Fri. 10/18	*C 17)* **5.2** Biot-Savart Law **T5** Quiver Plots	
Mon.10/21 Wed.10/23 Thurs. 10/24	*(C 21.6-7,.9)* **1.3.4-1.3.5, 1.5.2-1.5.3, 5.3.1-.3.2** Div & Curl B *(C 21.6-7,.9)* **5.3.3-.3.4** Applications of Ampere's Law	HW6

From Last Time

<u>Summary</u>

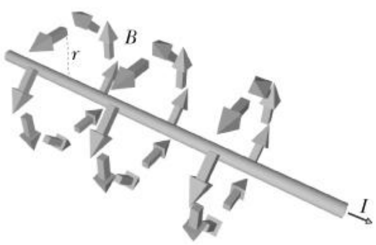
Biot-Savart Law

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{(dq'\vec{v}) \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

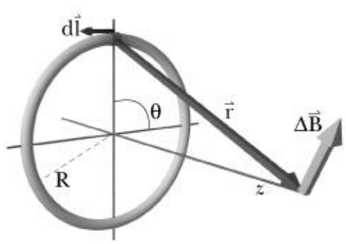
$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{(d\ell'\vec{I}) \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{(da'\vec{K}) \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

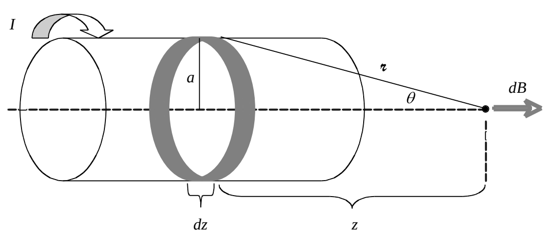
$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{(d\tau'\vec{J}) \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$



$$\vec{B} = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)(-\hat{y}) \quad \text{infinitely long: } B = \frac{\mu_0 I}{2\pi s}$$



On axis: $\vec{B}(z) = \frac{\mu_0}{2}\frac{IR^2}{(R^2+z^2)^{3/2}}\hat{z}$ at center $B_z = \mu_0 I/2R$



$$B = \frac{\mu_0 IN}{2L}\left[\frac{z_2'}{(R^2+z_2'^2)^{1/2}} - \frac{z_1'}{(R^2+z_1'^2)^{1/2}}\right]$$

$$= \frac{\mu_0 IN}{2L}(\cos\theta_2 - \cos\theta_1)$$

(with the observation location at the origin and where $z'_2 = z'_1 + L$)

Example: Flat Coil of Wire Disk

Another direction in which we could expand upon our ring solution is radially – say we want to know the on-axis field due to a flat coil of current.

$$d\vec{B}(z) = \frac{\mu_0}{2}\frac{(dI)r'^2}{(r'^2 + z^2)^{3/2}}\hat{z} \text{ (from the ring)}$$

Where

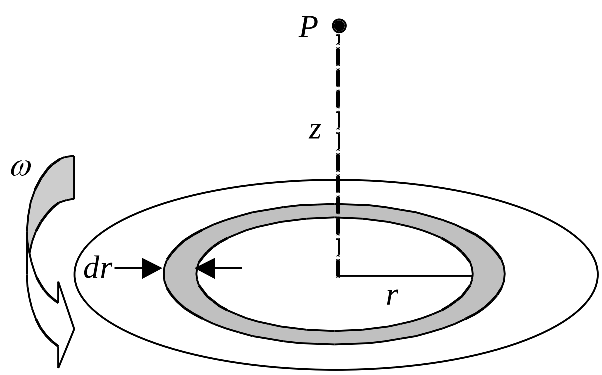
$$K = \frac{dI}{dl_\perp} = \frac{dI}{dr'} = \frac{NI}{R} \Rightarrow dI = \frac{NI}{R}dr'$$

$$\vec{B}(z) = \frac{\mu_0}{2}\frac{NI}{R}\int_0^R \frac{r'^2 dr'}{(r'^2 + z^2)^{3/2}}\hat{z}$$

Problem 5.47 (a) – Charged Rotating Disk

Find the magnetic field on the axis at a distance *a* above a disk of radius *R* with charge density σ rotating at an angular speed ω (clockwise when viewed from the point *P*).

