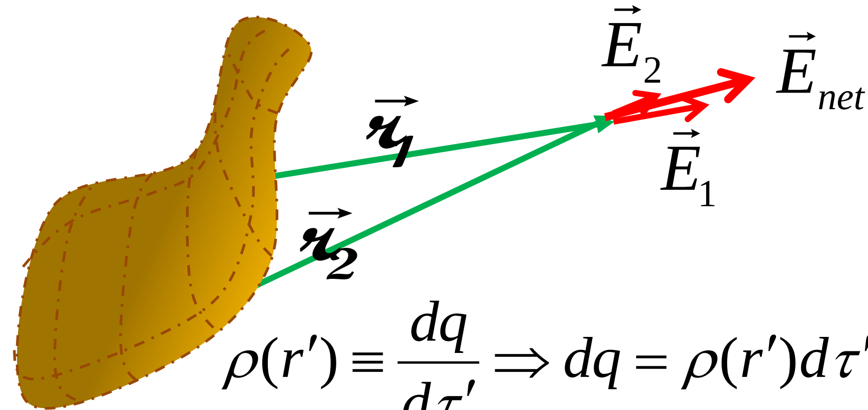
Wed. Fri.	(C 21.6-7,.9) 1.3.4-1.3.5, 1.5.2-1.5.3, 5.3.1-.3.2 Div & Curl B (C 21.6-7,.9) 5.3.3-.3.4 Applications of Ampere's Law	
Mon. Wed. Thurs. Fri.	1.6, 5.4.1-.4.2 Magnetic Vector Potential 5.4.3 Multipole Expansion of the Vector Potential Review	 HW8

Memory Lane: Electrostatics

Field of charge distributions

$$\vec{E}_1(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{\mathscr{r}_1^{\,2}}\hat{\mathscr{r}}$$

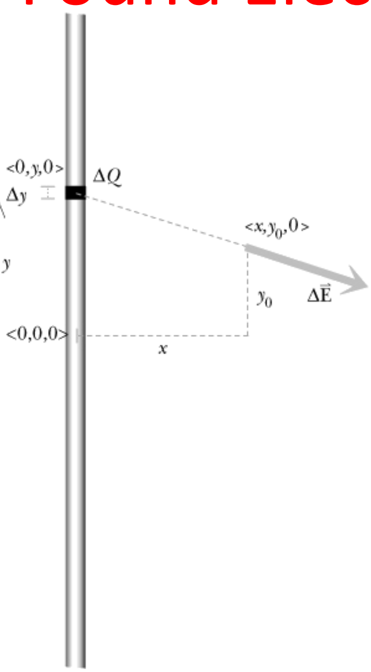


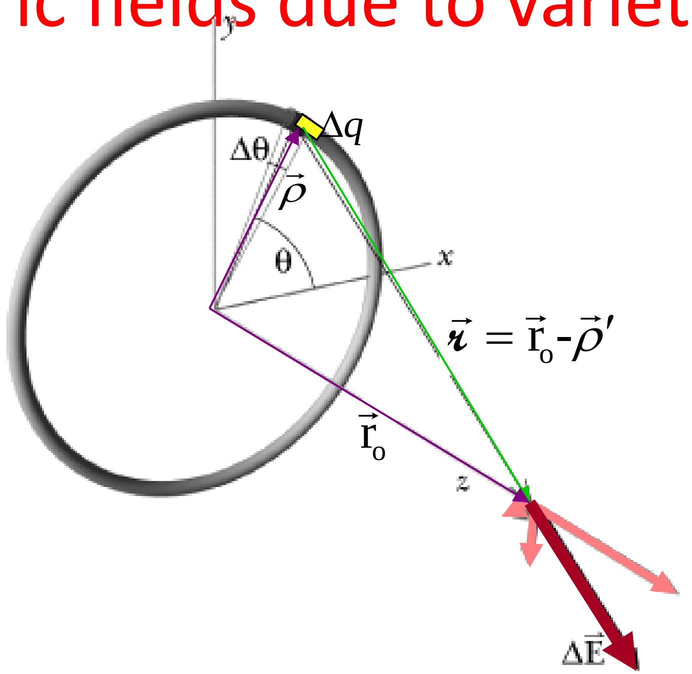
$$\rho(r') \equiv \frac{dq}{d\tau'} \Rightarrow dq = \rho(r')d\tau'$$

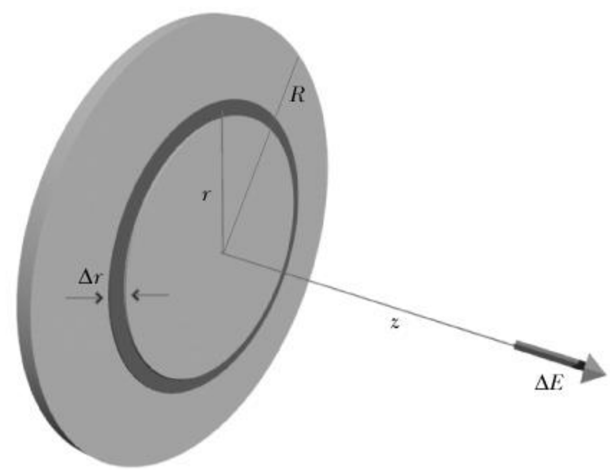
$$\vec{E}(\vec{r})_{net} = \sum_{i=1} \frac{1}{4\pi\varepsilon_o}\frac{q_i}{\mathscr{r}_i^{\,2}}\hat{\mathscr{r}}_i \xrightarrow[\lim q \to dq]{} \int_{charge} \frac{1}{4\pi\varepsilon_o}\frac{\hat{\mathscr{r}}}{\mathscr{r}_i^{\,2}}dq$$

$$\vec{E}(\vec{r}) = \int \frac{1}{4\pi\varepsilon_o}\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\rho(\vec{r}')d\tau'$$

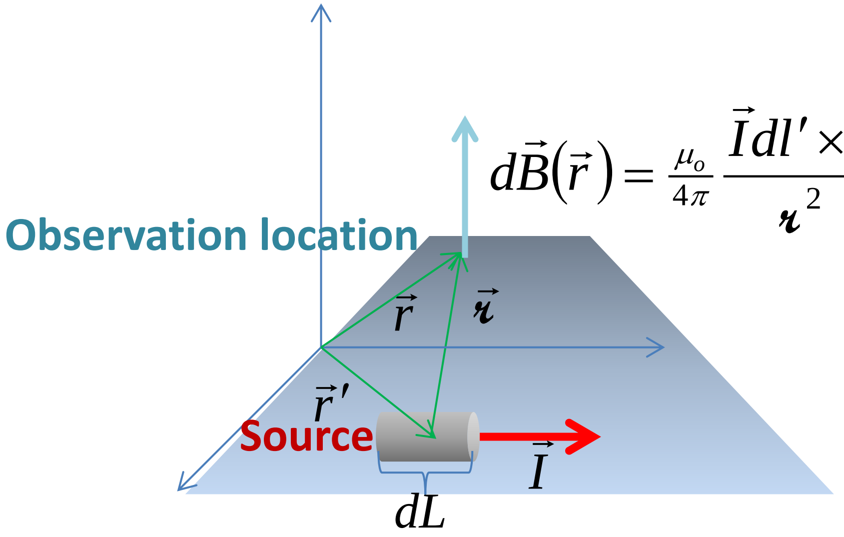
Found Electric fields due to variety of charge distributions







