Mon., 11/23 Tues., 11/24 Wed.,11/25 Thurs.,11/26 Fri., 11/27	**6.3, 6.4** Auxiliary Field & Linear Media **Thanksgiving Break** **Thanksgiving** **Thanksgiving Break**	HW9
Mon., 11/30 Wed., 12/2 Fri., 12/4	**7.1.1-7.1.3** *Ohm's Law & Emf* **7.1.3-7.2.2** *Emf & Induction* **7.2.3-7.2.5** *Inductance and Energy of B*	

Recall

Or, defining the density of magnetic dipoles as

$$\vec{M} \equiv \frac{d\vec{m}}{d\tau}$$

And dubbing it the "Magnetization,"

$$\vec{A}_{dip}(\vec{r}) = \int \frac{\mu_o}{4\pi} \frac{\vec{M} \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau$$

$$\vec{K}_b = \vec{M} \times \hat{n} \text{ and } \vec{J}_b = \vec{\nabla} \times \vec{M}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \left\{ \int_{volume} \frac{\vec{J}_b(\vec{r}')}{\mathscr{r}} d\tau' + \oint_{surface} \frac{\vec{K}_b(\vec{r}')}{\mathscr{r}} da' \right\}$$

So, if you want to find the magnetic field from the magnetization $\vec{M}$, there are two options:

1. Use $\vec{M}$ to find $\vec{A}$ using Equations 6.13-15, then use $\vec{B} = \vec{\nabla} \times \vec{A}$.
2. Find the bound currents $\vec{J}_b$ and $\vec{K}_b$ from $\vec{M}$, then use Ampere's law or the Biot-Savart law to find $\vec{B}$.

Summary

The Auxiliary Field (H)

Define the *auxiliary field* as

$$\vec{H} \equiv \frac{1}{\mu_0}\vec{B} - \vec{M}.$$

This is a useful definition because we can write a "Ampere's law" for $\vec{H}$ that only depends on the free current (<u>not</u> the bound current):

$$\vec{\nabla} \times \vec{H} = \frac{1}{\mu_0}\left(\vec{\nabla} \times \vec{B}\right) - \vec{\nabla} \times \vec{M} = \vec{J} - \vec{J}_b,$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_f.$$

The integral form of the law (use the Curl Theorem) is

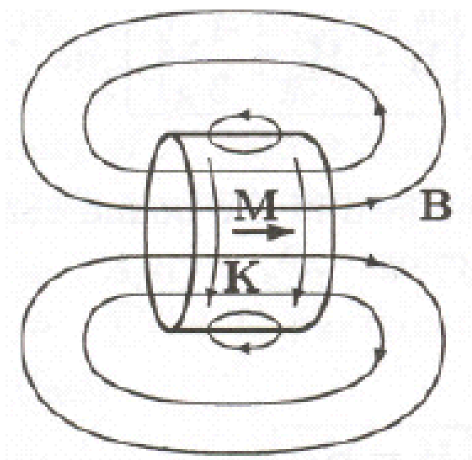
$$\oint \vec{H}\cdot d\vec{\ell} = I_{f,enc} \tag{6.20}$$

$\vec{H}$ is Not Exactly Analogous to $\vec{B}$

The free current density $\vec{J}_f$ does <u>not</u> completely determine $\vec{H}$ in the same way that the total current density $\vec{J}$ determines the electrostatic field $\vec{B}$!

$$\vec{\nabla}\cdot\vec{H} = \frac{1}{\mu_0}\left(\cancel{\vec{\nabla}\cdot\vec{B}}\right) - \vec{\nabla}\cdot\vec{M} = -\vec{\nabla}\cdot\vec{M} \tag{6.23}$$

For example, consider a cylinder with uniform magnetization $\vec{M}$ parallel to its axis. There are no free currents, so you might assume that $\vec{H}=0$ everywhere. However, that would mean that $\vec{B}=0$ outside, which is obviously wrong. In this case $\vec{\nabla}\cdot\vec{M}\neq 0$ at the ends of the cylinder, so "Ampere's law" is not sufficient to find $\vec{H}$. The bound surface current $\vec{K}_b = \vec{M}\times\hat{n}$ wraps around the cylinder, so it produces the same magnetic field as a short solenoid.