Consider a thin ring between *r* and *r* + *dr* as shown below. The charge per length on the ring is $\lambda = \sigma\, dr$ and each point on it is moving at a speed $v = \omega r$, so the current is $I = \lambda v = \sigma\omega r\, dr$.



By symmetry, the magnetic field is in the *z* direction. Using the result from Ex. 5.6 (Eq. 5.38), the contribution from the ring is

$$dB_z = \frac{\mu_0(\sigma\omega r\, dr)}{2}\frac{r^2}{(r^2 + z^2)^{3/2}}.$$

The magnetic field for the entire disk is

$$B_z = \frac{\mu_0\sigma\omega}{2}\int_0^R \frac{r^3\, dr}{(r^2 + z^2)^{3/2}}.$$

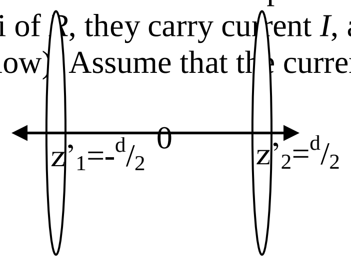
Let $u = r^2$, so $du = 2r\, dr$. This substitution (don't forget to change limits) gives

$$B_z = \frac{\mu_0 \sigma \omega}{4} \int_0^{R^2} \frac{u\,du}{\left(u+z^2\right)^{3/2}} = \frac{\mu_0 \sigma \omega}{4} \left[2\frac{\left(u+2z^2\right)}{\left(u+z^2\right)^{1/2}} \right]_0^R,$$

$$= \frac{\mu_0 \sigma \omega}{2} \left[\frac{\left(R^2+2z^2\right)}{\left(R^2+z^2\right)^{1/2}} - 2z \right].$$

Problem 5.46 – Helmholtz Coils

A nearly uniform field can be produced by using two coils. Suppose that the coils have radii of R, they carry current I, and the separation between them is d (see the figure below). Assume that the currents flow in the directions shown.



a. Find the magnetic field on the axis as a function of z, using the midway point as the origin ($z = 0$). Show that $\partial B/\partial z = 0$ at the midpoint. This means that, functionally, at least at that one location, the field is not changing. Of course, that could be because it's a minimum, maximum, or saddle point.

The magnetic field from both coils points up at points on the axis. The total magnetic field (first term from lower loop, second from upper loop) at the midpoint is

$$B_z = \frac{\mu_0 I R^2}{2} \left\{ \frac{1}{\left[R^2 + (d/2+z)^2\right]^{3/2}} - \frac{1}{\left[R^2 + (d/2-z)^2\right]^{3/2}} \right\},$$

Worth mentioning, but not worth doing the following

so the derivative is

$$\frac{\partial B_z}{\partial z} = \frac{\mu_0 I R^2}{2} \left\{ \frac{(-3/2)2(d/2+z)}{\left[R^2 + (d/2+z)^2\right]^{5/2}} - \frac{(-3/2)2(d/2-z)}{\left[R^2 + (d/2-z)^2\right]^{5/2}} \right\},$$

$$= \frac{3\mu_0 I R^2}{2} \left\{ \frac{-(d/2+z)}{\left[R^2 + (d/2+z)^2\right]^{5/2}} + \frac{(d/2-z)}{\left[R^2 + (d/2-z)^2\right]^{5/2}} \right\}.$$

It is easy to see that

$\left.\frac{\partial B_z}{\partial z}\right|_{z=0} = 0$. This is true regardless of the separation (d).

b. If the separation *d* is picked correctly, then $\partial^2 B/\partial z^2 = 0$ at the midpoint. No concavity suggests that it's neither a minimum nor a maximum – could be a saddle point, but that's hard to imagine. Locally, the field is constant. This configuration is known as *Helmholtz coils*. Determine the appropriate value of *d*.