Biot-Savart Law

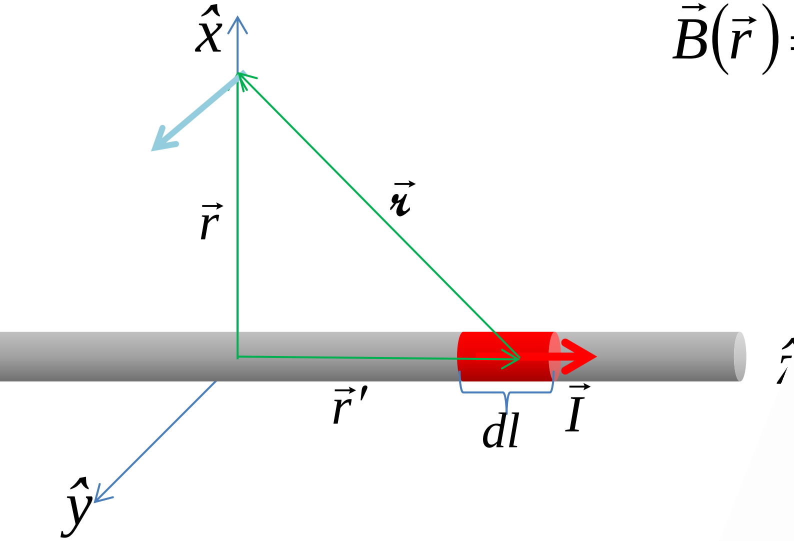


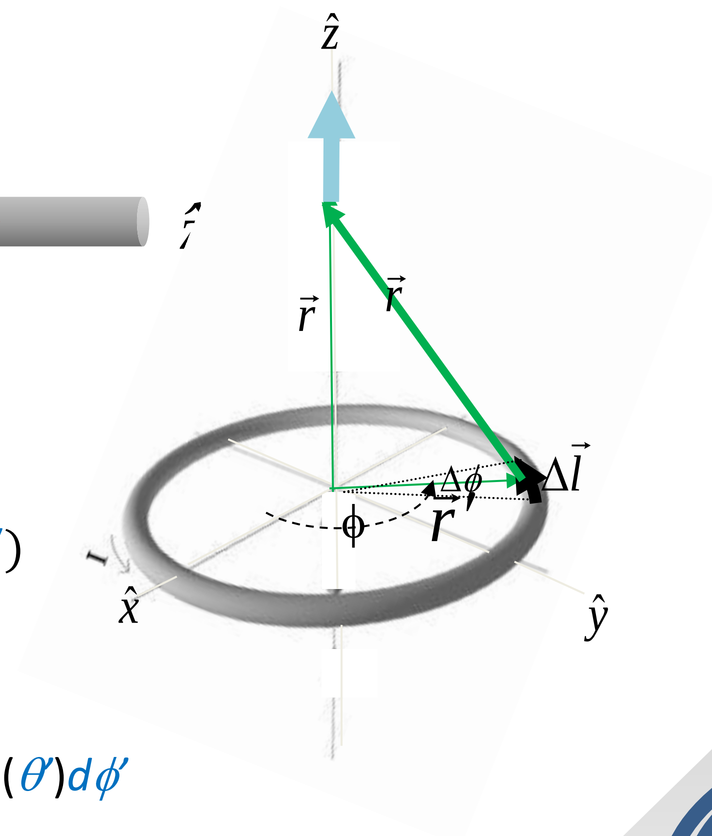
$$d\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{\vec{I}dl' \times \hat{\boldsymbol{\mathscr{r}}}}{\boldsymbol{\mathscr{r}}^2}$$

