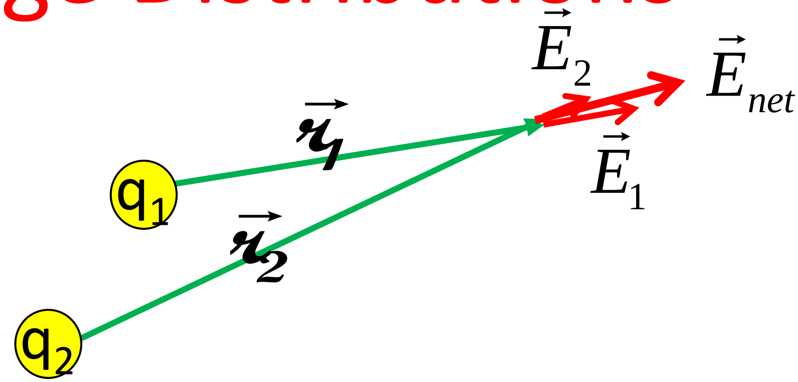
Fri.	(C 15) 2.1.4 Continuous Charge Distributions	
Mon.	(C 21.1-.5,.8) 1.2, 2.2.1-.2.2 Gauss & Div, T2 Numerical Quadrature	
Wed.	(C 21.1-.5,.8) 2.2.3 Using Gauss	
Thurs		HW1

Fields of Continuous Charge Distributions



Superposition of Fields

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \ldots = \sum_{i=1} \vec{E}_i$$

$$\vec{E}_{net} = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r_1^2}\hat{r}_1 + \frac{1}{4\pi\varepsilon_o}\frac{q_2}{r_2^2}\hat{r}_2 + \ldots = \sum_{i=1}\frac{1}{4\pi\varepsilon_o}\frac{q_i}{r_i^2}\hat{r}_i$$

In the limit of differentially small charge morsels

$$\vec{E}_{net} = \sum_{i=1}\frac{1}{4\pi\varepsilon_o}\frac{q_i}{r_i^2}\hat{r}_i \underset{\lim q \to dq}{\longrightarrow} \int_{charge}\frac{1}{4\pi\varepsilon_o}\frac{\hat{r}}{r_i^2}\,dq$$

Types of charge distributions

$$\vec{E}_{net} = \sum_{i=1} \frac{1}{4\pi\varepsilon_o} \frac{q_i}{\mathscr{r}_i^{\,2}} \hat{\mathscr{r}}_i \xrightarrow[\lim q \to dq]{} \int_{charge} \frac{1}{4\pi\varepsilon_o} \frac{\hat{\mathscr{r}}}{\mathscr{r}_i^{\,2}} dq$$

Line charge

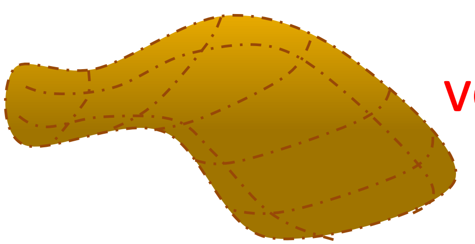
Linear charge density $\lambda(r') \equiv \frac{dq}{dl'} \Rightarrow dq = \lambda(r')dl'$

May vary with position

Surface charge

Surface charge density $\sigma(r') \equiv \frac{dq}{da'} \Rightarrow dq = \lambda(r')da'$

Volume charge

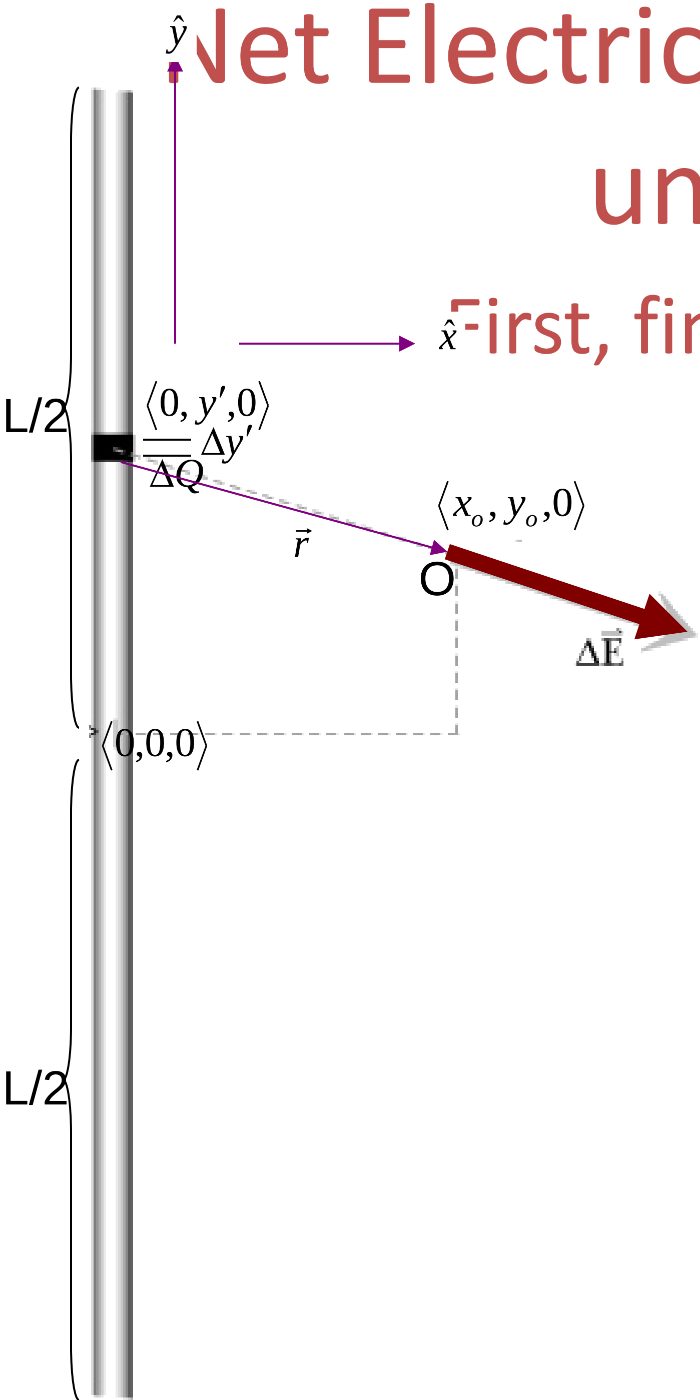


volume charge density $\rho(r') \equiv \frac{dq}{d\tau'} \Rightarrow dq = \rho(r')d\tau'$

Linear Charge Density Example in Excruciating Detail:

Field of Rod

Net Electric Field at point O due to a uniformly charged thin Rod



First, find field at O due to a morsel of the rod, ΔE.

$$\Delta\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{\mathscr{r}^2}\hat{\mathscr{r}} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{\mathscr{r}^2}\frac{\vec{\mathscr{r}}}{|\mathscr{r}|} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{\mathscr{r}^3}\vec{\mathscr{r}}$$

where

$$\vec{\mathscr{r}} = \langle \text{observation location} \rangle - \langle \text{charge location} \rangle$$

$$= \langle x_o, y_0, 0\rangle - \langle 0, y', 0\rangle = \langle x_o, (y_0 - y'), 0\rangle$$

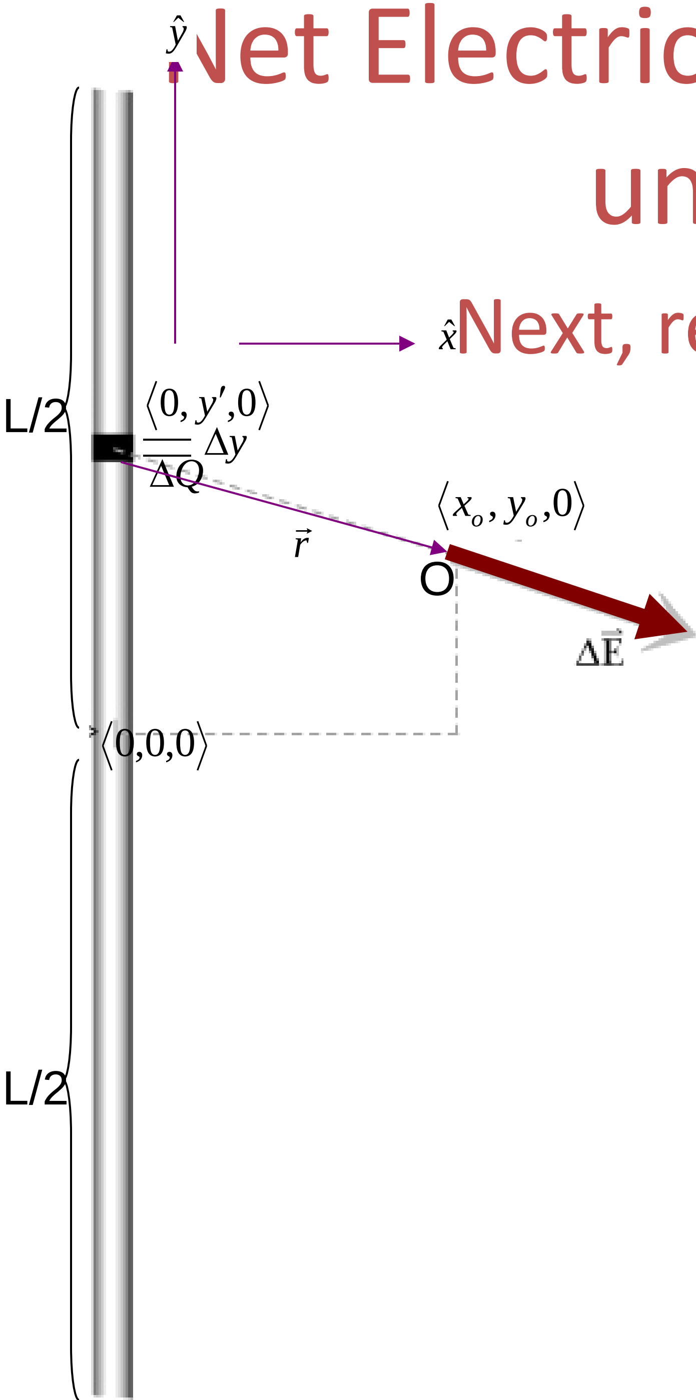
so,

$$|\mathscr{r}| = \left[x_o{}^2 + (y_0 - y')^2\right]^{1/2}$$

Thus

$$\Delta\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{\left[x_o{}^2 + (y_0 - y')^2\right]^{3/2}}\langle x_o, (y_0 - y'), 0$$

Net Electric Field at point O due to a uniformly charged thin Rod



Next, rephrase in terms of the position of the morsel, y'.

$$\Delta\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q}{\left[x_o{}^2 + (y_0 - y')^2\right]^{3/2}}\langle x_o, (y_0 - y'), 0\rangle$$

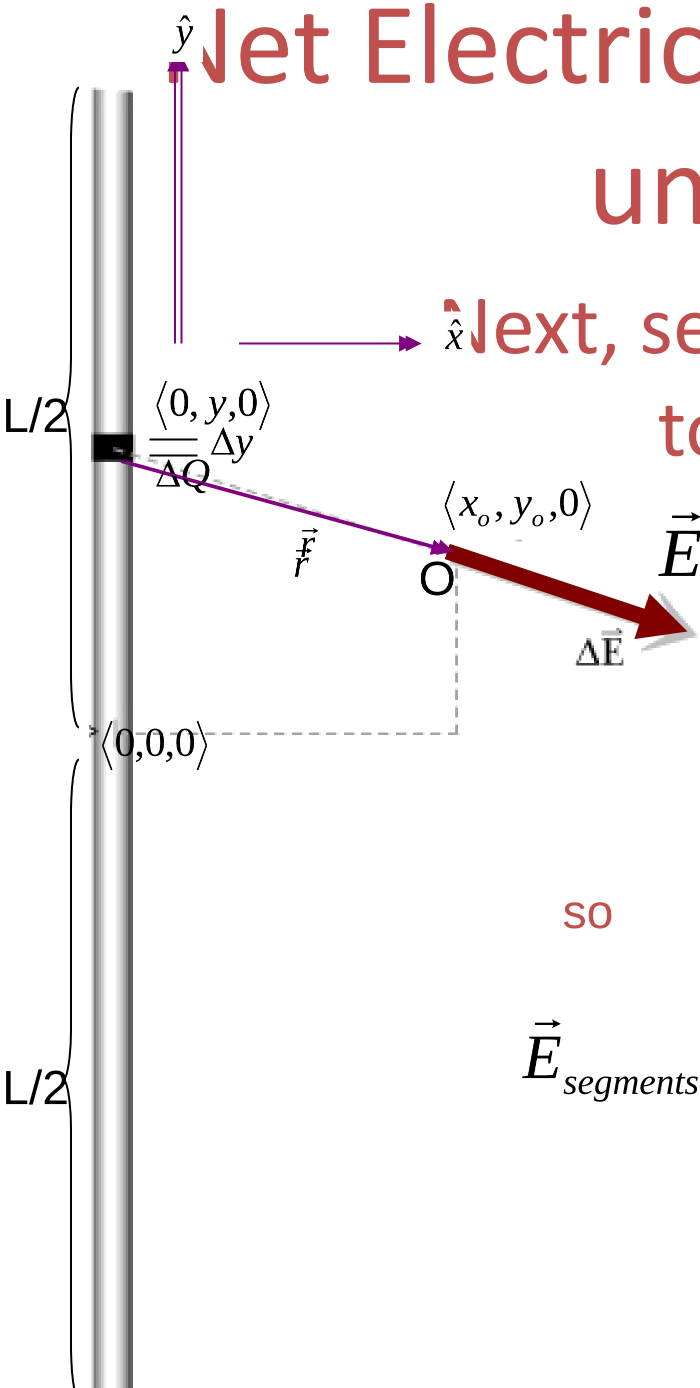
where

$$\frac{\Delta Q}{\Delta y'} = \frac{Q}{L} \Rightarrow \Delta Q = \frac{Q}{L}\Delta y'$$

so,

$$\Delta\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{(Q/L)\Delta y'}{\left[x_o{}^2 + (y_0 - y')^2\right]^{3/2}}\langle x_o, (y_0 - y'), 0\rangle$$