Differentiating again (product rule) gives

$$\frac{\partial^2 B_z}{\partial z^2} = \frac{3\mu_0 I R^2}{2}\left\{\frac{-1}{\left[R^2+(d/2+z)^2\right]^{5/2}} + \frac{-(d/2+z)\cdot(-5/2)2(d/2+z)}{\left[R^2+(d/2+z)^2\right]^{7/2}}\right.$$
$$\left. + \frac{-1}{\left[R^2+(d/2-z)^2\right]^{7/2}} + \frac{(d/2-z)\cdot(-5/2)2(d/2-z)(-1)}{\left[R^2+(d/2-z)^2\right]^{5/2}}\right\},$$

$$\frac{\partial^2 B_z}{\partial z^2} = \frac{3\mu_0 I R^2}{2}\left\{\frac{-1}{\left[R^2+(d/2+z)^2\right]^{5/2}} + \frac{5(d/2+z)^2}{\left[R^2+(d/2+z)^2\right]^{7/2}}\right.$$
$$\left. + \frac{-1}{\left[R^2+(d/2-z)^2\right]^{7/2}} + \frac{5(d/2-z)^2}{\left[R^2+(d/2-z)^2\right]^{5/2}}\right\}.$$

At the midpoint, the second derivative is

$$\left.\frac{\partial^2 B_z}{\partial z^2}\right|_{z=0} = \frac{3\mu_0 I R^2}{2}\left\{\frac{-2}{\left[R^2+(d/2)^2\right]^{5/2}} + \frac{2\cdot 5(d/2)^2}{\left[R^2+(d/2)^2\right]^{7/2}}\right\},$$

$$= 3\mu_0 I R^2\,\frac{\left[R^2+d^2/4\right]-5\left(d^2/4\right)}{\left[R^2+d^2/4\right]^{7/2}} = 3\mu_0 I R^2\,\frac{R^2-d^2}{\left[R^2+d^2/4\right]^{7/2}}.$$

Therefore, the condition $\left.\frac{\partial^2 B_z}{\partial z^2}\right|_{z=0} = 0$ is met if $d = R$. The magnetic field along the axis can be written as

$$B_z(z) = \left[\frac{\partial^3 B_z}{\partial z^3}\right]_{z=0} z^3 + O\left(z^4\right),$$

so the magnetic field changes very slowly when *z* is very small.