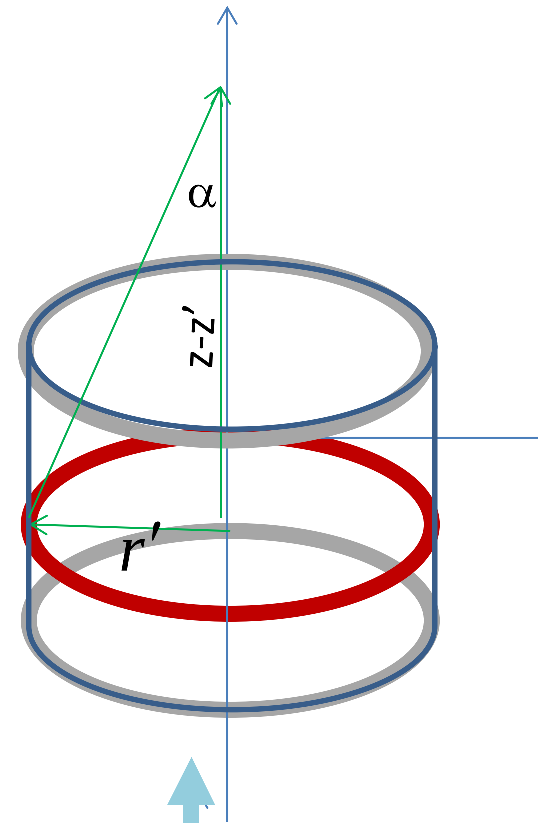
Magneto-statics

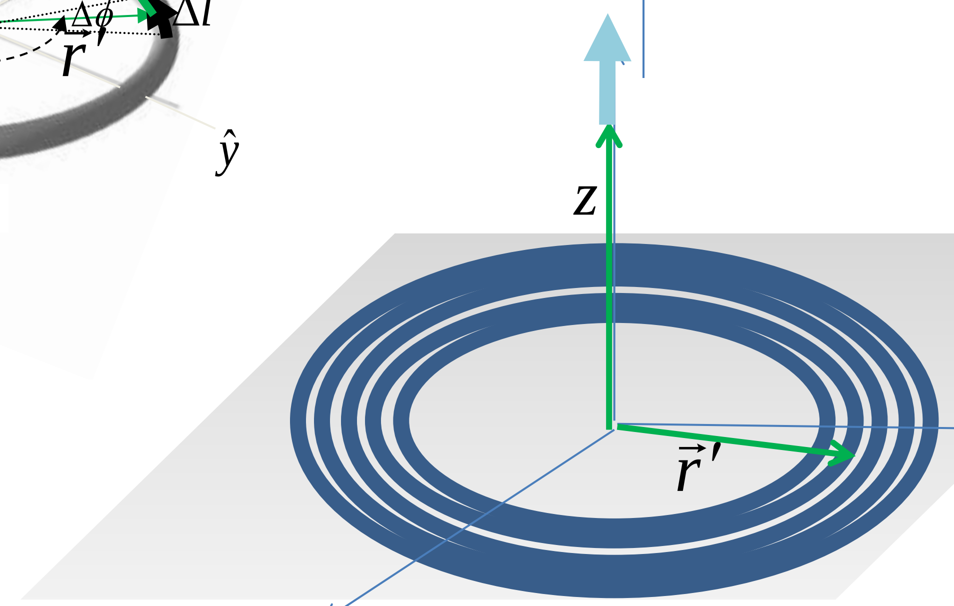
Fields of current distributions

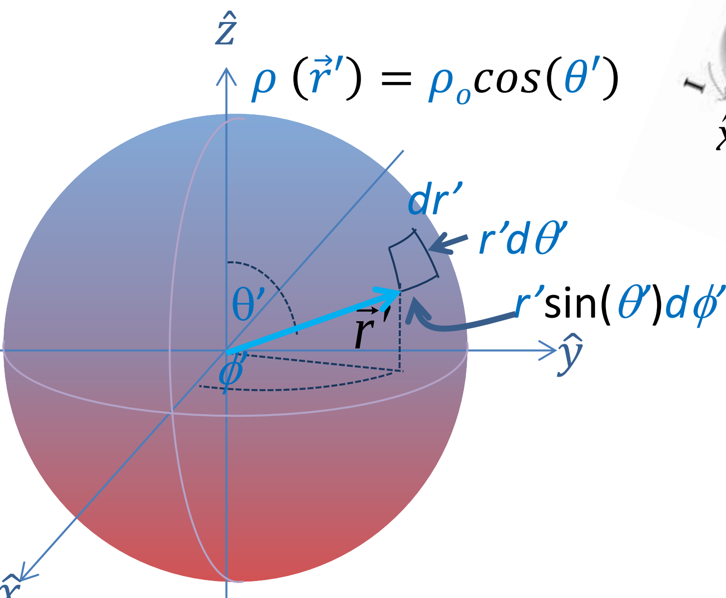
$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{I d\vec{\ell}' \times \hat{\boldsymbol{\mathscr{r}}}}{\boldsymbol{\mathscr{r}}^2}$$





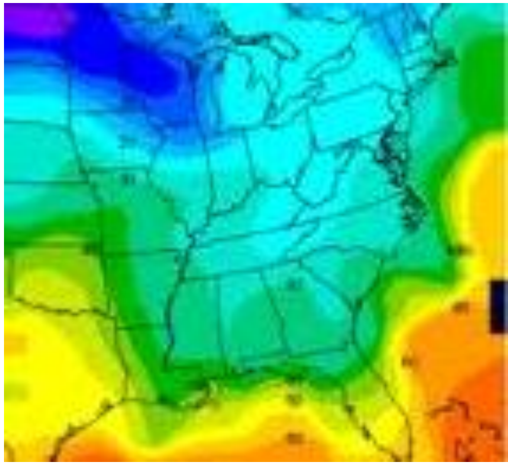






Del Operator

$$\vec{\nabla} \equiv \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$



Gradient – vector representing the local slope of a scalar field.

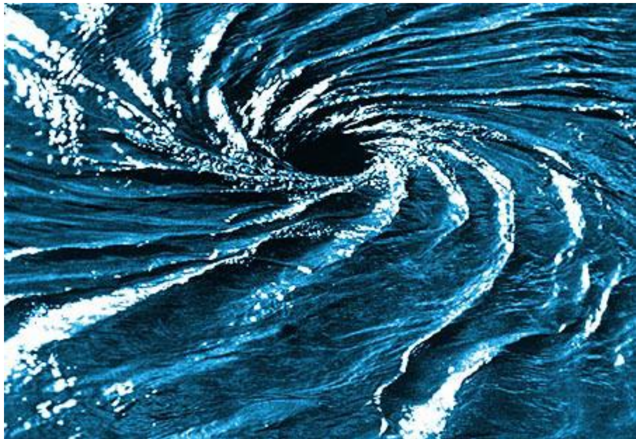
$$\vec{\nabla} T = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$$

Divergence – scalar representing in/out flow from a point in a vector field.

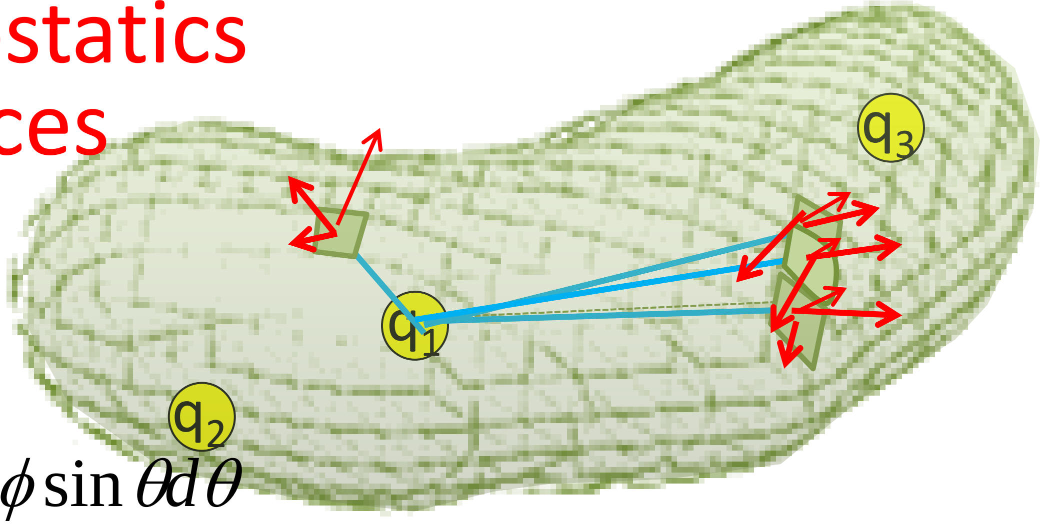
$$\vec{\nabla} \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl – vector representing circulation of a vector field.