Net Electric Field at point O due to a uniformly charged thin Rod



Next, setup sum over bits of field at O due to *all* morsels (all along wire).

$$\vec{E}_{wire} \approx \vec{E}_{segments} = \sum_{y=-L/2}^{y=L/2} \Delta\vec{E}$$

where

$$\Delta\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{(Q/L)\Delta y'}{\left[x_o{}^2 + (y_0 - y')^2\right]^{3/2}} \langle x_o, (y_0 - y'), 0 \rangle$$

so

$$\vec{E}_{segments} = \sum_{y'=-L/2}^{y'=L/2} \frac{1}{4\pi\varepsilon_0} \frac{(Q/L)\Delta y'}{\left[x_o{}^2 + (y_0 - y')^2\right]^{3/2}} \langle x_o, (y_0 - y'), 0 \rangle$$

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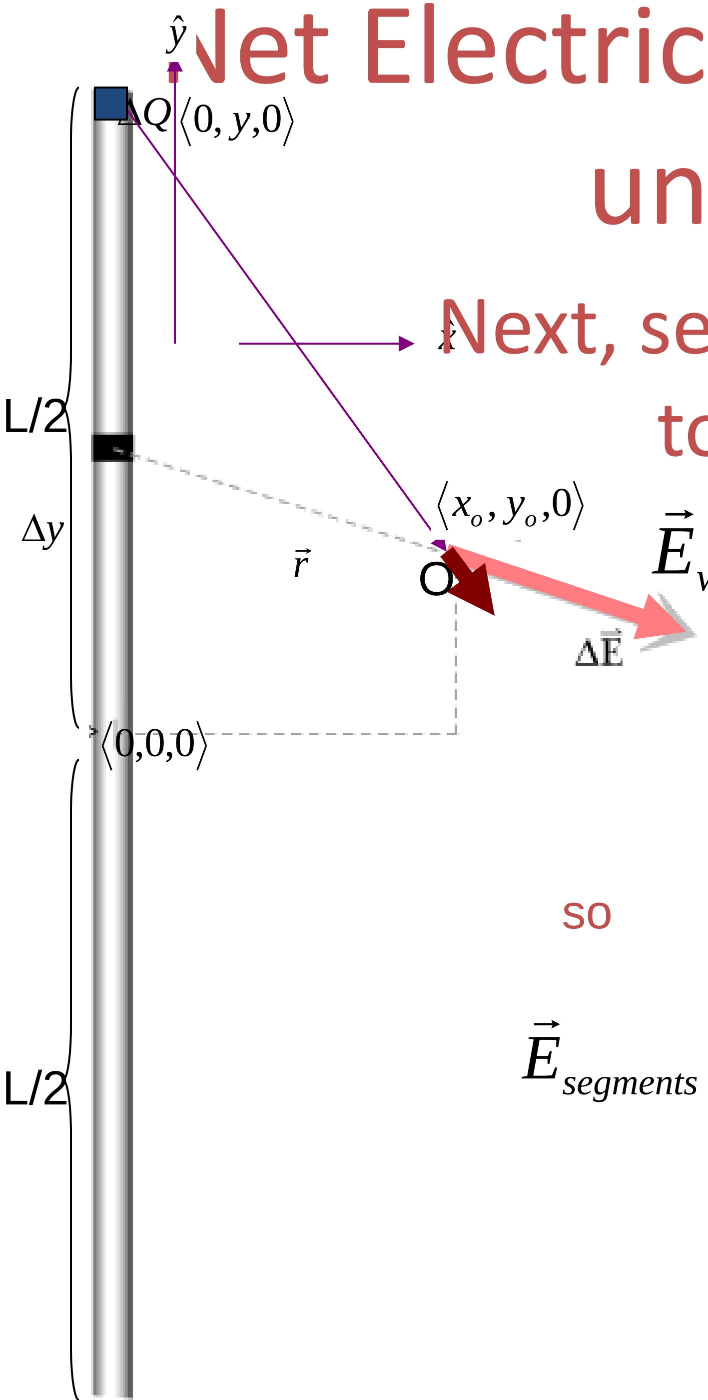

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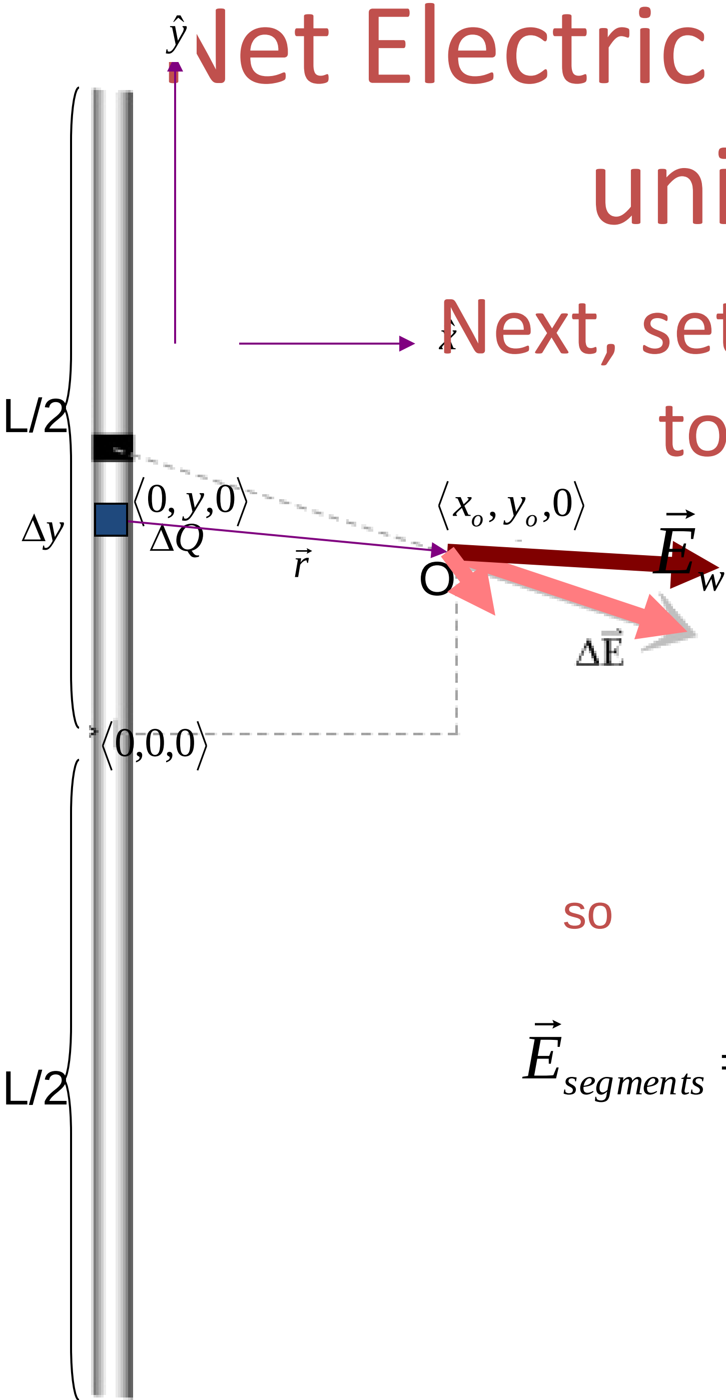

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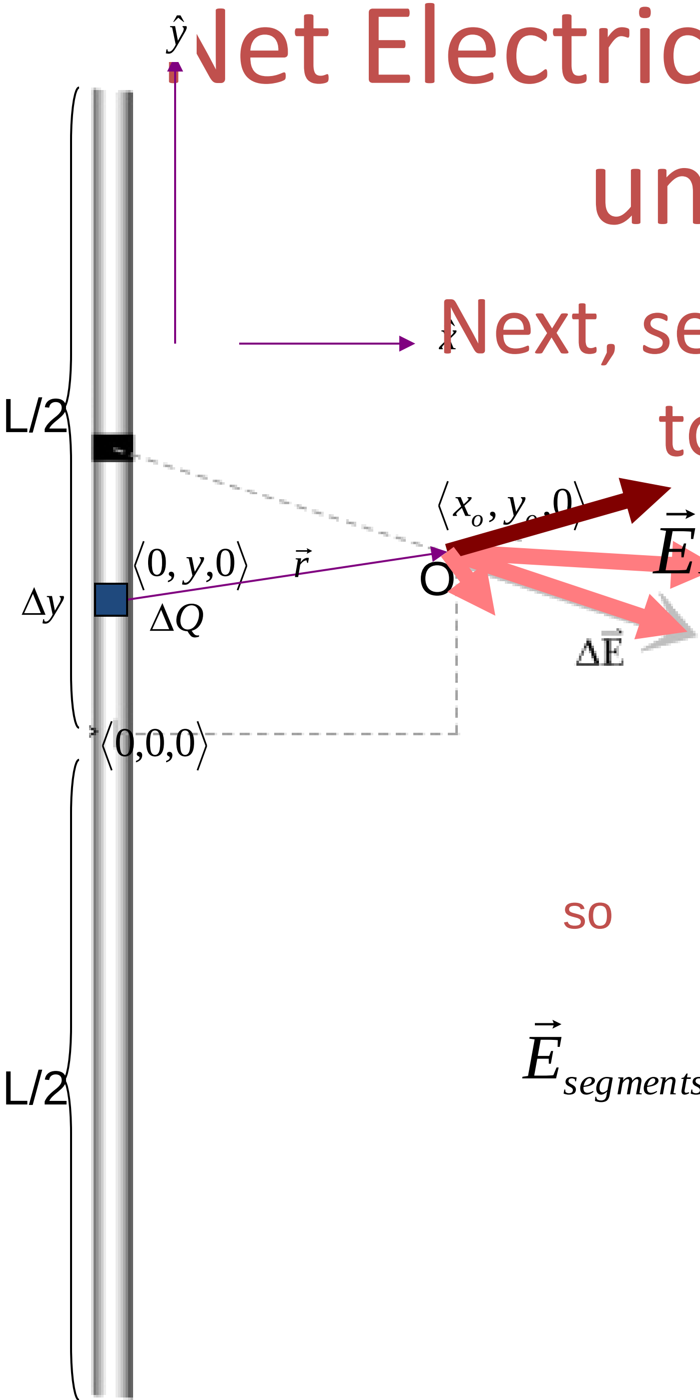

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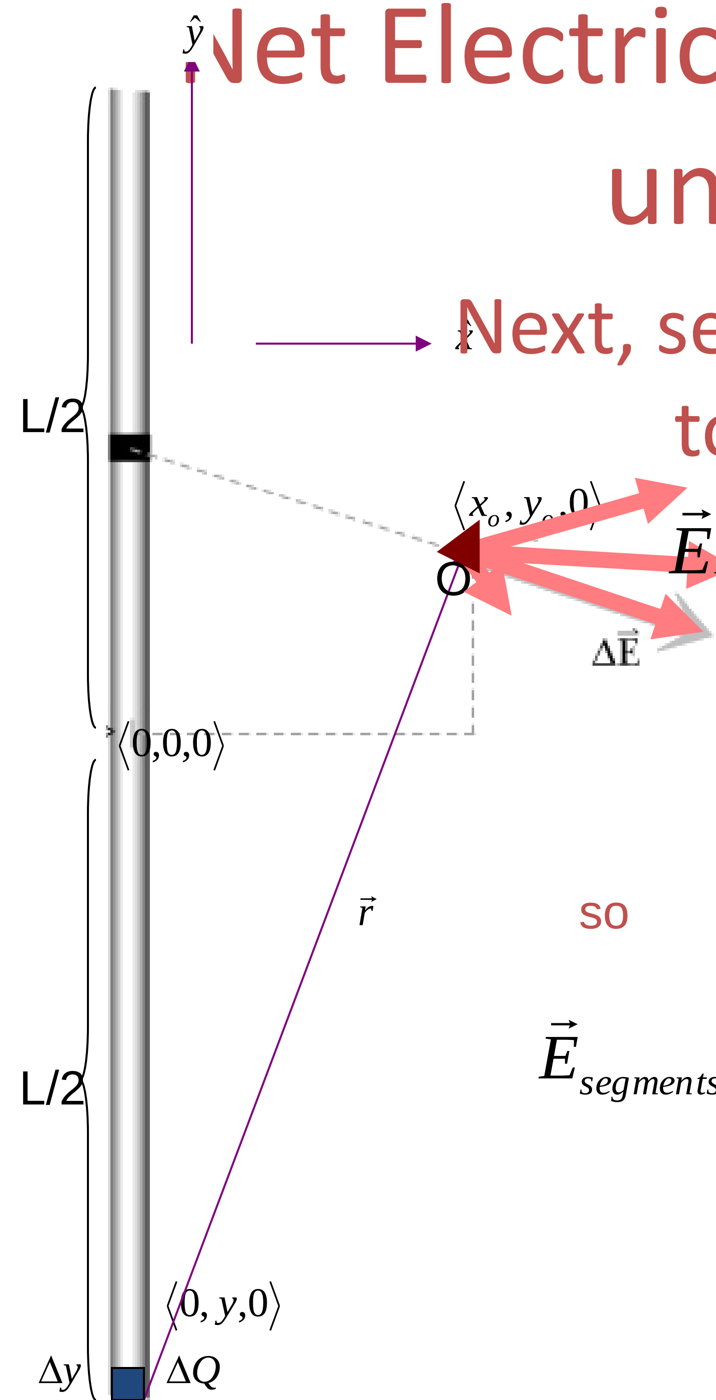
Net Electric Field at point O due to a uniformly charged thin Rod