<u>Vector Field Plots</u> – have students go over the handout

```
from pylab import *

# Set limits and number of points in grid
xmax = 2.0
xmin = -xmax
NX = 10
ymax = 2.0
ymin = -ymax
NY = 10

# Make grid and calculate vector components
x = linspace(xmin, xmax, NX)
y = linspace(ymin, ymax, NY)
X, Y = meshgrid(x, y)
S2 = X**2 + Y**2  # This is the radius squared
Bx = -Y/S2
By = +X/S2

figure()
QP = quiver(X,Y,Bx,By)
quiverkey(QP, 0.9, 1.05, 1.0, '1 mT', labelpos='N')

# Set the left, right, bottom, top limits of axes
dx = (xmax - xmin)/(NX - 1) # One less gap than points
dy = (ymax - ymin)/(NY - 1)
axis([xmin-dx, xmax+dx, ymin-dy, ymax+dy])

title('Magnetic Field of a Wire with I=50 A')
xlabel('x (cm)')
ylabel('y (cm)')
show()
```

```
from pylab import *
from numpy import *

q = 1.0e-6 # in coulombs
epsl0 = 8.85e-16 # in C**2/(N cm**2)
d = 3.0 # in cm

# Set limits and number of points in grid
xmax = 5.0
xmin = -xmax
NX = 10
ymax = 5.0
ymin = -ymax
NY = 10

# Make grid and calculate vector components
x = linspace(xmin, xmax, NX)
y = linspace(ymin, ymax, NY)
X, Y = meshgrid(x, y)
R1 = sqrt((X-d)**2 + Y**2)
Ex = q/(4.0*pi*epsl0) * (X-d)/R1**3
Ey = q/(4.0*pi*epsl0) * Y/R1**3

figure()
QP = quiver(X,Y,Ex,Ey)
quiverkey(QP, 0.9, 1.05, 2.0e7, '2e7 N/C', labelpos='N')

# Set the left, right, bottom, top limits of axes
dx = (xmax - xmin)/(NX - 1) # One less gap than points
dy = (ymax - ymin)/(NY - 1)
axis([xmin-dx, xmax+dx, ymin-dy, ymax+dy])

title('Electric Field of a Point Charge')
xlabel('x (cm)')
ylabel('y (cm)')
show()
```

```
from pylab import *
from numpy import *

q = 1.0e-6 # in coulombs
epsl0 = 8.85e-16 # in C**2/(N cm**2)

# Set limits and number of points in grid
xmax = 5.0
xmin = -xmax
NX = 10
ymax = 5.0
ymin = -ymax
NY = 10

# Make grid and calculate vector components
x = linspace(xmin, xmax, NX)
y = linspace(ymin, ymax, NY)
X, Y = meshgrid(x, y)
R = sqrt(X**2 + Y**2)
Ex = q/(4.0*pi*epsl0) * X/R**3
Ey = q/(4.0*pi*epsl0) * Y/R**3

figure()
QP = quiver(X,Y,Ex,Ey)
quiverkey(QP, 0.9, 1.05, 2.0e7, '2e7 N/C', labelpos='N')

# Set the left, right, bottom, top limits of axes
dx = (xmax - xmin)/(NX - 1) # One less gap than points
dy = (ymax - ymin)/(NY - 1)
axis([xmin-dx, xmax+dx, ymin-dy, ymax+dy])

title('Electric Field of a Point Charge')
xlabel('x (cm)')
ylabel('y (cm)')
show()
```

```
from pylab import *
from numpy import *

q = 1.0e-6 # in coulombs
epsl0 = 8.85e-16 # in C**2/(N cm**2)
d = 3.0 # in cm

# Set limits and number of points in grid
xmax = 5.0
xmin = -xmax
NX = 10
ymax = 5.0
ymin = -ymax
NY = 10

# Make grid and calculate vector components
x = linspace(xmin, xmax, NX)
y = linspace(ymin, ymax, NY)
X, Y = meshgrid(x, y)
R1 = sqrt((X-d)**2 + Y**2)
R2 = sqrt((X+d)**2 + Y**2)
Ex = q/(4.0*pi*epsl0) * ((X-d)/R1**3 - (X+d)/R2**3)
Ey = q/(4.0*pi*epsl0) * (Y/R1**3 - Y/R2**3)

figure()
QP = quiver(X,Y,Ex,Ey)
quiverkey(QP, 0.9, 1.05, 2.0e7, '2e7 N/C', labelpos='N')

# Set the left, right, bottom, top limits of axes
dx = (xmax - xmin)/(NX - 1) # One less gap than points
dy = (ymax - ymin)/(NY - 1)
axis([xmin-dx, xmax+dx, ymin-dy, ymax+dy])

title('Electric Field of a Dipole')
xlabel('x (cm)')
ylabel('y (cm)')
show()
```

""Is it possible to have a magnetic field in a two dimensional system?""
Casey P,
Hm.. what about that problem we just did for homework where there was a charge -q moving in the +x direction, then the mag field ended up being in the +zhat direction? I mean if you got the e-field involved you'd definitely have three dimensions but I think with just x and z it'd be two right?
Rachael Hach

"Can we take another look at relativistic electrodynamics? Specifically when magnetic fields are replaced by electric and vice versa."
Davies

"can we do an example problem that finds the magnetic field of a volume of a sphere or a cylinder?"
Jessica

"Can we talk a bit about the physical meaning of a varying current density? I'm a little confused about this since the Biot-Savart law is only supposed to apply to steady currents."
Freeman,
Yes I agree it'd be really cool if we could talk conceptually a little bit.
Rachael Hach

"Can we go over finding the magnetic field of a surface and volume current?"
Spencer

"Can we work through some of the integrals when using the Biot-Savart law with volumes?"
Connor W,

"Maybe if we have time we could also work out the extra credit problem (on the homework that we just submitted) in class, or at least talk about it."
Casey McGrath

"My question is similar to Antwain's. There are a lot of cross products. Like dl X r and DEL X B. When do you have to do the full cross product and when is it merely multiplication and changing directions? Is it about being perpendicular?"
Anton

"I'm still having some trouble understanding ex.5.5 and how dl x r is found."
Antwain

"I'm hoping we could do some examples of relativistic E/M, especially problems similar to the HW 5 problem in looking at systems from different points of view."
"We did a little bit with surface and volume current densities in the last homework. Is there a simple way to translate the Biot-Savart law to find the magnetic field generated by these currents?"
Ben Kid