However, if there is enough symmetry to use Ampere's law for the auxiliary field, then $\vec{\nabla}\cdot\vec{M}=0$. In these cases, you don't have to worry about the divergence of $\vec{H}$ (because there is none.)

Boundary Conditions

The integral version of Eq. 6.23 is $\oint \vec{H}\cdot d\vec{a} = -\oint \vec{M}\cdot d\vec{a}$.

$$H^{\perp}_{above} - H^{\perp}_{above} = -\left(M^{\perp}_{above} - M^{\perp}_{above}\right)$$

Eq. 6.20 gives

$$H^{\parallel}_{above} - H^{\parallel}_{above} = K_f,$$

where *H* and *K* are perpendicular. The directions are related by surface normal as

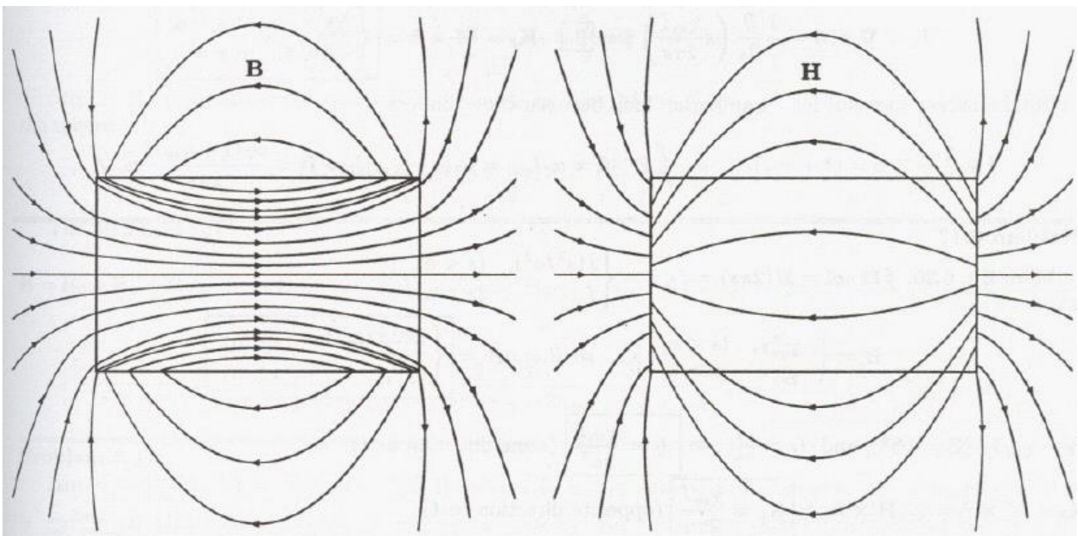
$$\vec{H}^{\parallel}_{above} - \vec{H}^{\parallel}_{above} = \vec{K}_f \times \hat{n}\,.$$

Whereas

$$B^{\perp}_{above} - B^{\perp}_{below} = 0$$
$$B^{\parallel}_{above} - B^{\parallel}_{below} = \mu_o \vec{K}_{tot} \times \hat{n}$$

For example

The diagrams below show the magnetic and auxiliary fields. They are the same outside of the cylinder where the magnetization is zero.



Examples/Exercises:

Example 6.2

A long copper rod of radius *R* carries a uniformly distributed (free) current *I*. Find the auxiliary field $\vec{H}$ everywhere.

Let the axis of the rod be the *z* axis. By symmetry, we know that $\vec{H} = H(s)\hat{\phi}$. Use a circle of radius *s* as an Amperian loop. Regardless of the radius, the line integral is

$$\oint \vec{H} \cdot d\vec{\ell} = H \cdot 2\pi s\,.$$

The free current enclosed is

$$I_{f,enc} = \begin{cases} \dfrac{\pi s^2}{\pi R^2} I & s < R, \\ I & s > R. \end{cases}$$

Apply the new form of Ampere's law, $\oint \vec{H} \cdot d\vec{\ell} = I_{f,enc}$, to get

$$I_{f,enc} = \begin{cases} \dfrac{Is}{2\pi R^2}\hat{\phi} & s < R, \\ \dfrac{I}{2\pi s}\hat{\phi} & s > R. \end{cases}$$

We can't say anything about the magnetization $\vec{M}$ inside the copper, except that it is in the opposite direction from $\vec{H}$ because copper is diamagnetic.