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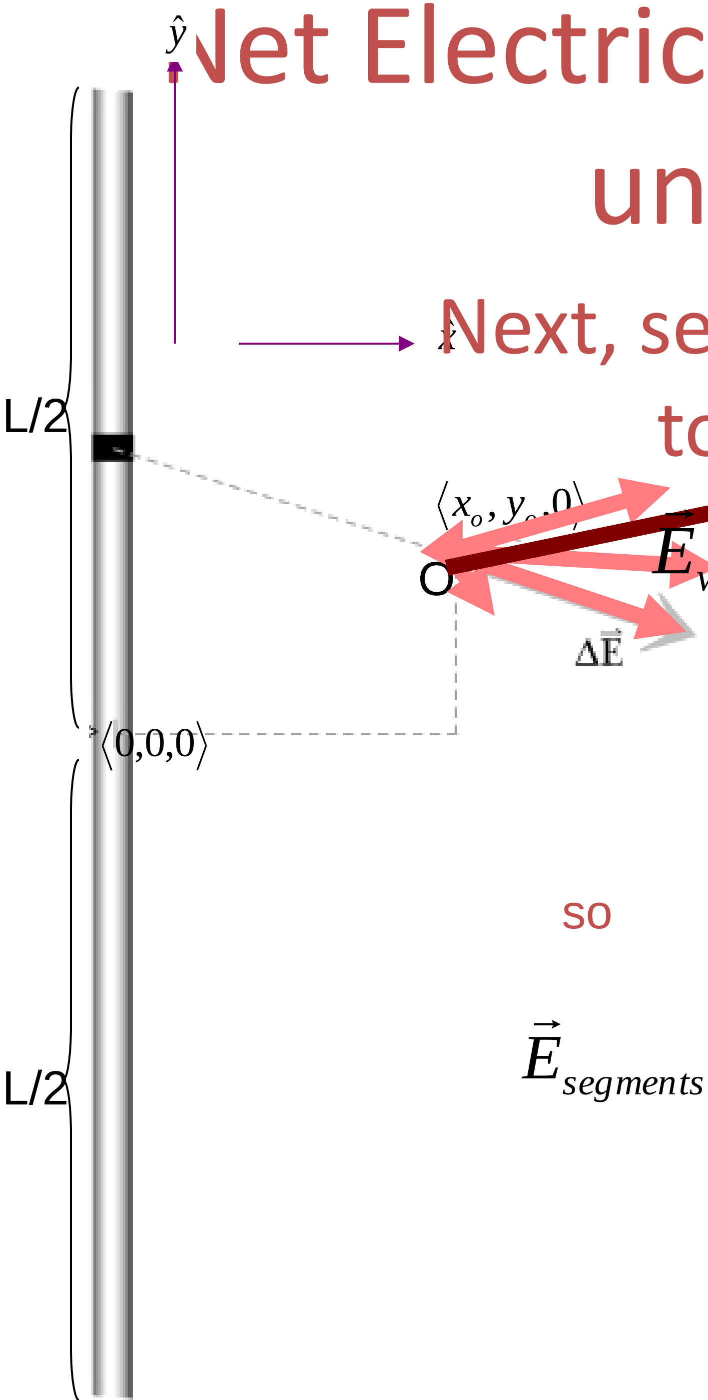

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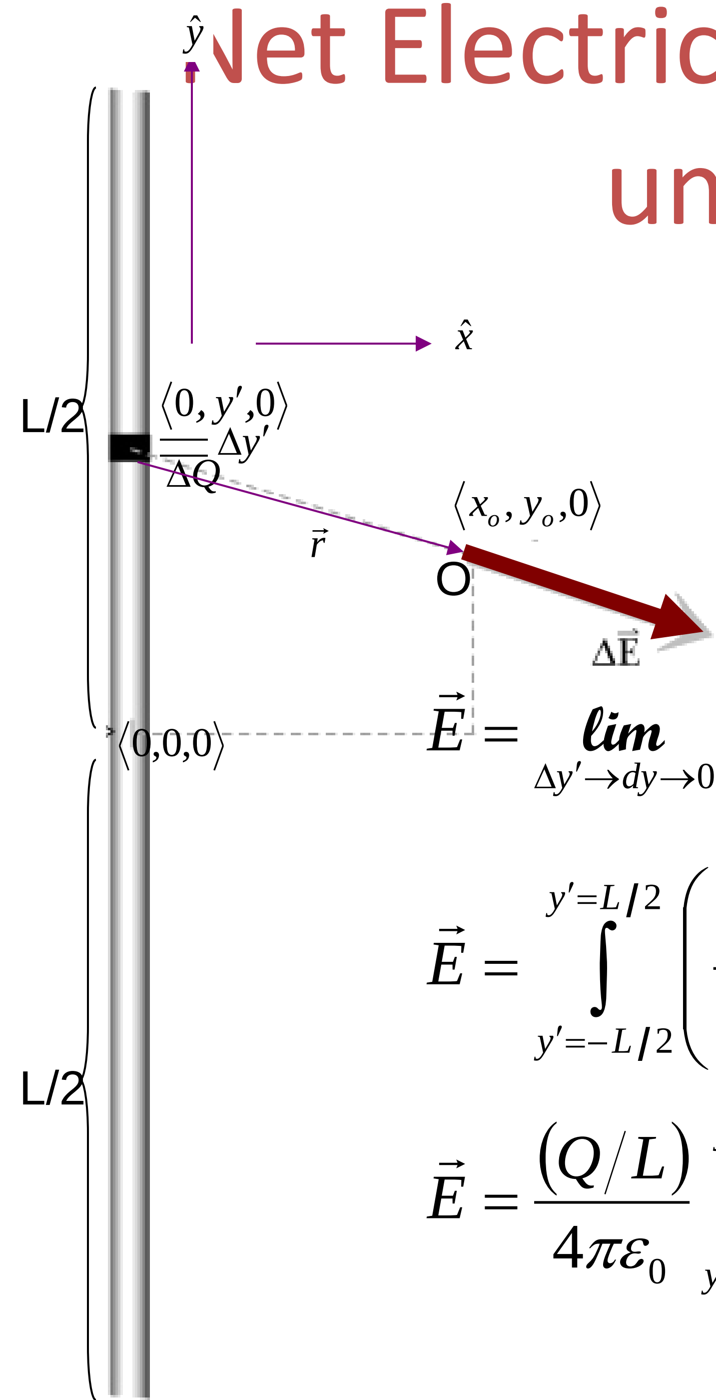
Net Electric Field at point O due to a uniformly charged thin Rod



Take Differential Limit: sum becomes integral

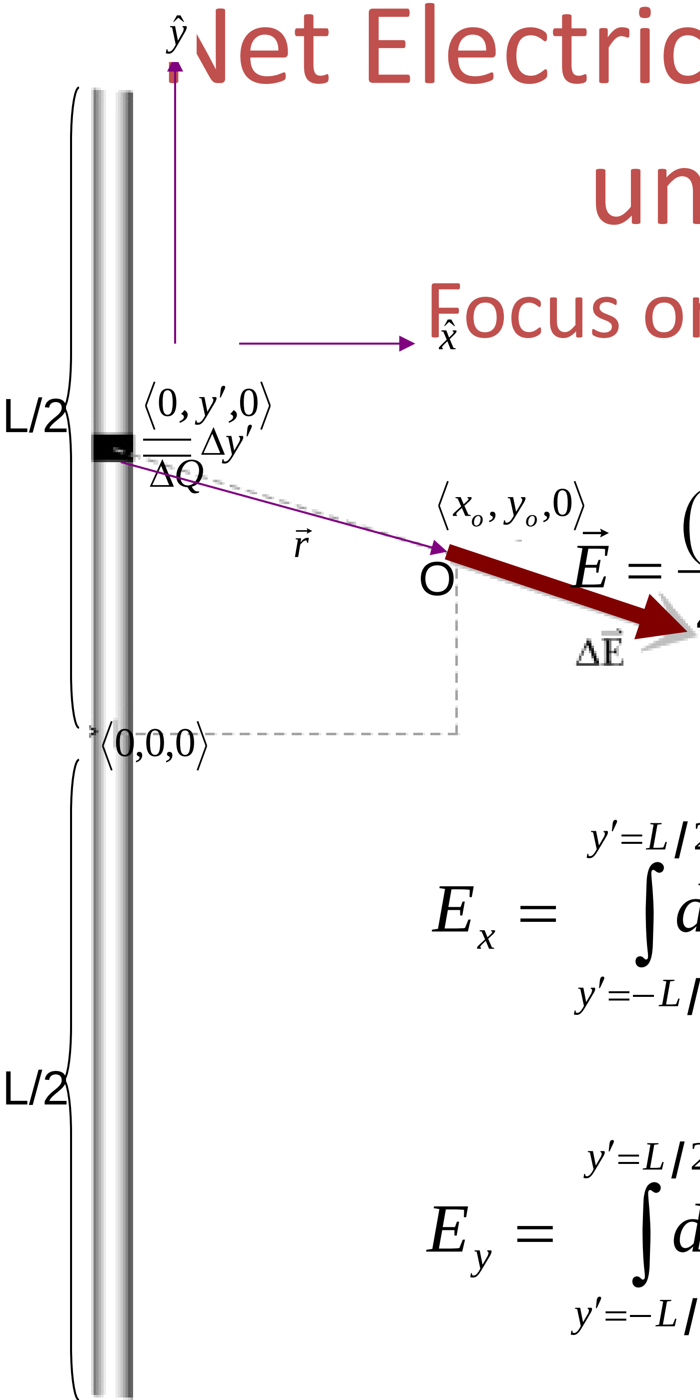
$$\vec{E} = \lim_{\Delta y' \to dy \to 0} \left(\sum_{y'=-L/2}^{y'=L/2} \frac{1}{4\pi\varepsilon_0} \frac{(Q/L)\Delta y'}{\left[x_o{}^2 + (y_0 - y')^2\right]^{3/2}} \langle x_o, (y_0 - y'), 0 \rangle \right)$$

$$\vec{E} = \int_{y'=-L/2}^{y'=L/2} \left(\frac{1}{4\pi\varepsilon_0} \frac{(Q/L)dy'}{\left[x_o{}^2 + (y_0 - y')^2\right]^{3/2}} \langle x_o, (y_0 - y'), 0 \rangle \right)$$

$$\vec{E} = \frac{(Q/L)}{4\pi\varepsilon_0} \int_{y'=-L/2}^{y'=L/2} \left(\frac{dy}{\left[x_o{}^2 + (y_0 - y')^2\right]^{3/2}} \langle x_o, (y_0 - y'), 0 \rangle \right)$$