$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_x & v_y & v_z \end{vmatrix} = \hat{x}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \ldots$$



Memory Lane: Electrostatics
Flux from Charge Sources



$$\oint \vec{E}_1 \cdot d\vec{a} = \Phi_{E1}$$

$$\Phi_{E1} = \oint E_1 da_{||}$$

$$\Phi_{E1} = \int_{\phi=-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\theta=0}^{2\pi} \frac{q_1}{4\pi\varepsilon_o \mathscr{r}^2} \mathscr{r}^2 d\phi \sin\theta d\theta$$

$$\Phi_{E1} = \frac{q_1}{4\pi\varepsilon_o} 4\pi$$

$$\oint \vec{E}_1 \cdot d\vec{a} = \Phi_{E1} = \frac{q_1}{\varepsilon_o}$$

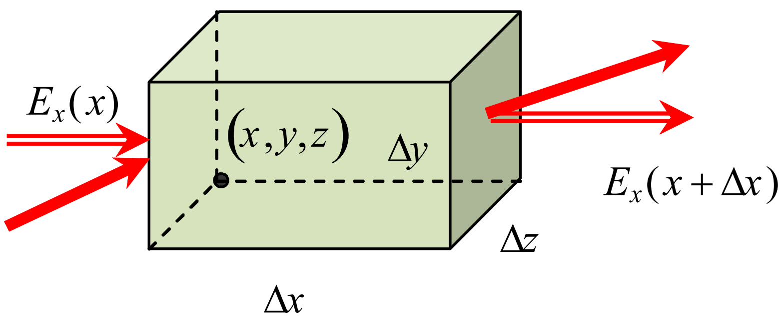
Ditto for q_2, q_3, …

$$\operatorname{div}\left(\vec{E}\right) \equiv \lim_{Vol\to 0} \frac{\Phi_E}{\mathrm{Vol}} = \lim_{Vol\to 0} \frac{1}{\varepsilon_o} \frac{Q_{encl}}{Vol}$$

$$\operatorname{div}\left(\vec{E}\right) = \lim_{Vol\to 0} \frac{\oint \vec{E} \cdot d\vec{a}}{Vol} = \frac{1}{\varepsilon_o}\rho$$

$$\oint \vec{E}_{net} \cdot d\vec{a} = \frac{Q_{net.enclosed}}{\varepsilon_o}$$ Simplified symmetric problems $$\vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_o}\rho$$

(integral form) Gauss's Law (derivative form)



Pause & Put together Gauss's Th'm

$$\oint \vec{E} \cdot d\vec{l} = 0$$

Motivated scalar potential $\Delta V = -\int \vec{E} \cdot d\vec{l}$

$$\vec{\nabla} \times \vec{E} = 0 \qquad -\vec{\nabla} V = \vec{E}$$

Gauss's Theorem – explicitly putting it together

$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enclosed}}{\varepsilon_o}$ but $Q_{encl} = \int \rho \mathrm{d}\tau$ And we'd gone off and proven $\vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_o} \rho$

Putting these together:

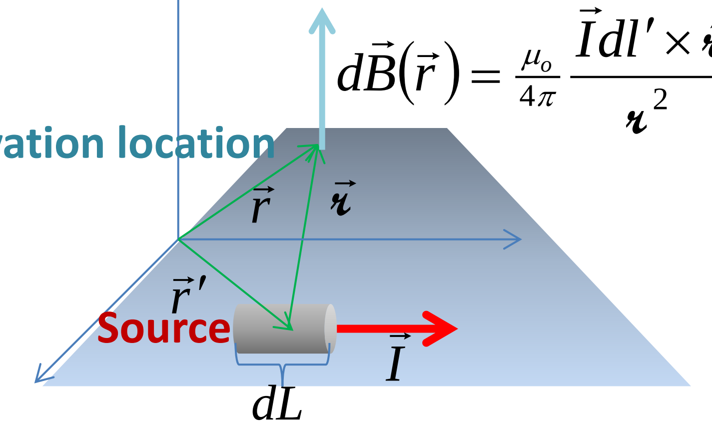
$$\oint \vec{E} \cdot d\vec{a} = \int \frac{\rho}{\varepsilon_o} d\tau$$

$$\oint \vec{E} \cdot d\vec{a} = \int \vec{\nabla} \cdot \vec{E} d\tau$$

Though we were thinking specifically about electric field while we did the math that got us to this relation, it's quite general and true for any vector field. So, as expressed in Ch. 1, for generic function F,

$$\oint \vec{F} \cdot d\vec{a} = \int \vec{\nabla} \cdot \vec{F} d\tau$$

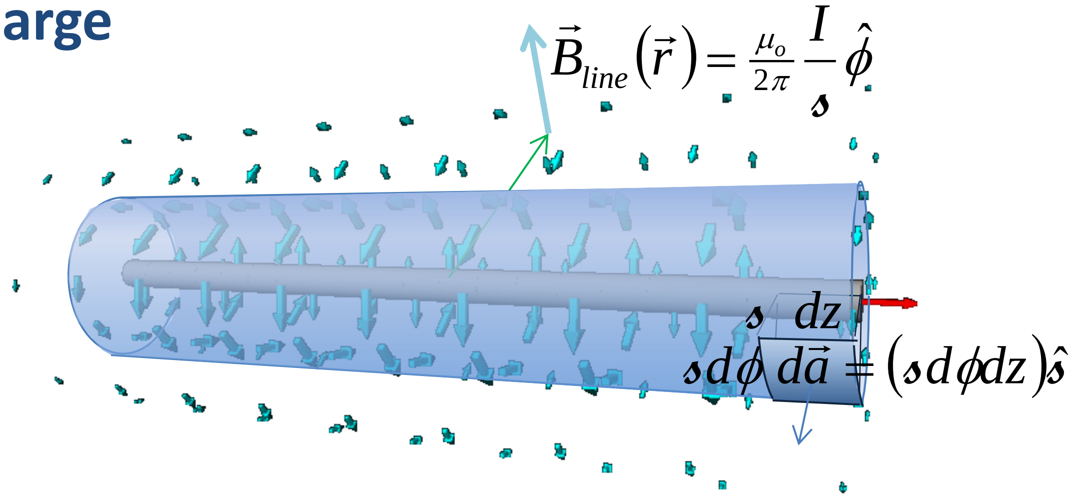
Magneto-Statics
Flux from Current Sources



$$\oint \left(d\vec{B}(\vec{r})\right) \cdot d\vec{a} = d\Phi_B = \oint \left(\frac{\mu_o}{4\pi}\frac{\vec{I}dl' \times \hat{\mathscr{r}}}{\mathscr{r}^2}\right) \cdot d\vec{a}$$

