Fri.	(C 21.6-7,.9) 5.3.3-.3.4 Applications of Ampere's Law	
Mon. Wed. Thurs. Fri.	1.6, 5.4.1-.4.2 Magnetic Vector Potential 5.4.3 Multipole Expansion of the Vector Potential Review	 HW8

Biot-Savart Law

$$\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi} \int \frac{\vec{J} \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$$

It Follows that

$$\vec{\nabla}_r \cdot \vec{B}(\vec{r}) = 0$$

or equivalently

$$\oint \vec{B}(\vec{r}) \cdot d\vec{a} = 0$$

Ampere's

$$\vec{\nabla}_r \times \vec{B}(\vec{r}) = \mu_o \vec{J}(\vec{r})$$

or equivalently

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I_{enc}$$

Shortcut to finding field if symmetry is right

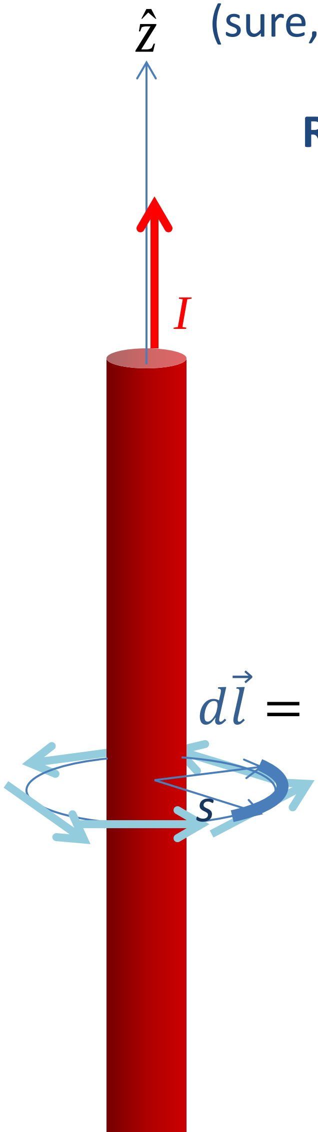
What these integrals do and don't say –
Over *whole* integral, not isolated segment

Using Ampere's Law

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I_{enc}$$

Simple Example: 'very long', straight wire of uniform current

(sure, we already know the answer, but just to see how it's done)



Reason direction from symmetry and known laws

Must be rotationally (about z-axis) symmetric and transnationally (along z-axis)symmetric

That and $\int \vec{B}(\vec{r}) \cdot d\vec{a} = 0$ means not radially in or out.

That and $\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi} \int \frac{\vec{J} \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$ means no z (parallel to current) component.

Leaves ϕ component. Right-hand rule determines direction.

$d\vec{l} = s d\varphi \hat{\varphi}$

Select Loop accordingly

Most convenient if magnitude and relative direction of B is constant over loop

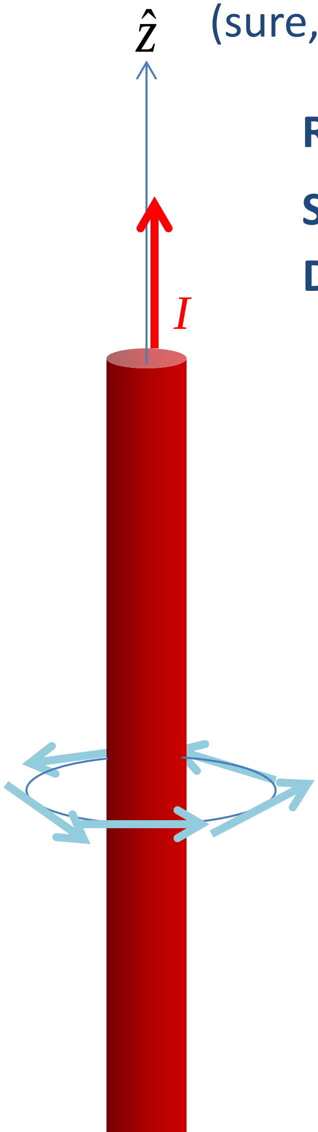
Do math

Using Ampere's Law

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I_{enc}$$

Simple Example: 'very long', straight wire of uniform current

(sure, we already know the answer, but just to see how it's done)