Net Electric Field at point O due to a uniformly charged thin Rod

Focus on x and y components individually

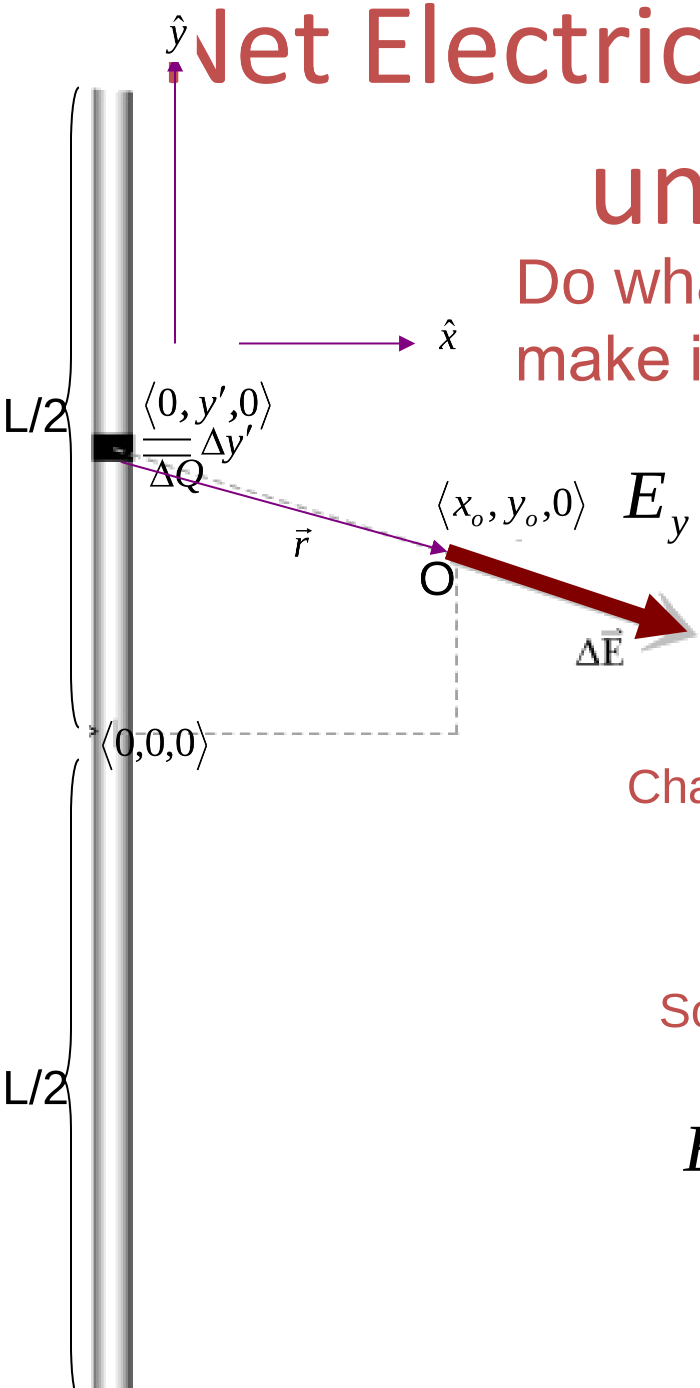


$$\vec{E} = \frac{(Q/L)}{4\pi\varepsilon_0}\int_{y'=-L/2}^{y'=L/2}\left(\frac{dy'}{\left[x_o^{\ 2}+(y_0-y')^2\right]^{3/2}}\langle x_o, (y_0-y'), 0\rangle\right)$$

$$E_x = \int_{y'=-L/2}^{y'=L/2} dE_x = \frac{(Q/L)x_o}{4\pi\varepsilon_0}\int_{y'=-L/2}^{y'=L/2}\frac{dy'}{\left[x_o^{\ 2}+(y_0-y')^2\right]^{3/2}}$$

$$E_y = \int_{y'=-L/2}^{y'=L/2} dE_y = \frac{(Q/L)}{4\pi\varepsilon_0}\int_{y'=-L/2}^{y'=L/2}\frac{(y_0-y')dy'}{\left[x_o^{\ 2}+(y_0-y')^2\right]^{3/2}}$$

Net Electric Field at point O due to a uniformly charged thin Rod



Do what you must to either *do* the integral, or make it recognizable as one that's done for you.

$$E_y = \int_{y'=-L/2}^{y'=L/2} dE_y = \frac{(Q/L)}{4\pi\varepsilon_0} \int_{y'=-L/2}^{y'=L/2} \frac{(y_0 - y')dy'}{\left[x_o{}^2 + (y_0 - y')^2\right]^{3/2}}$$

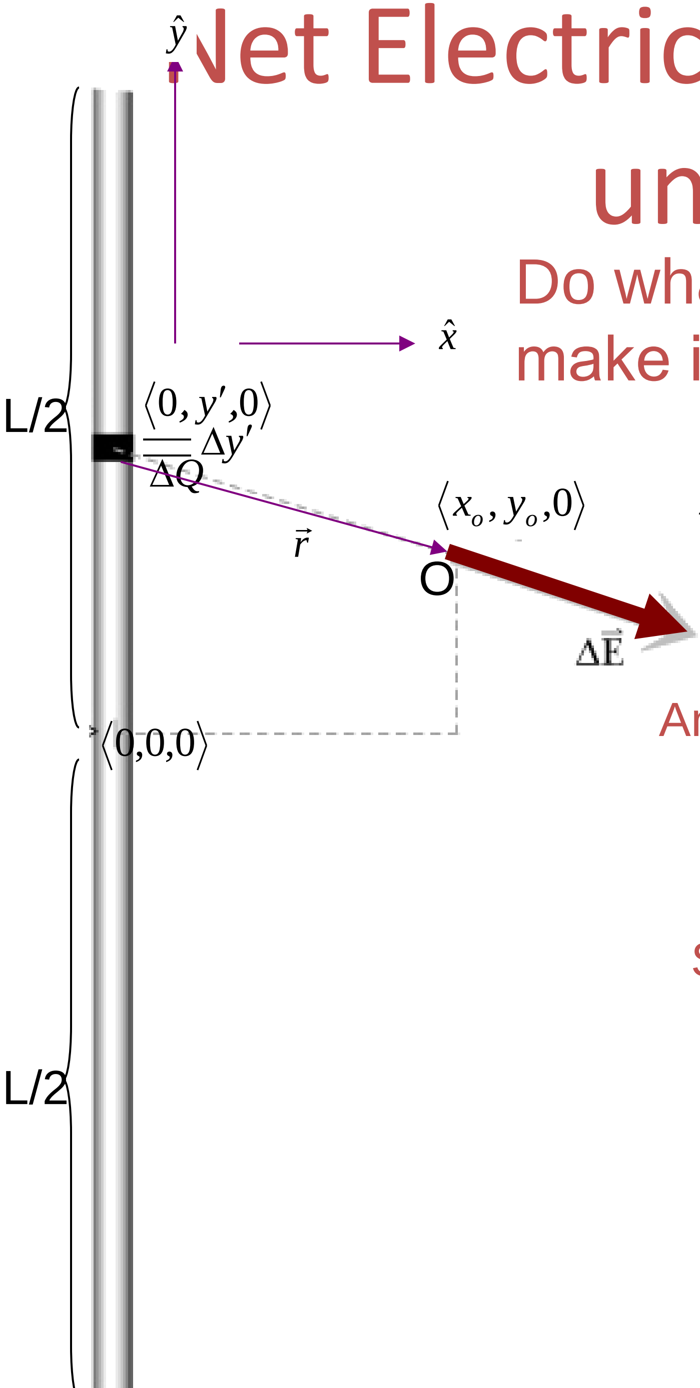
Change of variables:

$$\tilde{y} \equiv (y_o - y') \Rightarrow y' = y_o - \tilde{y} \quad \text{and} \quad dy' = -d\tilde{y}$$

So,

$$E_y = \frac{(Q/L)}{4\pi\varepsilon_0} \int_{\tilde{y}=y_o+L/2}^{\tilde{y}=y_o-L/2} \frac{-\tilde{y}d\tilde{y}}{\left[x_o{}^2 + \tilde{y}^2\right]^{3/2}}$$

Net Electric Field at point O due to a uniformly charged thin Rod



Do what you must to either *do* the integral, or make it recognizable as one that's done for you.

$$E_y = \frac{(Q/L)}{4\pi\varepsilon_0} \int_{\tilde{y}=y_o-L/2}^{\tilde{y}=y_o+L/2} \frac{\tilde{y}\,d\tilde{y}}{\left[x_o^{\;2} + \tilde{y}^{\,2}\right]^{3/2}}$$

Another change of variables:

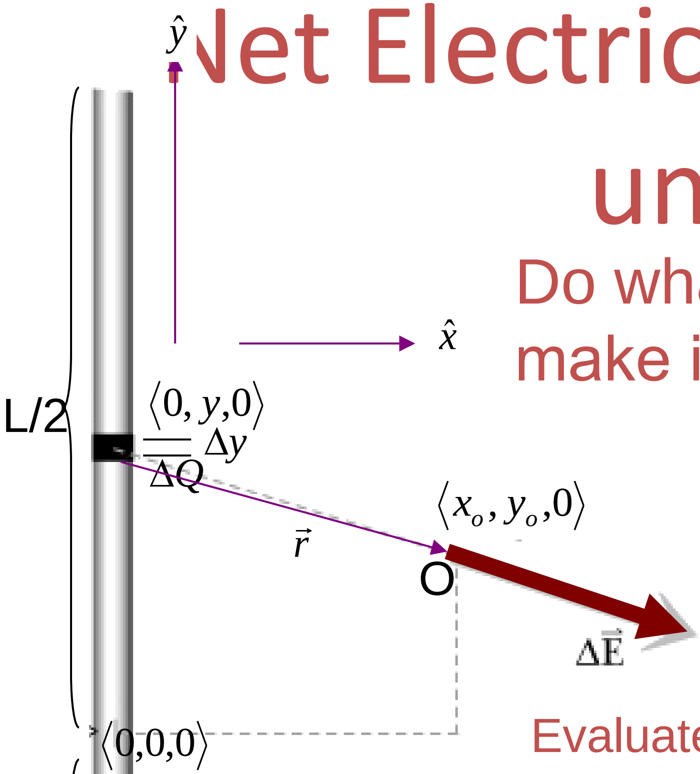
$$\breve{y} \equiv \tilde{y}^{\,2}$$

So,

$$E_y = \frac{(Q/L)}{4\pi\varepsilon_0} \int_{\breve{y}=(y_o-L/2)^2}^{\breve{y}=(y_o+L/2)^2} \frac{\frac{1}{2}\,d\breve{y}}{\left[x_o^{\;2} + \breve{y}\right]^{3/2}}$$

Net Electric Field at point O due to a uniformly charged thin Rod

Do what you must to either *do* the integral, or make it recognizable as one that's done for you.



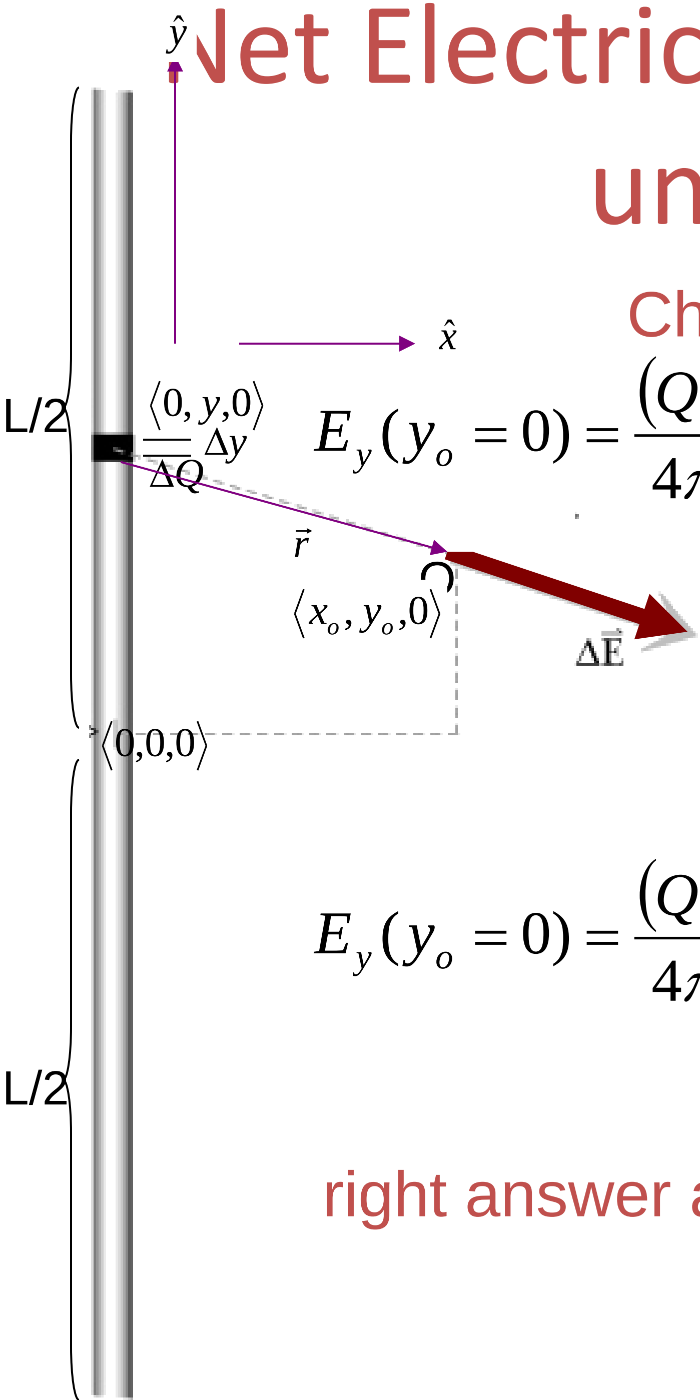
$$E_y = \frac{(Q/L)}{8\pi\varepsilon_0}\int_{\breve{y}=(y_o-L/2)^2}^{\breve{y}=(y_o+L/2)^2}\frac{d\breve{y}}{\left[{x_o}^2+\breve{y}\right]^{3/2}}$$