Not so easy; start with special case to *suggest / understand* general result

infinite line charge



Surface of any radius would work

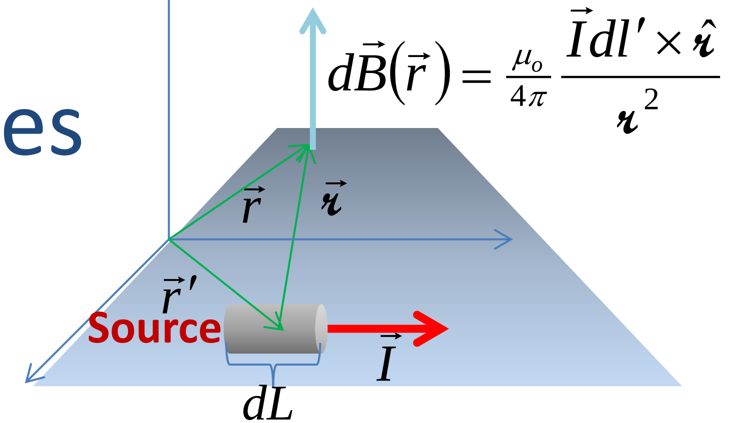
$$\oint \vec{B}(\vec{r})_{line} \cdot d\vec{a} = \Phi_B = \oint \left(\frac{\mu_o}{2\pi}\frac{I}{\mathscr{s}}\hat{\phi}\right) \cdot d\vec{a} = \oint \left(\frac{\mu_o}{2\pi}\frac{I}{\mathscr{s}}\hat{\phi}\right) \cdot (\mathscr{s}d\phi dz)\hat{\mathscr{s}} = \oint \left(\frac{\mu_o}{2\pi} I d\phi dz\right)\left(\hat{\phi} \cdot \hat{\mathscr{s}}\right) = 0$$

Similarly,

$$\vec{\nabla} \cdot \vec{B}_{line}(\vec{r}) = \frac{1}{s}\frac{\partial}{\partial s}(sB_s) + \frac{1}{s}\frac{\partial}{\partial \phi}(B_\phi) + \frac{\partial}{\partial z}(B_z) = \frac{1}{s}\frac{\partial}{\partial s}(s0) + \frac{1}{s}\frac{\partial}{\partial \phi}\left(\frac{\mu_o}{2\pi}\frac{I}{\mathscr{s}}\right) + \frac{\partial}{\partial z}(0) = 0$$

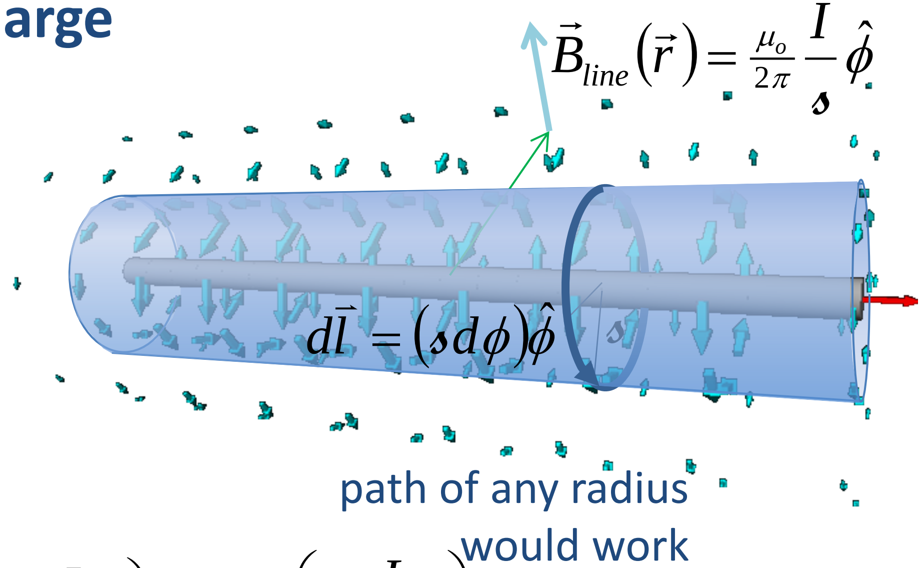
Magneto-Statics
Circulation from Current Sources

$$\oint \left(d\vec{B}(\vec{r})\right)\cdot d\vec{l} = \oint \left(\frac{\mu_o}{4\pi} \frac{\vec{I}dl' \times \hat{\mathscr{r}}}{\mathscr{r}^2} \right)\cdot d\vec{l}$$



Not so easy; start with special case to *suggest / understand* general result

infinite line charge



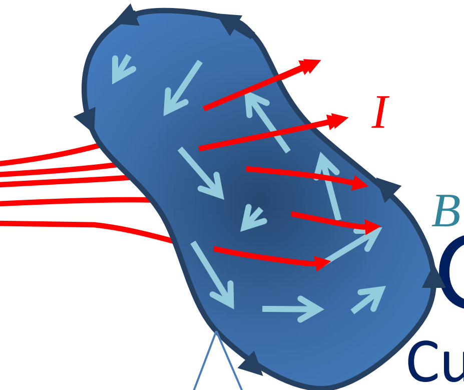
$$\oint \vec{B}(\vec{r})_{line} \cdot d\vec{a} = 0$$

$$\vec{\nabla} \cdot \vec{B}_{line}(\vec{r}) = 0$$

$$\oint \vec{B}(\vec{r})_{line} \cdot d\vec{l} = \oint \left(\frac{\mu_o}{2\pi} \frac{I}{\mathscr{s}} \hat{\phi} \right) \cdot d\vec{l} = \oint \left(\frac{\mu_o}{2\pi} \frac{I}{\mathscr{s}} \hat{\phi} \right) \cdot (\mathscr{s}d\phi)\hat{\phi} = \oint \left(\frac{\mu_o}{2\pi} I d\phi \right)\left(\hat{\phi}\cdot\hat{\phi}\right) = \frac{\mu_o}{2\pi} I 2\pi = \mu_o I$$

What of $Curl\left(\vec{B}_{line}(\vec{r})\right)$? Takes a little work to get right answer and understand.