Reason direction $\hat{B} = \hat{\phi}$

Select Loop accordingly $d\vec{l} = s\,d\varphi\,\hat{\varphi}$

Do math

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I_{enc}$$

$$\oint B(\vec{r})\hat{\varphi} \cdot s\,d\varphi\,\hat{\varphi} = \mu_o I$$

$$\oint B(\vec{r})\,s\,d\varphi = \mu_o I$$

$$B(\vec{r}) \oint s\,d\varphi = \mu_o I$$

$$B(\vec{r})\,2\pi s = \mu_o I$$

$$B(\vec{r}) = \frac{\mu_o I}{2\pi s}$$

$$\vec{B}(\vec{r}) = \frac{\mu_o I}{2\pi s}\hat{\varphi}$$

Using Ampere's Law

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I_{enc}$$

Simple Exercise: 'very long', straight cylinder of uniform *surface* current at radius *R*.

What's B inside and outside?

Reason direction

Select Loop accordingly

Do math

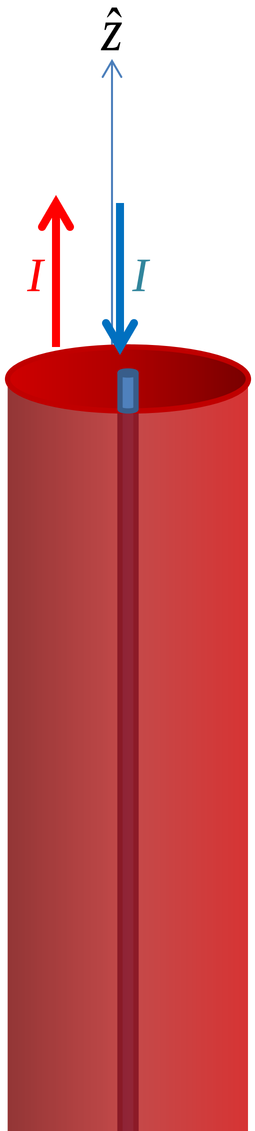
[redacted]

Using Ampere's Law

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I_{enc}$$

Exercise: 'very long', straight co-axial cable: cylinder and wire of opposite current

What's B inside and outside?



Reason direction

Select Loop accordingly

Do math

[redacted]

Using Ampere's Law

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I$$

Exercise: 'very long' cylinder with $\vec{J}(\vec{r}\,') = J_o \frac{s'}{R} \hat{z}$

What's B inside and outside?

Reason direction

Select Loop accordingly

Do math

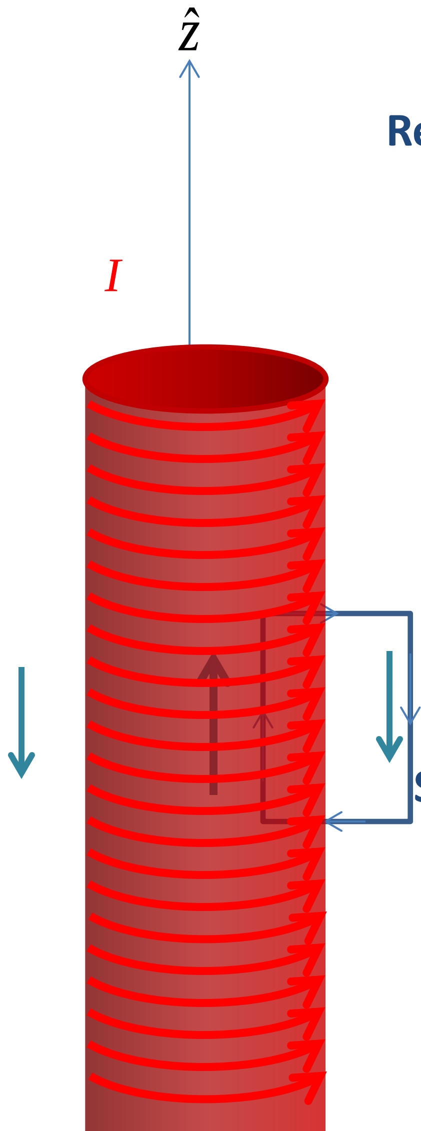
[redacted]

Using Ampere's Law

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I_{enc}$$

Example: 'very long' solenoid with $\vec{K} = I\frac{N}{L}\hat{\varphi}$

What's B inside and outside?



