fields on charges, so maybe, just for a bit of closure, it's worth writing out the Lorentz Force Law directly in terms of the sources.

$$\vec{F}(\vec{r},t)=q\vec{E}+q\vec{v}\times\vec{B}$$

$$\vec{F}(\vec{r},t)=\frac{1}{4\pi\varepsilon_o}\int q\left(\frac{\dot{\rho}(\vec{r}',t_r)\hat{\mathscr{r}}}{c\mathscr{r}}+\frac{\rho(\vec{r}',t_r)\hat{\mathscr{r}}}{\mathscr{r}^2}-\frac{\dot{\vec{J}}(\vec{r}',t_r)}{c^2\mathscr{r}}+\vec{v}(\vec{r},t)\times\left(\left(\frac{\dot{\vec{J}}(\vec{r}',t_r)}{c^3\mathscr{r}}+\frac{\vec{J}(\vec{r}',t_r)}{c^2\mathscr{r}^2}\right)\times\hat{\mathscr{r}}\right)\right)d\tau'$$

That the electro-magnetic force exerted on a point charge moving with a particular velocity.

Advanced (i.e., opposite of retarded) Solutions Maybe here is a good place to introduce, and summarily dismiss, an alternative solution. Back when Griffith's demonstrated that the retarded potentials solved the d'Alembertian equations, he noted that *advanced* potentials *also* solved them. From those advanced potentials would follow advanced fields and would follow a force expression like the one above, but in which the sources are considered at time a time $t_a \equiv t + \frac{\boldsymbol{\imath}}{c}$. That's right, Maxwell's Laws would allow that charge q would feel a push due to not what the sources *have* done but what they're *yet to do.* Of course this strikes us right in the funny bone as completely absurd – it doesn't conform with the fundamental notion of causality. So we dismiss it as unphysical.

It's worth pausing and emphasizing – obscurely contained with Maxwell's four Laws is the condition that there is a time offset between what the fields are doing and what the sources are doing; *however,* Maxwell's Equations alone don't enforce causality. That is a separate 5^{th} law that we impose.

Some other fun things we can do with these expressions are see what *really* makes B and E change with time, what *really* makes B and E curl. You'll do that in the homework.

Something else worth exploring is backing out the conditions under which our old Coulomb's Law and Biot-Savart Law hold.

Pr. 10.11

If we have a constant current density, then what does the general electric field expression reduced to?

$$\vec{J}(\vec{r}',t) = \vec{J}(\vec{r}',0)$$
$$\dot{\vec{J}}(\vec{r}',t) = 0$$

So, in

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\int\left(\frac{\dot{\rho}(\vec{r}',t_r)\hat{\boldsymbol{\imath}}}{c\boldsymbol{\imath}} + \frac{\rho(\vec{r}',t_r)\hat{\boldsymbol{\imath}}}{\boldsymbol{\imath}^2} - \frac{\dot{\vec{J}}(\vec{r}',t_r)}{c^2\boldsymbol{\imath}}\right)d\tau'$$

clearly, the third term vanishes. However, what does "constant current" imply about the other two terms? Recall the continuity equation links current and charge density.

$$-\vec{\nabla}\cdot\vec{J}(\vec{r}',t) = \dot{\rho}(\vec{r}',t)$$

But if the current density is time-independent, that tells us that the charge density varies at a constant rate

$$-\vec{\nabla}\cdot\vec{J}(\vec{r}',t) = -\vec{\nabla}\cdot\vec{J}(\vec{r}',0)$$
$$\dot{\rho}(\vec{r}',t) = \dot{\rho}(\vec{r}',0)$$

So,

$$\rho(\vec{r}',t) = \rho(\vec{r}',0) + \dot{\rho}(\vec{r}',0)t$$

Similarly,

$$\rho(\vec{r}',t_r) = \rho(\vec{r}',0) + \dot{\rho}(\vec{r}',0)t_r\,, \qquad \dot{\rho}(\vec{r}',t_r) = \dot{\rho}(\vec{r}',0)$$