Ampere's Law



$$\oint \vec{B}\cdot d\vec{l} = \mu_o I_{piercing}$$

Goes Differential: Curl

Curl=circulation density (per area encircled)

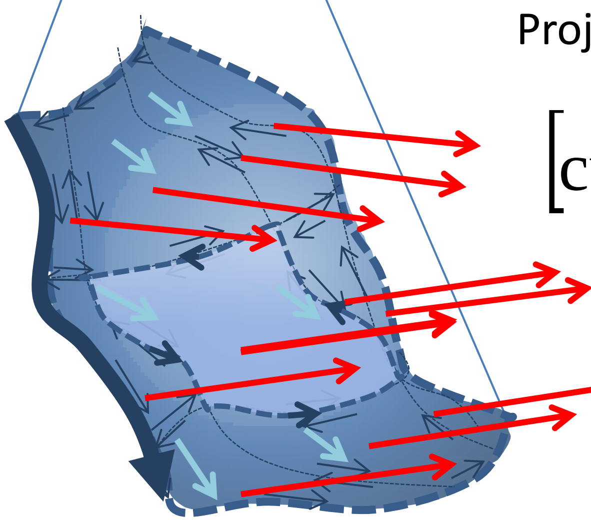
Zoom in to differential scale:

Break area into differential area 'patches'

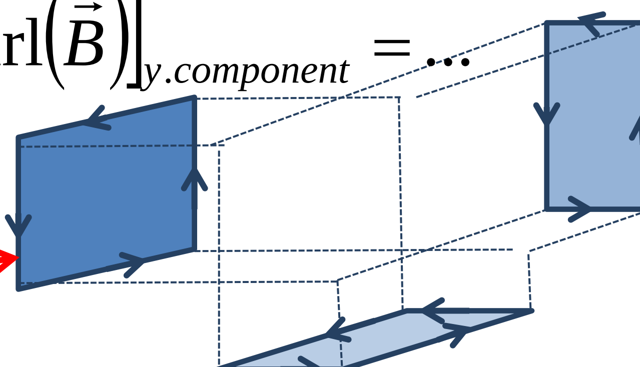
Break closed path into paths around differential patches

(ultimately all internal path legs cancel with each other)

Project onto coordinate planes

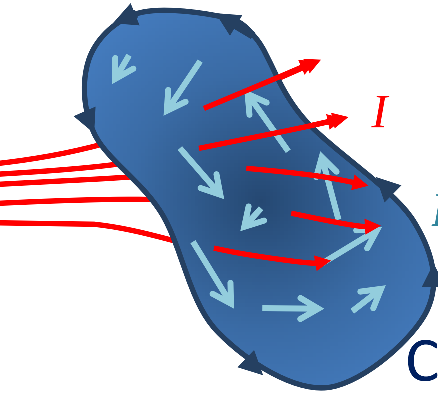


$$\left[\text{curl}\left(\vec{B}\right)\right]_{y.component} = \ldots$$



$$\left[\text{curl}\left(\vec{B}\right)\right]_{x.component} = \ldots$$

$$\left[\text{curl}\left(\vec{B}\right)\right]_{z.component} \equiv \lim_{\Delta A_{z.patch}\to 0}\left(\frac{\oint \vec{B}\cdot d\vec{\ell}_{z.patch}}{\Delta A_{z.patch}}\right)$$

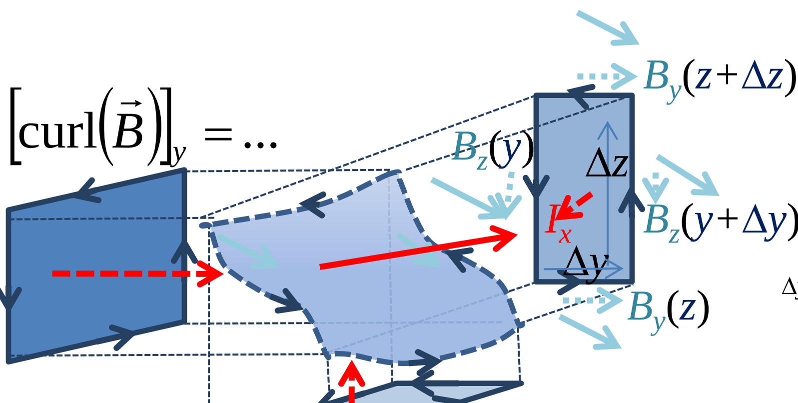


Ampere's Law

$$\oint \vec{B}\cdot d\vec{l} = \mu_o I_{piercing}$$

Goes Differential: Curl

Curl=circulation density (per area encircled)



$$\left[\mathrm{curl}\left(\vec{B}\right)\right]_y = \ldots$$

$$\left[\mathrm{curl}\left(\vec{B}\right)\right]_z = \ldots$$

$$\left[\mathrm{curl}\left(\vec{B}\right)\right]_x = \lim_{\Delta A_{x.patch}\to 0}\left(\frac{\oint \vec{B}\cdot d\vec{\ell}_{x.patch}}{\Delta A_{x.patch}}\right) = \lim_{\Delta A_{x.patch}\to 0}\left(\frac{\mu_o I_{x.patch}}{\Delta A_{x.patch}}\right)$$

$$\lim_{\Delta y\Delta z\to 0}\left[\frac{B_y(z)\Delta y + B_z(y+\Delta y)\Delta z + B_y(z+\Delta z)(-\Delta y) + B_z(y)(-\Delta z)}{\Delta y\Delta z}\right]$$

$$\lim_{\Delta y\Delta z\to 0}\left[\frac{B_z(y+\Delta y)\Delta z - B_z(y)\Delta z}{\Delta y\Delta z} - \frac{B_y(z+\Delta z)\Delta y - B_y(z)\Delta y}{\Delta y\Delta z}\right]$$

$$\left[\mathrm{curl}\left(\vec{B}\right)\right]_x = \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = \mu_o\frac{dI_x}{dA_{x.patch}} = \mu_o J_x$$

Similarly, $\left[\mathrm{curl}\left(\vec{B}\right)\right]_y = \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} = \mu_o J_y$ and $\left[\mathrm{curl}\left(\vec{B}\right)\right]_z = \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = \mu_o J_z$

$$\mathrm{curl}\left(\vec{B}\right) = \left\langle \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}, \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}, \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right\rangle = \mu_o\left\langle J_x, J_y, J_z\right\rangle$$

or $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

Magneto-Statics
Divergence and Circulation from Current Sources

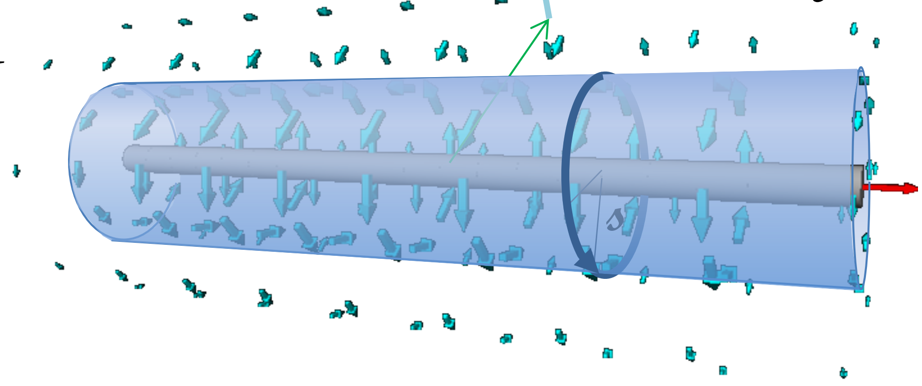
Derivation specific to constant, Infinite Line-Current