Reason direction

Must be rotationally (about z-axis) symmetric and transnationally (along z-axis)symmetric

That and $\int \vec{B}(\vec{r}) \cdot d\vec{a} = 0$ means no radial component.

That and $\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi}\int \frac{\vec{J}\times\hat{\mathscr{r}}}{\mathscr{r}^2}d\tau'$ means no ϕ (parallel to current) component.

Leaves *z* component. Right-hand rule determines direction.

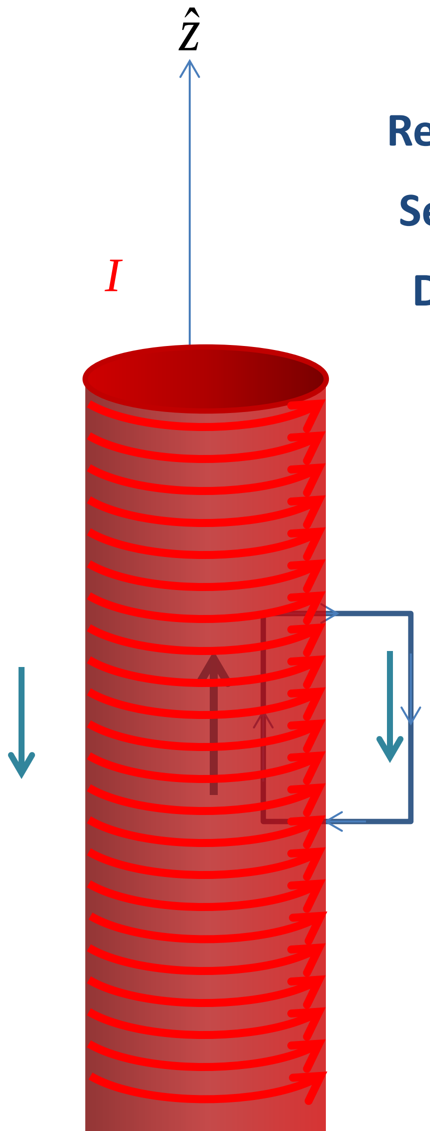
Select Loop accordingly

Do math

Using Ampere's Law

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I_{enc}$$

Example: 'very long' solenoid with $\vec{K} = I\frac{N}{L}\hat{\varphi}$



What's B inside and outside?

Reason direction $\hat{B} = \hat{z}$ inside $\hat{B} = -\hat{z}$ outside.

Select Loop accordingly

Do math $\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I$

$$\int_{inside} \vec{B}(\vec{r}) \cdot d\vec{z} + \int_{top} \vec{B}(\vec{r}) \cdot d\vec{s} + \int_{outside} \vec{B}(\vec{r}) \cdot d\vec{z} + \int_{bottom} \vec{B}(\vec{r}) \cdot d\vec{s}$$

perpendicular

$$B_{in}(\vec{r})\Delta z + B_{out}(\vec{r})\Delta z = \mu_o \int K dz = \mu_o \int I\frac{N}{L} dz = \mu_o I\frac{N}{L}\Delta z$$

$$B_{in}(\vec{r}) + B_{out}(\vec{r}) = \mu_o I\frac{N}{L} \qquad \vec{B}_{in} = \mu_o I\frac{N}{L}\hat{z}$$

Since answer is independent of distance in or out, B_{in} and B_{out} must be constants. B_{out} *just* outside must be same as B_{out} *infinitely* far away. The only B_{out} that a finite object will have at infinity is 0.