Evaluate Integral:

$$E_y = \frac{(Q/L)}{8\pi\varepsilon_0}\left(-2\frac{1}{\left[{x_o}^2+\breve{y}\right]^{1/2}}\right)\Bigg|_{\breve{y}=(y_o-L/2)^2}^{\breve{y}=(y_o+L/2)^2}$$

$$E_y = \frac{(Q/L)}{4\pi\varepsilon_0}\left(\frac{1}{\left[{x_o}^2+(y_o-L/2)^2\right]^{1/2}}-\frac{1}{\left[{x_o}^2+(y_o+L/2)^2\right]^{1/2}}\right)$$

Net Electric Field at point O due to a uniformly charged thin Rod



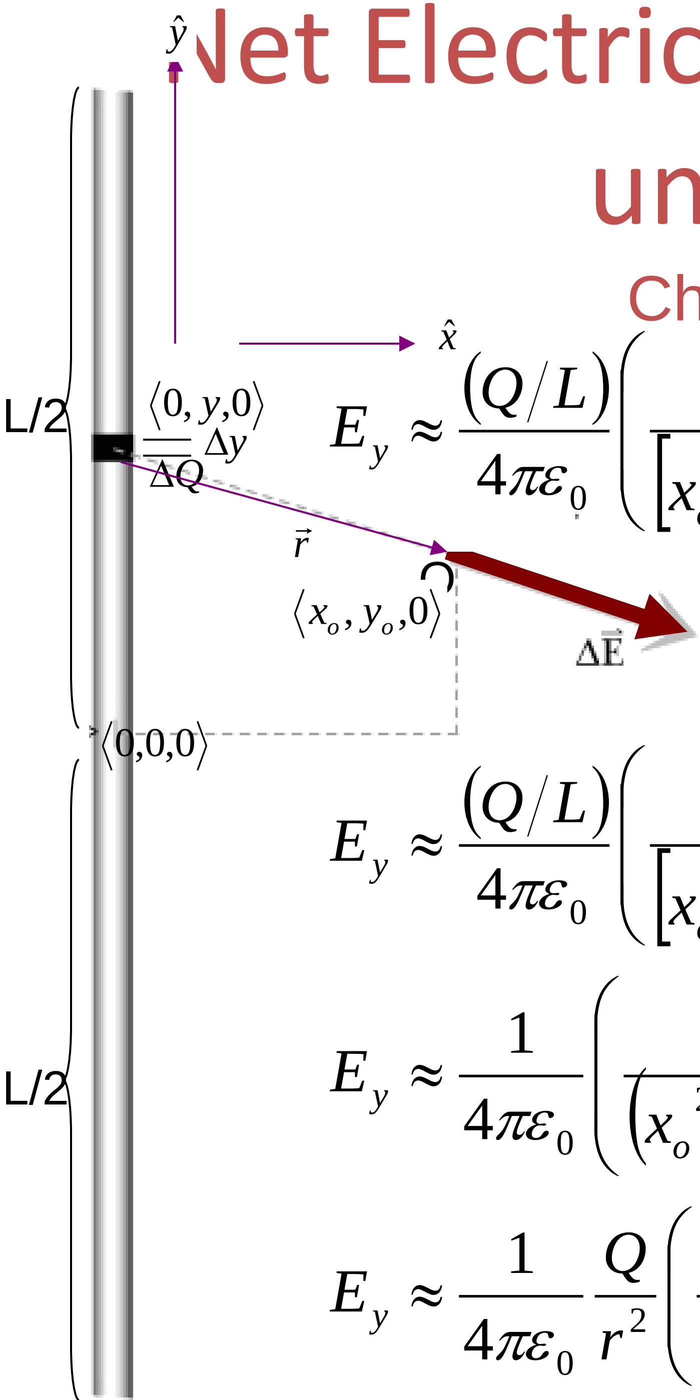
Check: right answer at $y_o = 0$?

$$E_y(y_o=0)=\frac{(Q/L)}{4\pi\varepsilon_0}\left(\frac{1}{\left[x_o^{\,2}+(y_o-L/2)^2\right]^{1/2}}-\frac{1}{\left[x_o^{\,2}+(y_o+L/2)^2\right]^{1/2}}\right)$$

$$E_y(y_o=0)=\frac{(Q/L)}{4\pi\varepsilon_0}\left(\frac{1}{\left[x_o^{\,2}+(L/2)^2\right]^{1/2}}-\frac{1}{\left[x_o^{\,2}+(L/2)^2\right]^{1/2}}\right)=0$$

right answer at $y_o = 0$? Yes.

Net Electric Field at point O due to a uniformly charged thin Rod



Check: right answer at y_o>>L?

$$E_y \approx \frac{(Q/L)}{4\pi\varepsilon_0}\left(\frac{1}{\left[x_o^{\,2}+y_o^{\,2}-y_oL\right]^{1/2}}-\frac{1}{\left[x_o^{\,2}+y_o^{\,2}+y_oL\right]^{1/2}}\right)$$

Apply binomial expansion which says if ε<<1, then $(1+\varepsilon)^n \approx 1+n\varepsilon$

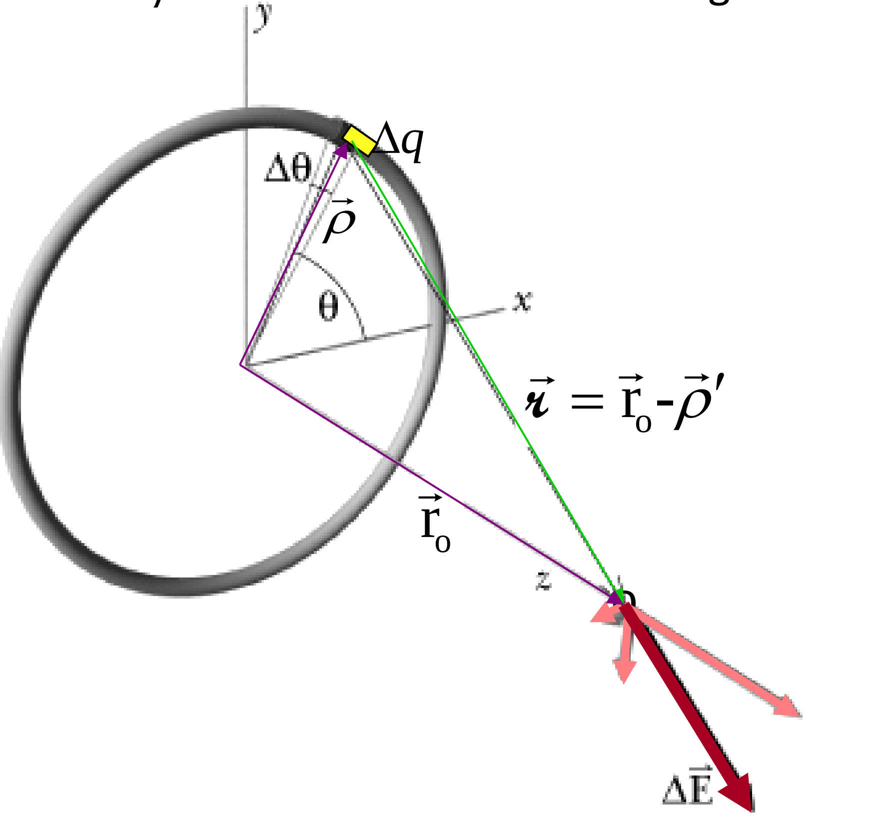
$$E_y \approx \frac{(Q/L)}{4\pi\varepsilon_0}\left(\frac{1}{\left[x_o^{\,2}+y_o^{\,2}\right]^{1/2}}\frac{y_oL}{2\left(x_o^{\,2}+y_o^{\,2}\right)}-\frac{1}{\left[x_o^{\,2}+y_o^{\,2}\right]^{1/2}}\frac{-y_oL}{2\left(x_o^{\,2}+y_o^{\,2}\right)}\right)$$

$$E_y \approx \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{\left(x_o^{\,2}+y_o^{\,2}\right)}\frac{y_o}{\left[x_o^{\,2}+y_o^{\,2}\right]^{1/2}}\right)$$

$$E_y \approx \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\left(\frac{y_o}{|r|}\right)$$

Exercise

Problem 2.5: Find the electric field a distance *z* above the center of a circular loop of radius ρ that carries a uniform charge of linear density λ.



note: Cylindrical Symmetry suggests Cylindrical Coordinates

[redacted]

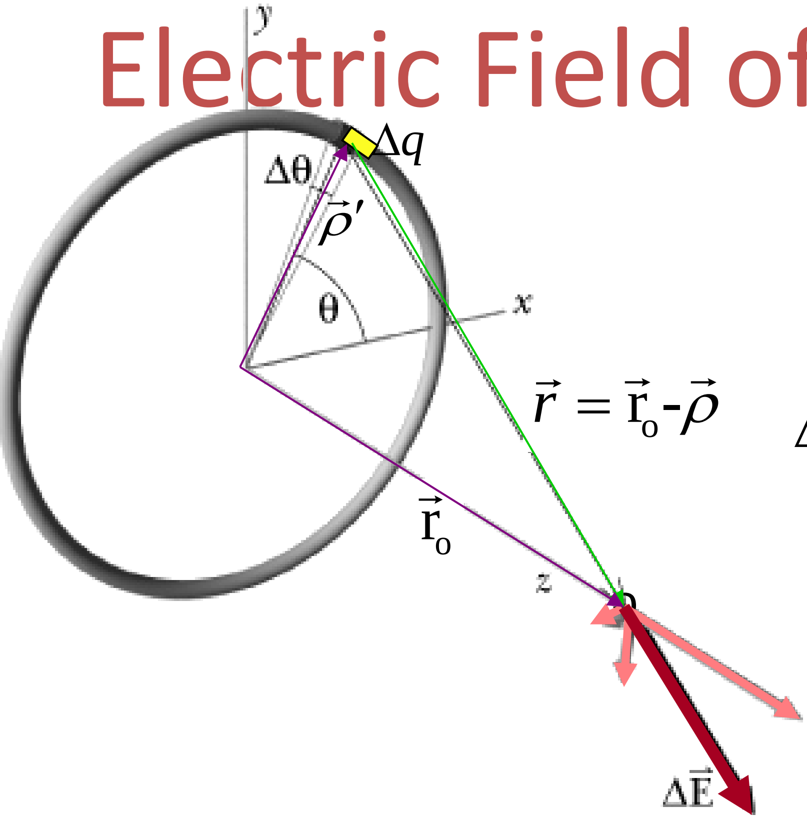
Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

Step 2: write an expression for ΔE

Step 3: Add up all ΔE's to get the total E

Step 4: Check results

Electric Field of a Uniformly Charged Ring



$$\Delta\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{\Delta q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_o}\frac{\Delta q}{r^3}\vec{r}$$

where

$$\vec{r} = \vec{r}_o - \vec{\rho}' = \langle 0,0,z\rangle - \langle \rho\cos\theta', \rho\sin\theta', 0\rangle$$

$$\vec{r} = \langle -\rho'\cos\theta, -\rho'\sin\theta, z\rangle$$

so

$$|\vec{r}| = \sqrt{(\rho'\cos\theta')^2 + (\rho'\sin\theta')^2 + z^2}$$

$$|\vec{r}| = \sqrt{\rho'^2(\cos^2\theta' + \sin^2\theta') + z^2}$$

$$|\vec{r}| = \sqrt{\rho'^2 + z^2}$$

thus

$$\Delta\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{\Delta q}{(\rho'^2 + z^2)^{3/2}}\langle -\rho'\cos\theta', -\rho'\sin\theta', z\rangle$$

Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

Step 2: write an expression for ΔE

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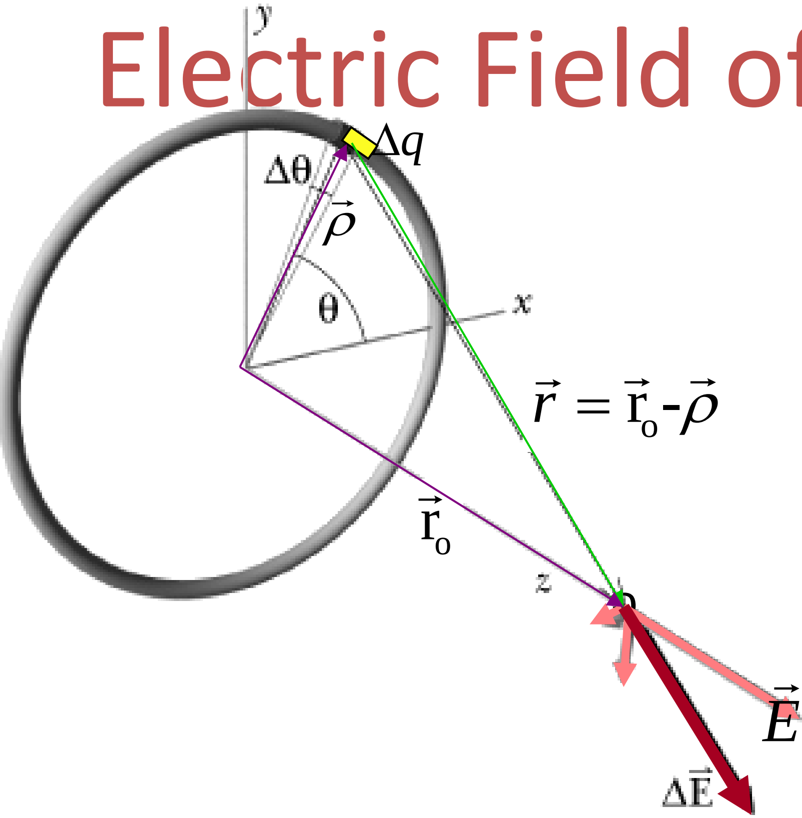
Electric Field of a Uniformly Charged Ring



$$\vec{E} = \sum_{ring} \Delta\vec{E}$$

where

$$\Delta\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{\Delta q}{\left(\rho'^2 + z^2\right)^{3/2}}\langle -\rho'\cos\theta', -\rho'\sin\theta', z\rangle$$

so

$$\vec{E} = \sum_{\theta'=0}^{\theta'=2\pi}\frac{1}{4\pi\varepsilon_o}\frac{\Delta q}{\left(\rho'^2 + z^2\right)^{3/2}}\langle -\rho'\cos\theta', -\rho'\sin\theta', z\rangle$$

Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

Step 2: write an expression for ΔE

Step 3: Add up all ΔE's to get the total E

Step 4: Check results

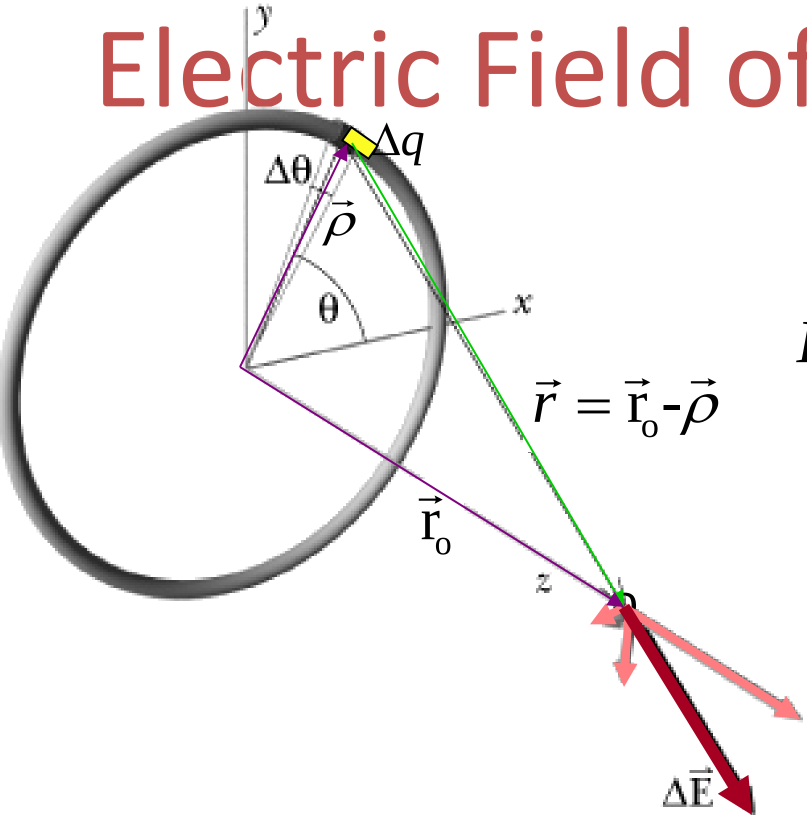
To make an integral, need a Δθ.

$$\frac{\Delta q}{\Delta\theta'\rho'} = \frac{q}{2\pi\rho'} \Rightarrow \Delta q = \frac{1}{2\pi}\Delta\theta'$$

thus

$$\vec{E} = \sum_{\theta'=0}^{\theta'=2\pi}\frac{1}{4\pi\varepsilon_o}\frac{\frac{q}{2\pi}\Delta\theta'}{\left(\rho'^2 + z^2\right)^{3/2}}\langle -\rho'\cos\theta', -\rho'\sin\theta', z\rangle$$

Electric Field of a Uniformly Charged Ring



$$\vec{E} = \sum_{\theta=0}^{\theta=2\pi} \frac{1}{4\pi\varepsilon_o} \frac{\frac{q}{2\pi}\Delta\theta}{\left(\rho^2 + z_o{}^2\right)^{3/2}} \left\langle -\rho\cos\theta, -\rho\sin\theta, z_o \right\rangle$$

$$\vec{E} = \lim_{\Delta\theta \to 0} \sum_{\theta=0}^{\theta=2\pi} stuff\,\Delta\theta = \int_{\theta=0}^{\theta=2\pi} stuff\, d\theta$$

$$\vec{E} = \int_{\theta=0}^{\theta=2\pi} \frac{1}{4\pi\varepsilon_o} \frac{\frac{q}{2\pi}}{\left(\rho^2 + z^2\right)^{3/2}} \left\langle -\rho\cos\theta, -\rho\sin\theta, z \right\rangle d\theta$$

$$\vec{E}_x = \frac{1}{4\pi\varepsilon_o} \frac{\frac{q}{2\pi}(-\rho)}{\left(\rho^2 + z^2\right)^{3/2}} \int_{\theta=0}^{\theta=2\pi} \cos\theta\, d\theta = 0$$

$$\vec{E}_y = \frac{1}{4\pi\varepsilon_o} \frac{\frac{q}{2\pi}(-\rho)}{\left(\rho^2 + z^2\right)^{3/2}} \int_{\theta=0}^{\theta=2\pi} \sin\theta\, d\theta = 0$$

$$\vec{E}_z = \frac{1}{4\pi\varepsilon_o} \frac{\frac{q}{2\pi} z}{\left(\rho^2 + z^2\right)^{3/2}} \int_{\theta=0}^{\theta=2\pi} d\theta = \frac{1}{4\pi\varepsilon_o} \frac{\frac{q}{2\pi} z}{\left(\rho^2 + z^2\right)^{3/2}} 2\pi$$

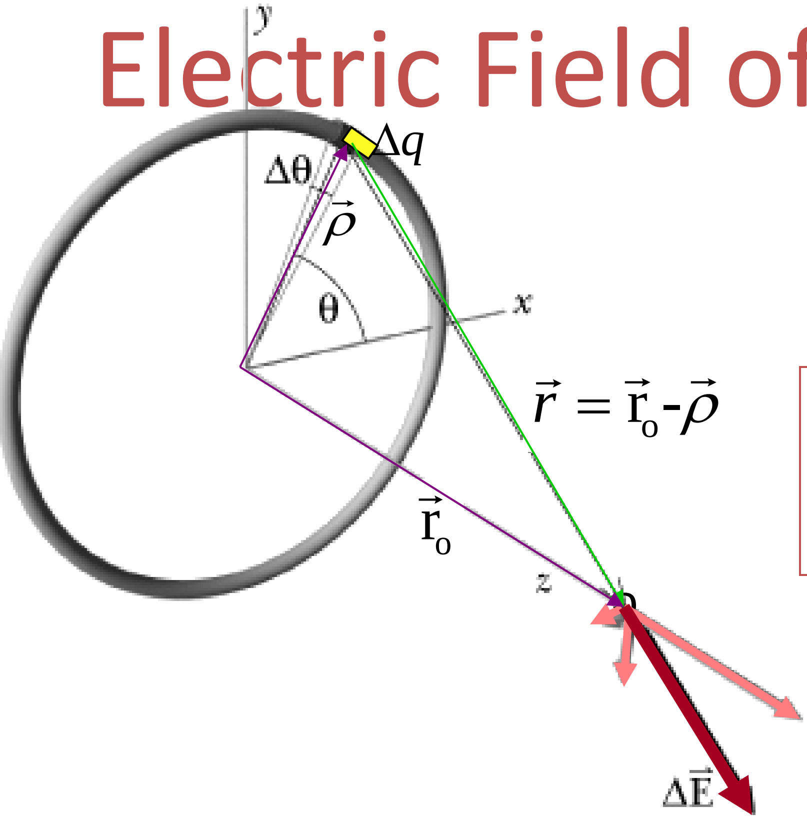
Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

Step 2: write an expression for ΔE

Step 3: Add up all ΔE's to get the total E

Step 4: Check results

Electric Field of a Uniformly Charged Ring



$$\vec{E}_x = 0 \qquad \vec{E}_y = 0 \qquad \vec{E}_z = \frac{1}{4\pi\varepsilon_o}\frac{qz_o}{\left(\rho^2 + z_o^2\right)^{3/2}}$$

Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

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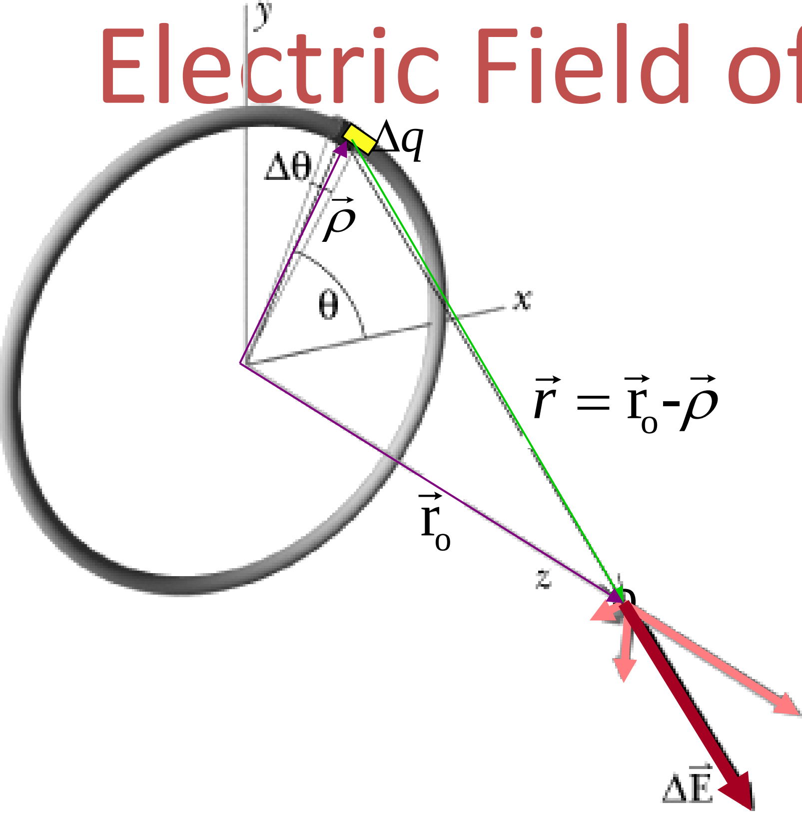
Step 4: Check results

Electric Field of a Uniformly Charged Ring



$$\vec{E}_x = 0 \qquad \vec{E}_y = 0 \qquad \vec{E}_z = \frac{1}{4\pi\varepsilon_o}\frac{qz_o}{\left(\rho^2 + z_o{}^2\right)^{3/2}}$$

Why?

Units?

Logic?

Limits?

Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

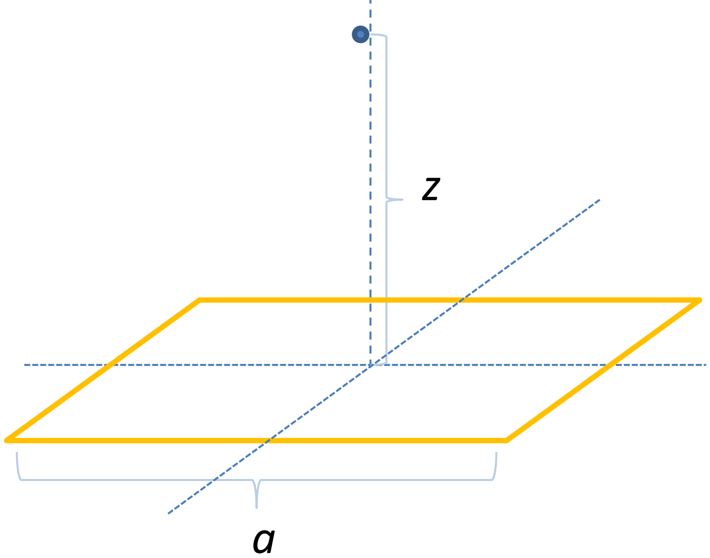
Step 2: write an expression for ΔE

Step 3: Add up all ΔE's to get the total E

Step 4: Check results

Exercise

Problem 2.4 Find the electric field a distance *z* above the center of a square loop (side a) carrying a uniform line charge l.