Plugging these things into the electric-field expression gives

$$\vec{E}(r,t)=\frac{1}{4\pi\varepsilon_o}\int\left(\frac{\dot{\rho}(\vec{r}',0)\hat{\mathscr{r}}}{c\mathscr{r}}+\frac{\left[\rho(\vec{r}',0)+\dot{\rho}(\vec{r}',0)t_r\right]\hat{\mathscr{r}}}{\mathscr{r}^2}\right)d\tau'$$

$$\vec{E}=\frac{1}{4\pi\varepsilon_o}\int\left(\frac{\dot{\rho}(\vec{r}',0)\hat{\mathscr{r}}}{c\mathscr{r}}+\frac{\rho(\vec{r}',0)\hat{\mathscr{r}}}{\mathscr{r}^2}+\frac{\left(\dot{\rho}(\vec{r}',0)\left(t-\frac{\mathscr{r}}{c}\right)\right)\hat{\mathscr{r}}}{\mathscr{r}^2}\right)d\tau'$$

$$\vec{E}=\frac{1}{4\pi\varepsilon_o}\int\left(\frac{\dot{\rho}(\vec{r}',0)\hat{\mathscr{r}}}{c\mathscr{r}}+\frac{\rho(\vec{r}',0)\hat{\mathscr{r}}}{\mathscr{r}^2}+\frac{\left[\dot{\rho}(\vec{r}',0)\,t\right]\hat{\mathscr{r}}}{\mathscr{r}^2}-\frac{\left[\dot{\rho}(\vec{r}',0)\right]\hat{\mathscr{r}}}{c\mathscr{r}}\right)d\tau'$$

$$\vec{E}=\frac{1}{4\pi\varepsilon_o}\int\left(\frac{\rho(\vec{r}',0)\hat{\mathscr{r}}}{\mathscr{r}^2}+\frac{\left[\dot{\rho}(\vec{r}',0)\,t\right]\hat{\mathscr{r}}}{\mathscr{r}^2}\right)d\tau'$$

$$\vec{E}=\frac{1}{4\pi\varepsilon_o}\int\left(\frac{\left[\rho(\vec{r}',0)+\dot{\rho}(\vec{r}',0)\,t\right]\hat{\mathscr{r}}}{\mathscr{r}^2}\right)d\tau'$$

$$\vec{E}(r,t)=\frac{1}{4\pi\varepsilon_o}\int\left(\frac{\left[\rho(\vec{r}',t)\right]\hat{\mathscr{r}}}{\mathscr{r}^2}\right)d\tau'$$

It reduces to the old Coulomb's Law: if you've got a constant current, i.e., magneto'statics' then the electric field *now* is rather miraculously due to the charge distribution *now* (even though it's changing linearly with time.)

Pr. 10.12 Similarly shows that if the current density varies slowly/ we're looking at fairly close things, so we can approximate $\vec{J}(\vec{r}',t_r)\approx\vec{J}(\vec{r}',t)+\left(t_r-t\right)\dot{\vec{J}}(\vec{r}',t_r)$

$$\vec{B}(\vec{r},t)=\frac{\mu_o}{4\pi}\int\left(\frac{\dot{\vec{J}}(\vec{r}',t_r)}{c\mathscr{r}}+\frac{\vec{J}(\vec{r}',t_r)}{\mathscr{r}^2}\right)\times\hat{\mathscr{r}}\,d\tau'$$

Reduces to

$\vec{B}(\vec{r},t)\approx\frac{\mu_o}{4\pi}\int\left(\frac{\vec{J}(\vec{r}',t)}{\mathscr{r}^2}\right)\times\hat{\mathscr{r}}\,d\tau'$, i.e., the old Biot-Savart Law.

"I didn't quite follow how 10.26 satisfied the nonhomogeneous wave eqn on pg 446, can we step through that?" Jessica

"I am having a lot of trouble finding the limits od integrals for time retarded system when finding potentials, can we go over this?" Davies
I'd also appreciate seeing this. Casey McGrath

"Can we go through the time derivation of B. After equation. 10.36" Antwain

"Along with Rae and Jessica I think it might be good to go over the proof they mentioned. Also when Griffiths says the same proof applies to the advanced time is he saying we use the exact same proof since the derivatives of ta and tr are identical?" Ben Kid

"Can we do an example where we calculate the retarded vector potential from a current that changes with time like Ex 10.2, except with a more complicated geometry like a circle?" Jessica
I'd also like to do a problem like Example 10.2. Spencer

"Similar to Jessica's question: what is the significance of the fact that the retarded potential satisfies the non-homogeneous wave equation? Does anything similarly correspond to the advanced potential?" Rachael Hach
I think that at least part of the significance of the retarded potential satisfying the inhomogeneous wave equation is that it shows that the potential propagates through space as we expect it to in that its changes in time at different locations are necessarily dependent on the source terms on the right side of equation 10.16
Ben Kid

"Can we go over where exactly the advanced potentials violate causality?" Casey P,
Remember that the term on the left side of the equations depends on t, so the advanced potentials calculate the potential at a given time based on the current or charge distribution in the future.
What I'm not sure about is why the heck he mentions them. Will they be come useful later or is Griffiths just acting cool? Freeman,