$$\vec{B}_{line}(\vec{r}) = \frac{\mu_o}{2\pi}\frac{I}{s}\hat{\phi}$$

$$\oint \vec{B}(\vec{r})_{line} \cdot d\vec{l} = \mu_o I$$

Stokes' Theorem

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$



$$\oint \vec{B}(\vec{r})_{line} \cdot d\vec{a} = 0$$

Gauss's Theorem

$$\vec{\nabla} \cdot \vec{B}_{line}(\vec{r}) = 0$$

Note: both follow from applying Biot-Savart, which holds only for steady currents

Pause and put together Stoke's:

Now for more general (more mathematical / less intuitive) proof

Stoke’s Theorem – explicitly putting it together

$\oint \vec{B}(\vec{r})_{line} \cdot d\vec{l} = \mu_o I$ but $\vec{I} = \int \vec{J} da$ And we’d gone off and proven $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

Putting these together:

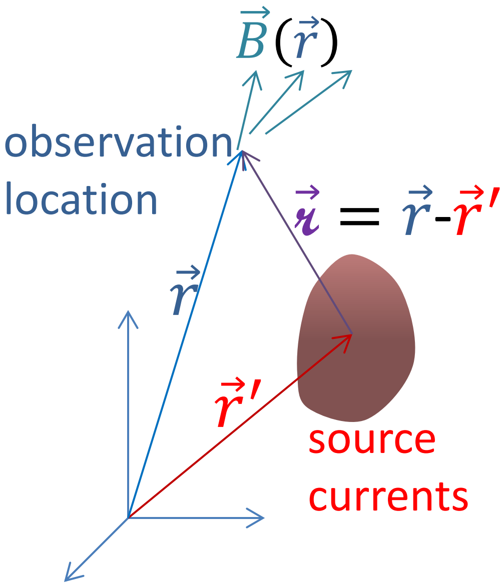
$$\oint \vec{B}(\vec{r})_{line} \cdot d\vec{l} = \int \mu_o \vec{J} \cdot da$$

$$\oint \vec{B}(\vec{r})_{line} \cdot d\vec{l} = \int \left(\vec{\nabla} \times \vec{B}\right) \cdot da$$

Though we were thinking specifically about magnetic field while we did the math that got us to this relation, it’s quite general and true for any vector field. So, as expressed in Ch. 1, for generic function F,

$$\oint \vec{F} \cdot d\vec{l} = \int \left(\vec{\nabla} \times \vec{F}\right) \cdot d\vec{a}$$

Derive Divergence of B



$$\vec{\nabla}_r \cdot \vec{B}(\vec{r}) = \vec{\nabla}_r \cdot \left(\frac{\mu_o}{4\pi}\int \frac{\vec{J}\times\hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'\right)$$

using

$$\vec{I}dl' = \frac{\vec{I}}{da_\perp} dl' da_\perp = \vec{J}d\tau'$$

Can slip del inside integral since *not* taking derivative with respect to integration variable

$$\vec{\nabla}_r \cdot \vec{B}(\vec{r}) = \frac{\mu_o}{4\pi}\int \vec{\nabla}_r \cdot \left(\vec{J}\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) d\tau'$$

Use Product Rule (6): $\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})$

taking derivative to see how field varies from one observation location to another, not for changes in source locations

$$\vec{\nabla}_r \times \left(\vec{J}\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\cdot\left(\cancel{\vec{\nabla}_r \times \vec{J}}\right) - \vec{J}\cdot\left(\vec{\nabla}_r \times \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)$$

$\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$ has no curl (can write out in Cartesian to convince, or see in spherical)

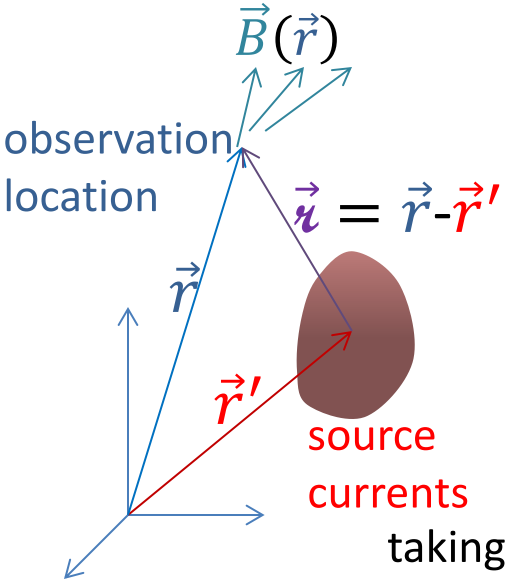
$$\vec{\nabla}_r \times \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = \frac{1}{r}\frac{1}{sin\theta}\frac{\partial}{\partial\varphi}\left(\frac{1}{\mathscr{r}^2}\right)\hat{\theta} - \frac{1}{r}\frac{\partial}{\partial\varphi}\left(\frac{1}{\mathscr{r}^2}\right)\hat{\varphi} = 0$$

$$\vec{\nabla}_r \cdot \vec{B}(\vec{r}) = \frac{\mu_o}{4\pi}\int 0 d\tau' = 0$$

So by Gauss' Theorem

$$\int \vec{\nabla}_r \cdot \vec{B}(\vec{r}) d\tau = \int \vec{B}(\vec{r}) \cdot d\vec{a} = 0$$

Derive Ampere's Differential form



$$\vec{\nabla}_r \times \vec{B}(\vec{r}) = \vec{\nabla}_r \times \left(\frac{\mu_o}{4\pi}\int \frac{\vec{J}\times\hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'\right)$$

using

$$\vec{I}dl' = \frac{\vec{I}}{da_\perp} dl' da_\perp = \vec{J}d\tau'$$

Can slip del inside integral since *not* taking derivative with respect to integration variable

$$\vec{\nabla}_r \times \vec{B}(\vec{r}) = \frac{\mu_o}{4\pi}\int \vec{\nabla}_r \times \left(\vec{J}\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) d\tau'$$