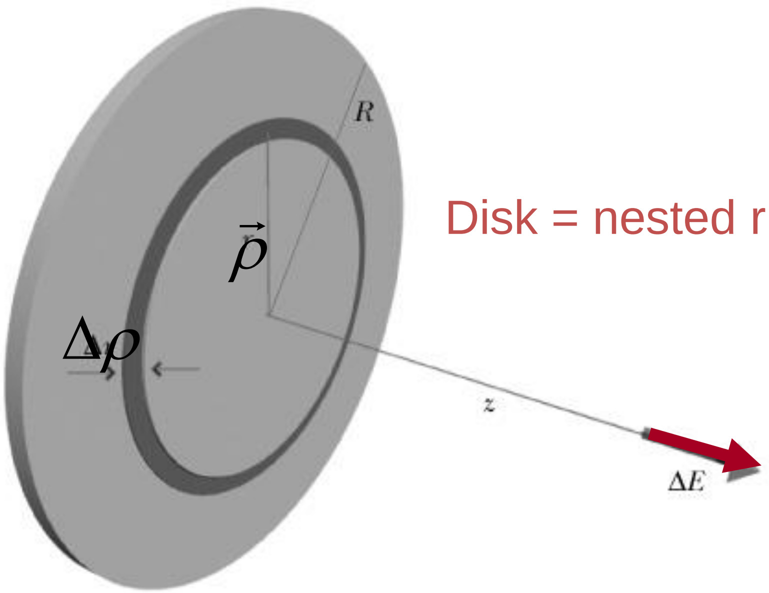


[redacted]

Surface Charge Density Example in Excruciating Detail:

Field of Disc

Electric Field of a Uniformly Charged Disk



Disk = nested rings

$$\Delta E_z = \frac{1}{4\pi\varepsilon_0}\frac{q_{ring} z_o}{\left(\rho^2 + z_o{}^2\right)^{3/2}}$$

where

$$q_{ring} = Q\frac{(\text{area of ring})}{(\text{area of disk})} = Q\frac{2\pi\rho\Delta\rho}{\pi R^2}$$

Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

Step 2: write an expression for ΔE

Step 3: Add up all ΔE's to get the total E

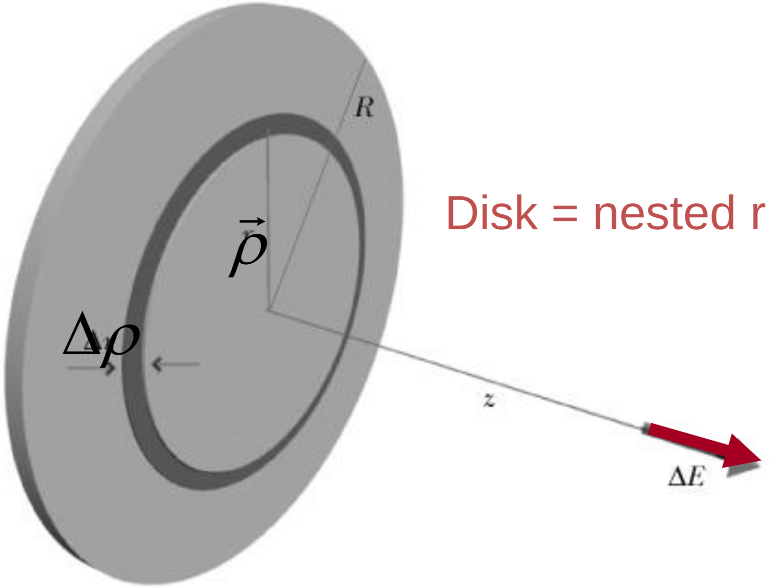
Step 4: Check results

so

$$\Delta E_z = \frac{1}{4\pi\varepsilon_0}\frac{\left(Q\dfrac{2\pi\rho\Delta\rho}{\pi R^2}\right)z}{\left(\rho^2 + z^2\right)^{3/2}}$$

$$\Delta E_z = \frac{1}{2\varepsilon_0}\frac{Q}{\pi R^2}\frac{z\rho\Delta\rho}{\left(\rho^2 + z^2\right)^{3/2}}$$

Electric Field of a Uniformly Charged Disk



Disk = nested rings

$$E_z = \sum_{disk} \Delta E_z$$

where

$$\Delta E_z = \frac{1}{2\varepsilon_0} \frac{Q}{\pi R^2} \frac{z_o \rho \Delta\rho}{\left(\rho^2 + z_o{}^2\right)^{3/2}}$$

so

$$E_z = \sum_{\rho=0}^{\rho=R} \frac{1}{2\varepsilon_0} \frac{Q}{\pi R^2} \frac{z_o \rho \Delta\rho}{\left(\rho^2 + z_o{}^2\right)^{3/2}}$$

$$E_z = \frac{1}{2\varepsilon_0} \frac{Q z_o}{\pi R^2} \int_{\rho=0}^{\rho=R} \frac{\rho d\rho}{\left(\rho^2 + z_o{}^2\right)^{3/2}}$$

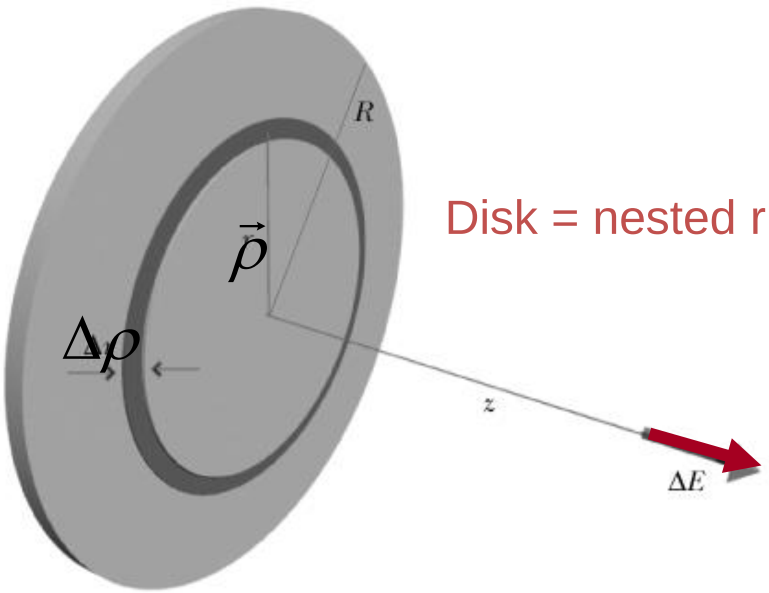
Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

Step 2: write an expression for ΔE

Step 3: Add up all ΔE's to get the total E

Step 4: Check results

Electric Field of a Uniformly Charged Disk



Disk = nested rings

$$E_z = \frac{1}{2\varepsilon_0}\frac{Qz_o}{\pi R^2}\int_{\rho=0}^{\rho=R}\frac{\rho d\rho}{\left(\rho^2+z_o^{\,2}\right)^{3/2}}$$

Change of variables

$$u \equiv \rho^2 + z_o^{\,2}$$

So limits become

$$u_{\min} = z_o^{\,2} \qquad u_{\max} = R^2 + z_o^{\,2}$$

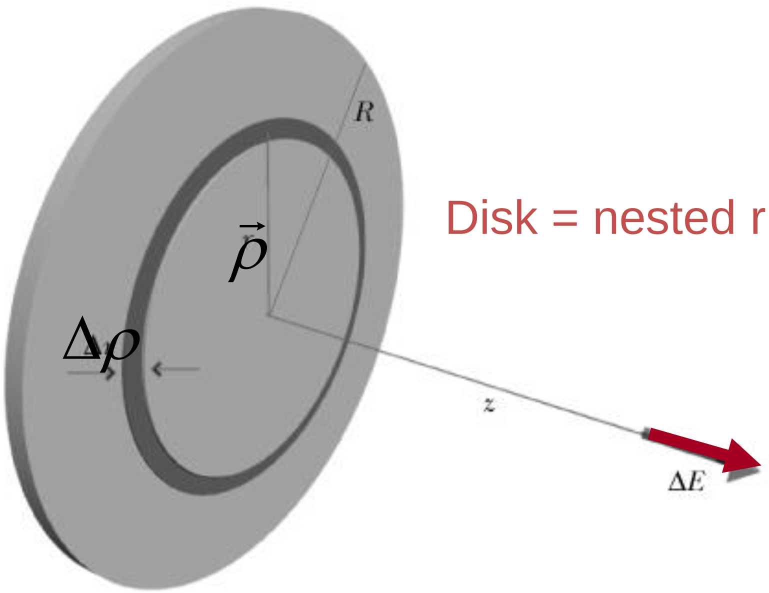
Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

Step 2: write an expression for ΔE

Step 3: Add up all ΔE's to get the total E

Step 4: Check results

Electric Field of a Uniformly Charged Disk



Disk = nested rings

$$E_z = \frac{1}{2\varepsilon_0}\frac{Qz_o}{\pi R^2}\int_{\rho=0}^{\rho=R}\frac{\rho d\rho}{\left(\rho^2+z_o{}^2\right)^{3/2}}$$

Change of variables

$$u \equiv \rho^2 + z_o{}^2$$

So limits become

$$u_{\min} = z_o{}^2 \qquad u_{\max} = R^2 + z_o{}^2$$

Differential bit becomes

$$du \equiv 2\rho d\rho \Rightarrow \rho d\rho = \tfrac{1}{2}du$$

Integral becomes

Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

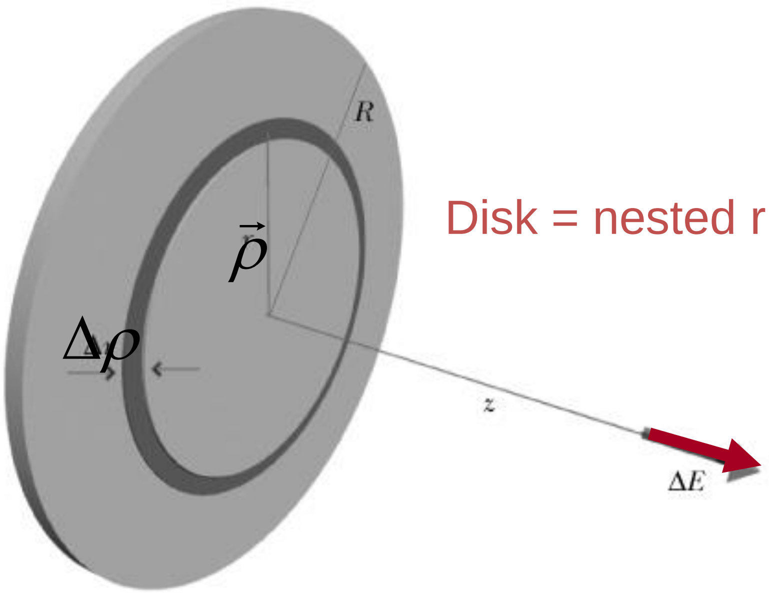
Step 2: write an expression for ΔE

Step 3: Add up all ΔE's to get the total E

Step 4: Check results

$$E_z = \frac{1}{4\varepsilon_0}\frac{Qz_o}{\pi R^2}\int_{u=z_o{}^2}^{u=R^2+z_o{}^2}\frac{du}{u^{3/2}} = \frac{1}{4\varepsilon_0}\frac{Qz_o}{\pi R^2}\left(\frac{-2}{u^{1/2}}\right)\Bigg|_{u=z_o{}^2}^{u=R^2+z_o{}^2}$$

Electric Field of a Uniformly Charged Disk



Disk = nested rings

$$E_z = \frac{1}{2\varepsilon_0}\frac{Qz_o}{\pi R^2}\left(\frac{1}{z_o} - \frac{1}{\left(R^2 + z_o{}^2\right)^{1/2}}\right)$$

Units?

Logic?

Limits?

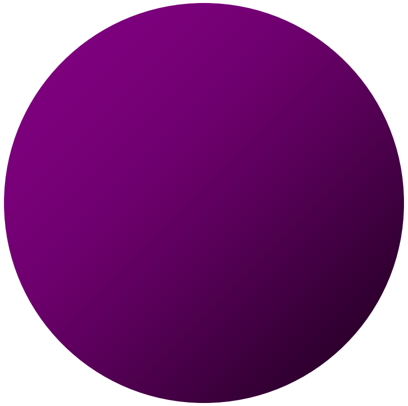
Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

Step 2: write an expression for ΔE

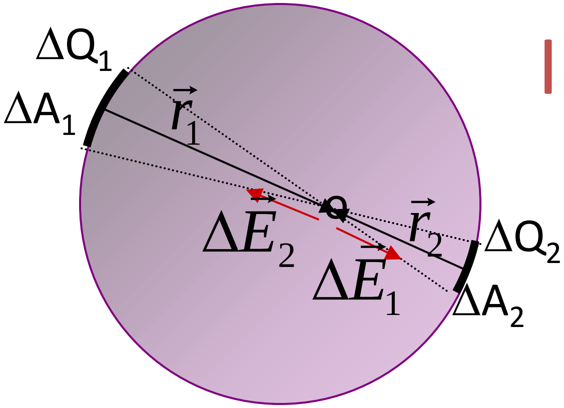
Step 3: Add up all ΔE's to get the total E