Using Ampere's Law

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I_{enc}$$

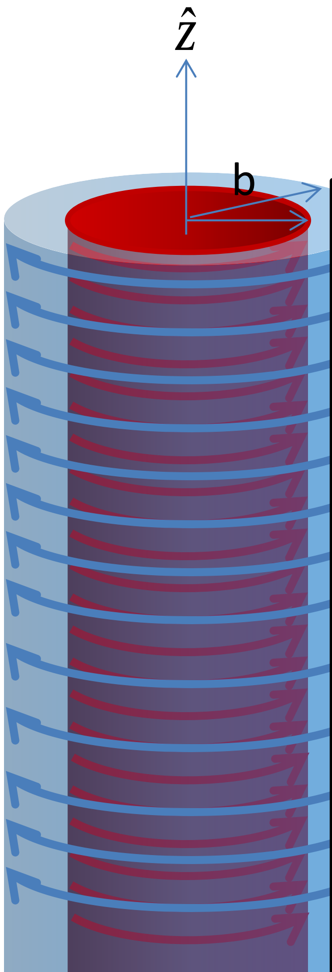
Exercise: co-axial solenoids of opposite current direction

What's B in (s<a), in between (a<s<b), and out (b<s)?

Reason direction

Select Loop accordingly

Do math



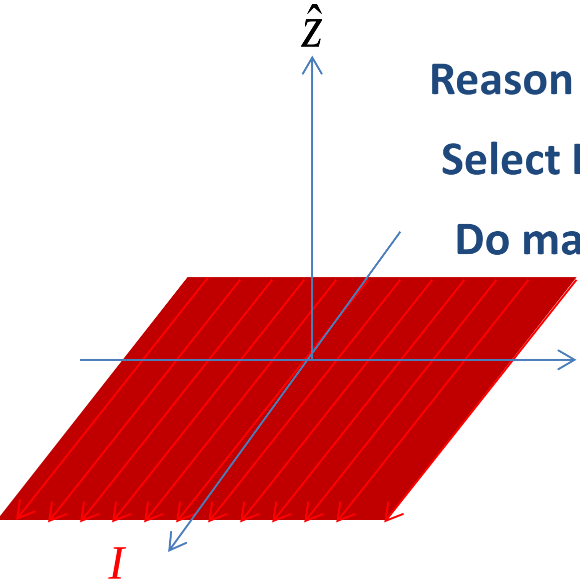
[redacted]

Using Ampere's Law

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I_{enc}$$

Example: 'very long' sheet with $\vec{K} = I\frac{N}{L}\hat{x}$

What's B above and below?



Reason direction

Select Loop accordingly

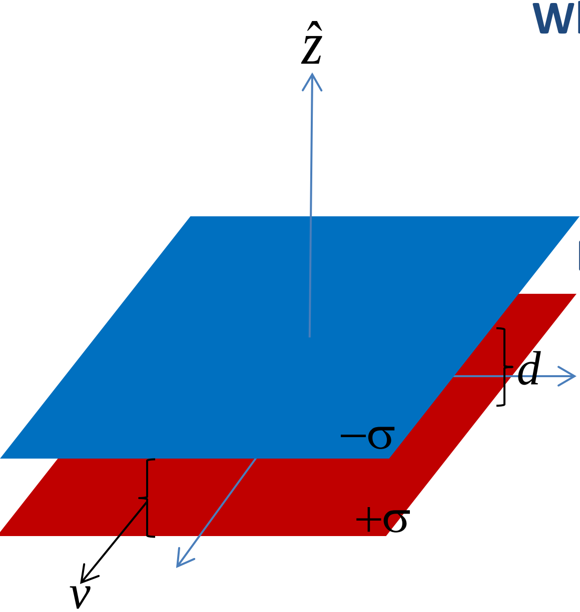
Do math

Using Ampere's Law

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I_{enc}$$

Exercise: parallel plate capacitor with charge densities $+\sigma$ and $-\sigma$, moving with speed v in x direction

What's B above, between, and below?