Use Product Rule (8): $\vec{\nabla}\times(\vec{A}\times\vec{B}) = (\vec{B}\cdot\vec{\nabla})\vec{A} - (\vec{A}\cdot\vec{\nabla})\vec{B} + \vec{A}(\vec{\nabla}\cdot\vec{B}) - \vec{B}(\vec{\nabla}\cdot\vec{A})$

taking derivative to see how field varies from one observation location to another, not for changes in source locations

$$\vec{\nabla}_r \times \left(\vec{J}\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) = \left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\cdot\vec{\nabla}_r\right)\vec{J} - \left(\vec{J}\cdot\vec{\nabla}_r\right)\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} + \vec{J}\left(\vec{\nabla}_r\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) - \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\left(\vec{\nabla}_r\cdot\vec{J}\right) = -\left(\vec{J}\cdot\vec{\nabla}_r\right)\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} + \vec{J}\left(\vec{\nabla}_r\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)$$

$$\vec{\nabla}_r \times \vec{B}(\vec{r}) = \frac{\mu_o}{4\pi}\left(\int -\left(\vec{J}\cdot\vec{\nabla}_r\right)\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\, d\tau' + \int \vec{J}\left(\vec{\nabla}_r\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) d\tau'\right)$$

$$\int +\left(\vec{J}\cdot\vec{\nabla}_{r'}\right)\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\, d\tau' \qquad \int \vec{J}\left(4\pi\delta^3(\vec{r})\right) d\tau'$$

$$\vec{\nabla}_r\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = 4\pi\delta^3(\vec{r})$$

Motivation

by Gauss's theorem

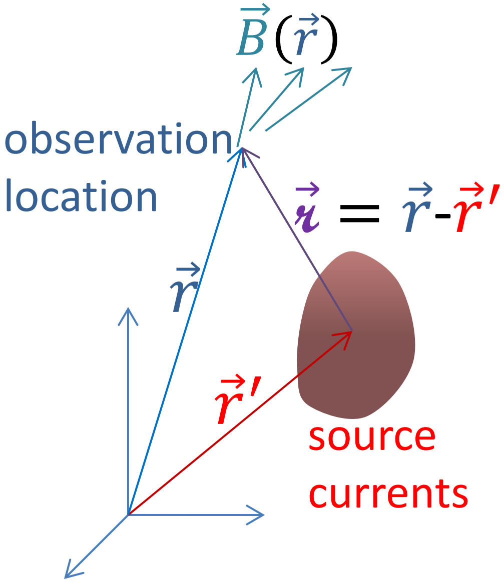
$$\int\left(\vec{\nabla}\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)\right)d\tau = \int\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)\cdot d\vec{a}$$

$$= \int\left(\frac{1}{\mathscr{r}^2}\right)\mathscr{r}^2 \sin\theta d\theta d\phi = 4\pi$$

Looking at just one component $\qquad 4\pi\vec{J}(\vec{r})$

$$\left(\left(\vec{J}\cdot\vec{\nabla}_{r'}\right)\frac{x-x'}{\mathscr{r}^3}\right)\hat{x} = \left(\vec{\nabla}_{r'}\cdot\left(\vec{J}\frac{x-x'}{\mathscr{r}^3}\right) - \frac{x-x'}{\mathscr{r}^3}\left(\vec{\nabla}_{r'}\cdot\vec{J}\right)\right)\hat{x}$$

Derive Ampere's Differential form



$$\vec{\nabla}_r \times \vec{B}(\vec{r}) = \vec{\nabla}_r \times \left(\frac{\mu_o}{4\pi} \int \frac{\vec{J} \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau' \right)$$

using

$$\vec{I}dl' = \frac{\vec{I}}{da_\perp} dl' da_\perp = \vec{J} d\tau'$$

$$\vec{\nabla}_r \times \vec{B}(\vec{r}) = \frac{\mu_o}{4\pi} \left(\int \left(\vec{J} \cdot \vec{\nabla}_{r'} \right) \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \, d\tau' \;\; + 4\pi \vec{J}(\vec{r}) \right)$$

where

$$\left(\left(\vec{J} \cdot \vec{\nabla}_{r'} \right) \frac{x - x'}{\mathscr{r}^3} \right) \hat{x} = \left(\vec{\nabla}_{r'} \cdot \left(\vec{J} \frac{x - x'}{\mathscr{r}^3} \right) - \frac{x - x'}{\mathscr{r}^3} \left(\vec{\nabla}_{r'} \cdot \vec{J} \right) \right) \hat{x}$$

Ditto for other two components

For now, with electro-magnetic *statics*

$$\vec{\nabla}_{r'} \cdot \vec{J} = -\frac{d\rho}{dt} \;\; = 0$$

Gauss's theorem

$$\int \vec{\nabla}_{r'} \cdot \left(\vec{J} \frac{x - x'}{\mathscr{r}^3} \right) d\tau' = \int \left(\vec{J} \frac{x - x'}{\mathscr{r}^3} \right) \cdot d\vec{a}' = 0$$

If area *fully encloses* current, then no current penetrates area

Ditto for other two components

$$\vec{\nabla}_r \times \vec{B}(\vec{r}) = \frac{\mu_o}{4\pi} \left(0 + 4\pi \vec{J}(\vec{r}) \right)$$

$$\vec{\nabla}_r \times \vec{B}(\vec{r}) = \mu_o \vec{J}(\vec{r})$$

So by Stokes' Theorem

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o \int \vec{J}(\vec{r}) \cdot d\vec{a}' = \mu_o I$$

Limitations: holds when Biot-Savart holds - statics

Maxwell's laws for electro-statics

$$\vec{\nabla}_r \times \vec{B}(\vec{r}) = \mu_o \vec{J}(\vec{r})$$

Ampere's $$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_o I$$

$$\vec{\nabla}_r \cdot \vec{B}(\vec{r}) = 0$$

$$\int \vec{B}(\vec{r}) \cdot d\vec{a} = 0$$

$$\vec{\nabla}_r \times \vec{E}(\vec{r}) = 0$$

$$\oint \vec{E}(\vec{r}) \cdot d\vec{l} = 0$$

$$\vec{\nabla}_r \cdot \vec{E}(\vec{r}) = \frac{1}{\varepsilon_0}\rho(\vec{r})$$

$$\int \vec{E}(\vec{r}) \cdot d\vec{a} = \frac{1}{\varepsilon_o} Q$$ Gauss's

For arguably symmetric fields,
useful for finding fields

Wed. Fri.	(C 21.6-7,.9) 1.3.4-1.3.5, 1.5.2-1.5.3, 5.3.1-.3.2 Div & Curl B (C 21.6-7,.9) 5.3.3-.3.4 Applications of Ampere's Law	
Mon. Wed. Thurs. Fri.	1.6, 5.4.1-.4.2 Magnetic Vector Potential 5.4.3 Multipole Expansion of the Vector Potential Review	 HW8