Outside of the wire, $\vec{M} = 0$, so the magnetic field for $s > R$ is

$$\vec{B} = \mu_0 \vec{H} = \frac{\mu_0 I}{2\pi s}\hat{\phi}.$$

Problem 6.13 (b)(c)

Suppose the magnetic field inside of a large piece of magnetized material is $\vec{B}_0$, so that $\vec{H}_0 = (1/\mu_0)\vec{B}_0 - \vec{M}$.

a. A long, needle-shaped cavity running parallel to $\vec{M}$ is hollowed out of the material. Find the magnetic field at the center of the cavity in terms of $\vec{B}_0$ and $\vec{M}$. Also, find $\vec{H}$ at the center of the cavity in terms of $\vec{H}_0$ and $\vec{M}$.

Let the *z* axis run through the center of the cavity. The unit normal $\hat{n}$ points inward, so the bound surface current is $\vec{K} = -M\hat{\phi}$. This is like the current of a solenoid (recall from yesterday that $nI \to K = M$), so it produces a magnetic field $\mu_0 \vec{M}$ (in the –*z* direction). The total magnetic field at the center of the cavity is $\vec{B} = \vec{B}_0 - \mu_0 \vec{M}$.

The auxiliary field at the center of the cavity (where $\vec{M} = 0$) is

$$\vec{H} = (1/\mu_0)\vec{B} - \cancel{\vec{M}} = (1/\mu_0)\left(\vec{B}_0 - \mu_0 \vec{M}\right) = \vec{H}_0.$$

b. Do the same for a thin, wafer-shaped cavity perpendicular to $\vec{M}$.

There is a bound surface charge that flow in the $-\hat{\phi}$, but it is small (b/c the wafer is thin) and far away from the center. Therefore, the magnetic field is $\vec{B} \approx \vec{B}_0$.

The auxiliary field at the center of the cavity (where $\vec{M} = 0$) is

$$\vec{H} = (1/\mu_0)\vec{B} - \cancel{\vec{M}} = (1/\mu_0)\vec{B}_0 = \vec{H}_0 + \vec{M}.$$

Summary

Linear Materials

Going to get into some of the quantitative stuff today, then we can get into some of the qualitative Monday.

For linear material, the magnetization can be written as

$$\vec{M} = \chi_m \vec{H} ,$$

where χ_m is the *magnetic* susceptibility. If $\chi_m > 0$, the material is *paramagnetic* and the magnetization is parallel to $\vec{B}$. If $\chi_m < 0$, then the material is *diamagnetic* and the magnetization is opposite to $\vec{B}$.

Note that this is in terms of the auxiliary field $\vec{H}$ instead of the magnetic field $\vec{B}$. We can write the magnetic field as

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M}\right) = \mu_0 \left(1 + \chi_m\right)\vec{H} = \mu \vec{H} ,$$

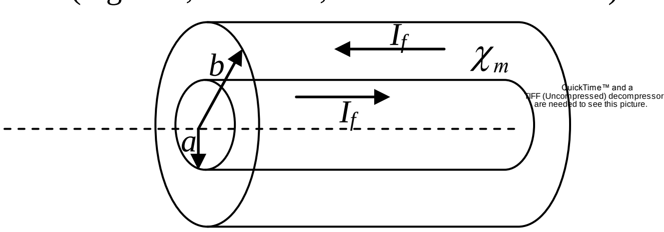
where $\mu \equiv \mu_0 \left(1 + \chi_m\right)$ is the *permeability*.

Examples/Exercises:

Returned to the rod, assumed a magnetic susceptibility for it and found B inside and outside. Note that B outside does not reference the susceptibility or bound current, thus there must be no net bound current inside an outer loop. Find bound current distribution and see that that is so.