Reason direction [redacted]

Select Loop accord[redacted]

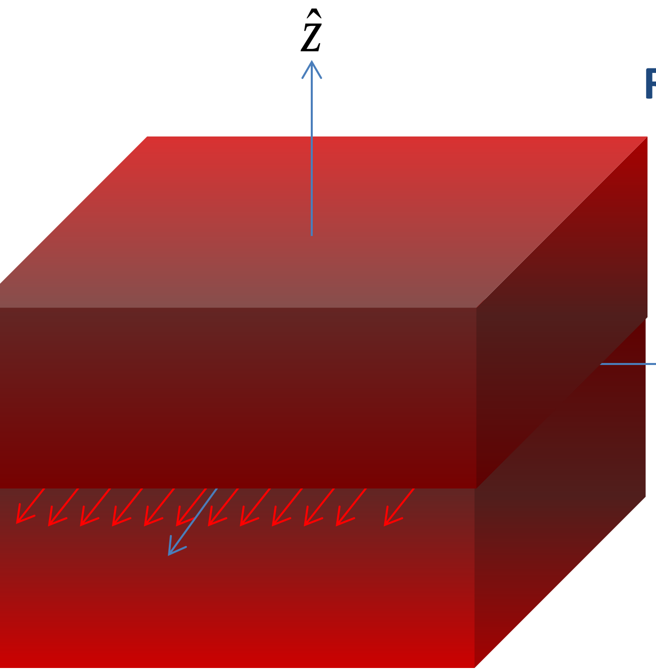
Do math [redacted]

Using Ampere's Law

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I_{enc}$$

Exercise: 'very long' sheet with $\vec{J} = J_o|z'|\hat{x}$

What's B above and below?



Reason direction

Select Loop accordingly

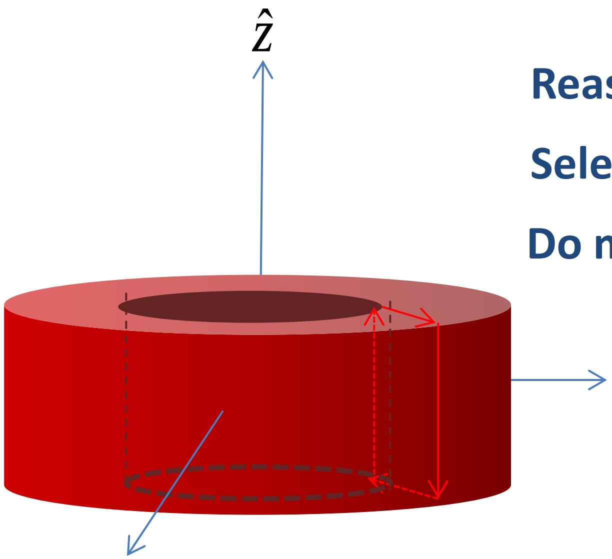
Do math

Using Ampere's Law

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I_{enc}$$

Example: torus

What's B above and below?

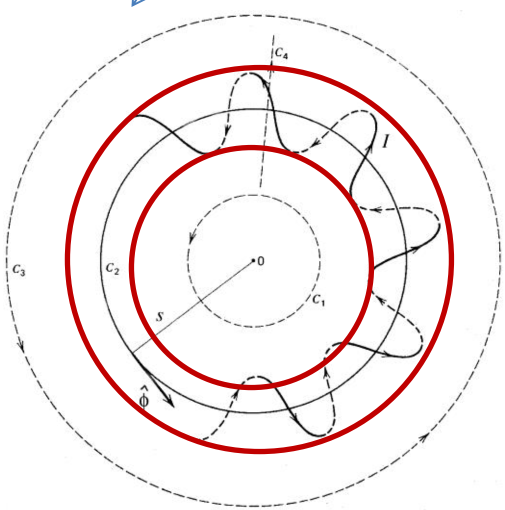


Reason direction

Select Loop accordingly

Do math

[redacted]



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Using Ampere's Law

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I_{enc}$$

Examples