Step 4: Check results

Electric Field of a Uniformly Charged Spherical Shell



Electric Field of a Uniformly Charged Spherical Shell

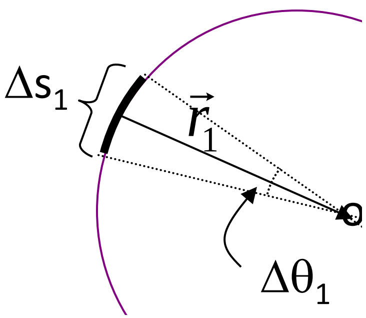


Inside

$$|\Delta E_1| = \tfrac{1}{4\pi\varepsilon_o}\left|\frac{\Delta Q_1}{r_1^{\,2}}\right|$$

where

$$\frac{\Delta Q_1}{Q} = \frac{\Delta A_1}{A} \Rightarrow \Delta Q_1 = Q\,\frac{\Delta A_1}{A}$$



where

$$\Delta A_1 = \pi\left({}^{s_1}\!/_2\right)^2$$

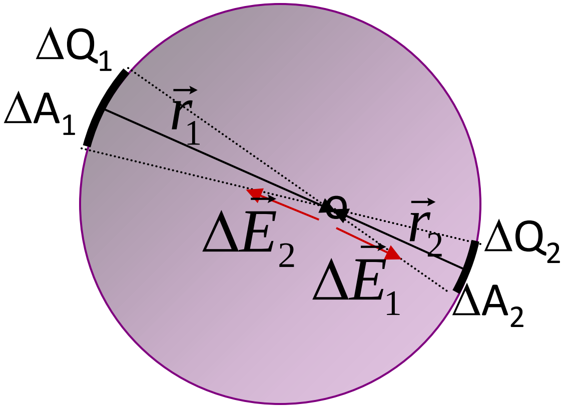
where

$$s = r_1 \Delta\theta_1$$

so

$$|\Delta E_1| = \tfrac{1}{4\pi\varepsilon_o}\left|\frac{\left(Q\,\frac{\pi\left({}^{r_1\Delta\theta_1}\!/_2\right)^2}{A}\right)}{r_1^{\,2}}\right| = \tfrac{1}{4\pi\varepsilon_o}\left|\frac{Q\pi\left(\Delta\theta_1\right)^2}{4A}\right|$$

Electric Field of a Uniformly Charged Spherical Shell

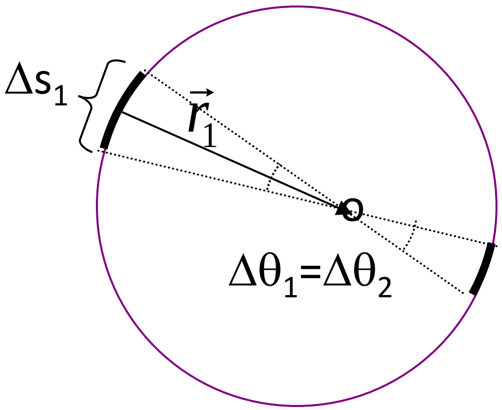


Inside $E_{shell} = 0$

$$|\Delta E_1| = \frac{1}{4\pi\varepsilon_o}\left|\frac{Q\pi(\Delta\theta_1)^2}{4A}\right|$$

Ditto for ΔE_2

$$|\Delta E_2| = \frac{1}{4\pi\varepsilon_o}\left|\frac{Q\pi(\Delta\theta_2)^2}{4A}\right|$$



but

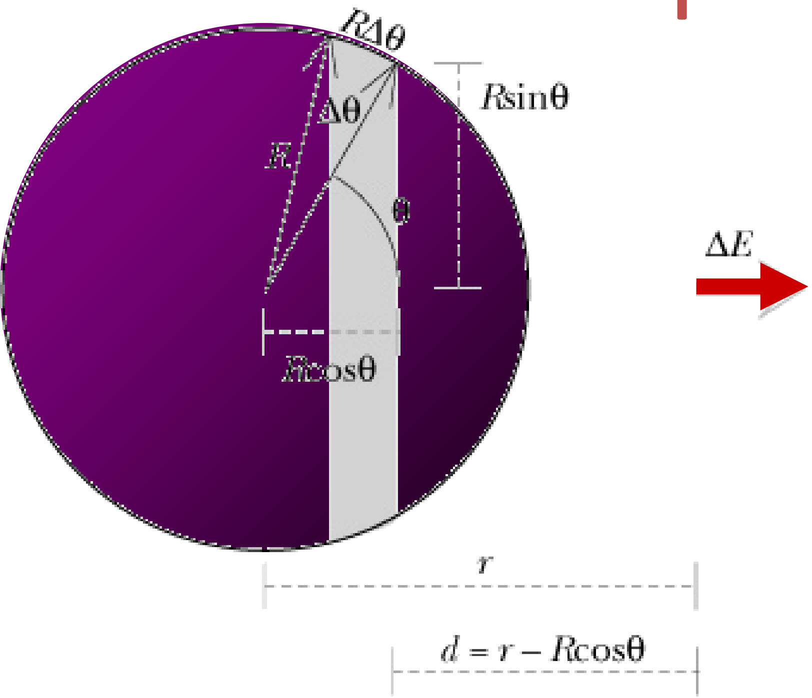
$$\Delta\theta_1 = \Delta\theta_2$$

so

$$|\Delta E_2| = \frac{1}{4\pi\varepsilon_o}\left|\frac{Q\pi(\Delta\theta_1)^2}{4A}\right| = |\Delta E_1|$$

Thus, the two are not just opposite direction, but also equal magnitude, so they cancel. This is true for ALL pairs of patches of the surface – they ALL CANCEL.

Electric Field of a Uniformly Charged Spherical Shell



Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

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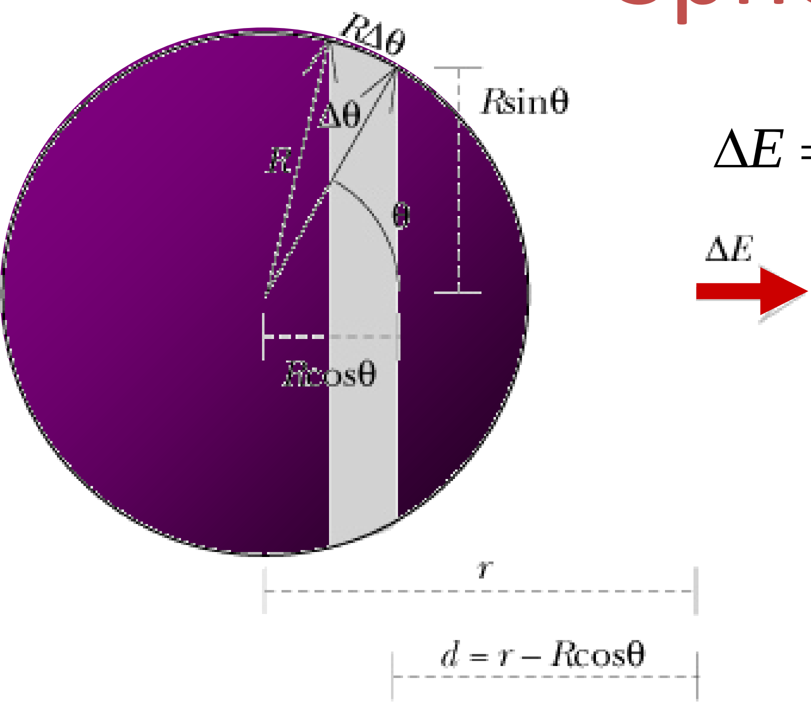
Step 4: Check results

Electric Field of a Uniformly Charged Spherical Shell



$$\Delta E = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q d}{\left[(R\sin\theta)^2 + d^2\right]^{3/2}}$$

For a ring

where

$$\Delta Q = Q\frac{(\text{area of ring})}{(\text{area of sphere})} = Q\frac{2\pi R^2 \sin\theta\Delta\theta}{4\pi R^2}$$

$$\Delta Q = \frac{Q\sin\theta}{2}\Delta\theta$$

and

$$d = r - R\cos\theta$$

Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

Step 2: write an expression for ΔE

Step 3: Add up all ΔE's to get the total E

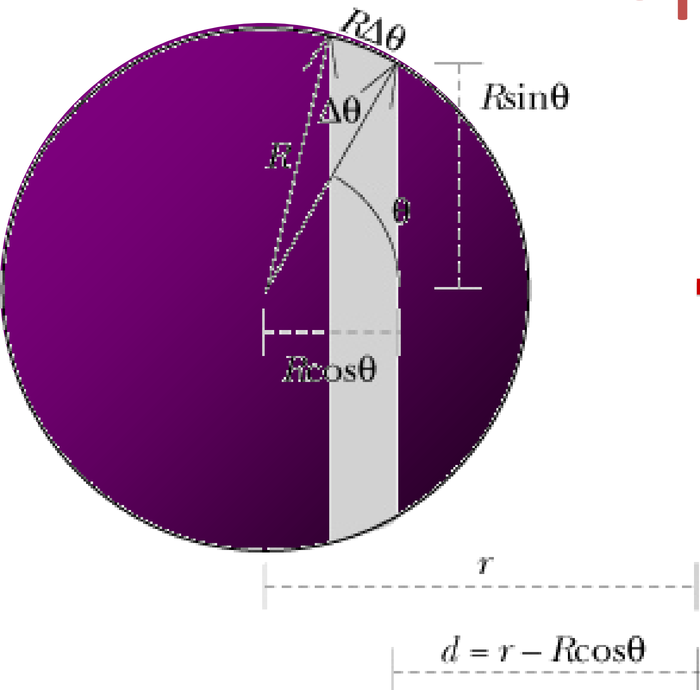
Step 4: Check results

so

$$\Delta E = \frac{1}{4\pi\varepsilon_0}\frac{(r - R\cos\theta)}{\left[(R\sin\theta)^2 + (r - R\cos\theta)^2\right]^{3/2}}\frac{Q\sin\theta}{2}\Delta\theta$$

$$\Delta E = \frac{1}{4\pi\varepsilon_0}\frac{(r - R\cos\theta)}{\left[R^2 + r^2 - 2Rr\cos\theta\right]^{3/2}}\frac{Q\sin\theta}{2}\Delta\theta$$

Electric Field of a Uniformly Charged Spherical Shell



ΔE

$$E = \sum_{sphere} \Delta E$$

where

$$\Delta E = \frac{1}{4\pi\varepsilon_0} \frac{(r - R\cos\theta)}{\left[R^2 + r^2 - 2Rr\cos\theta\right]^{3/2}} \frac{Q\sin\theta}{2} \Delta\theta$$

so

$$E = \sum_{\theta=0}^{\theta=\pi} \frac{1}{4\pi\varepsilon_0} \frac{(r - R\cos\theta)}{\left[R^2 + r^2 - 2Rr\cos\theta\right]^{3/2}} \frac{Q\sin\theta}{2} \Delta\theta$$

so

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{2} \int_0^{\pi} \frac{(r - R\cos\theta)}{\left[R^2 + r^2 - 2Rr\cos\theta\right]^{3/2}} \sin\theta \, d\theta$$

Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

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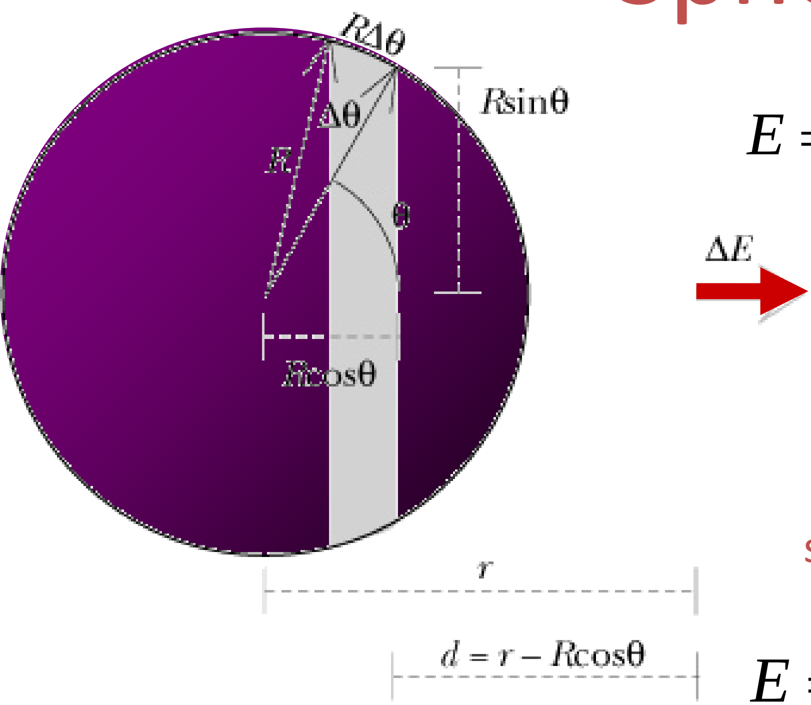
Electric Field of a Uniformly Charged Spherical Shell



$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2}\int_0^{\pi}\frac{(r - R\cos\theta)}{\left[R^2 + r^2 - 2Rr\cos\theta\right]^{3/2}}\sin\theta\, d\theta$$

Change of variables

$$u \equiv \cos\theta \qquad du/d\theta = -\sin\theta$$

$$\theta = 0 \rightarrow u = 1 \qquad \theta = \pi \rightarrow u = -1$$

so

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2}\int_1^{-1}\frac{(r - Ru)}{\left[R^2 + r^2 - 2Rru\right]^{3/2}}\, du$$

Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

so

Step 2: write an expression for ΔE

Step 3: Add up all ΔE's to get the total E

Step 4: Check results

Note:

$$(r - Ru) = \frac{(R^2 + r^2 - 2Rru) - (R^2 - r^2)}{2r}$$

Can thus simplify integrand