Problem 6.16

A coaxial cable consists of two very long cylindrical tubes (radii *a* and *b*), separated by linear insulating material of magnetic susceptibility χ_m. A current *I* flows down the inner conductor and returns along the outer one. In each case, the current distributes itself uniformly over the surface. Find the magnetic field between the tubes. As a check, calculate the magnetization and bound currents, and confirm that (together, of course, with the free currents) they generate the correct field.



Let the axis of the cylinders be the *z* axis. By symmetry, we know that $\vec{H} = H(s)\hat{\phi}$. Use a circle of radius *s* as the Amperian loop. For any radius, the line integral is

$\oint \vec{H} \cdot d\vec{\ell} = H \cdot 2\pi s$. The <u>free</u> current enclosed is I_f for $a < s < b$, so $\vec{H} = (I/2\pi s)\hat{\phi}$ between the cylinders and zero elsewhere. The magnetic field between the tubes is

$$\vec{B} = \mu_0 (1 + \chi_m) \vec{H} = \mu_0 (1 + \chi_m) \frac{I}{2\pi s} \hat{\phi}.$$

The magnetization of the insulating material is

$$\vec{M} = \chi_m \vec{H} = \frac{\chi_m I}{2\pi s} \hat{\phi},$$

so the bound currents are

$$\vec{J}_b = \vec{\nabla} \times \vec{M} = \frac{1}{s} \frac{\partial}{\partial s} (s M_\phi) \hat{z} = \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\chi_m I}{2\pi s} \right) \hat{z} = 0.$$

and

$$\vec{K}_b = \vec{M} \times \hat{n} = \begin{cases} \dfrac{\chi_m I}{2\pi a} \hat{z} & \text{at } s = a \quad (\hat{n} = -\hat{s}), \\ -\dfrac{\chi_m I}{2\pi b} \hat{z} & \text{at } s = b \quad (\hat{n} = +\hat{s}). \end{cases}$$

Use the same Amperian loop as before. The line integral is $\oint \vec{B} \cdot d\vec{\ell} = B \cdot 2\pi s$ .The <u>total</u> current enclosed for $a < s < b$ is

$$I_{enc} = I + \frac{\chi_m I}{2\pi a} (2\pi a) = (1 + \chi_m) I,$$

so

$$\vec{B} = \mu_0 (1 + \chi_m) \frac{I}{2\pi s} \hat{\phi}.$$

Problem 6.19

On the basis of the naïve model presented in Sect. 6.1.3, estimate the magnetic susceptibility of a diamagnetic metal such as copper. Compare your answer with the empirical value in Table 6.1 and comment on any discrepancy.

Consider just the valence electron of each atom. The change in the magnetic dipole moment for a single atom is

$$\Delta \vec{m} = -\frac{e^2 R^2}{4 m_e} \vec{B}. \tag{6.8}$$

The magnetization is

$$\vec{M} = \frac{\Delta \vec{m}}{V} = -\frac{(e^2 R^2 / 4 m_e) \vec{B}}{\frac{4}{3} \pi R^3} = -\left(\frac{3 e^2}{16 \pi m_e R} \right) \vec{B},$$

where V is the volume of the atom. We also know that

$$\vec{M} = \chi_m \vec{H} = \chi_m \frac{\vec{B}}{\mu_0 \left(1 + \chi_m\right)} \approx \left(\frac{\chi_m}{\mu_0}\right) \vec{B}, \qquad (6.29,\ 6.30)$$

where the approximation is valid if $\chi_m << 1$. The susceptibility is

$$\chi_m = -\frac{3\mu_0 e^2}{16\pi m_e R} = -\left(\frac{\mu_0}{4\pi}\right) \frac{3e^2}{4m_e R}.$$

Use $R = 1 \times 10^{-10}\,\mathrm{m}$ (an angstrom), to get

$$\chi_m = -\left(10^{-7}\,\mathrm{N/A^2}\right) \frac{3\left(1.6 \times 10^{-19}\,\mathrm{C}\right)^2}{4\left(9.11 \times 10^{-31}\,\mathrm{kg}\right)\left(10^{-10}\,\mathrm{m}\right)} = -2 \times 10^{-5}.$$

According to Table 6.1, $\chi_m = -9.7 \times 10^{-6} \approx -1 \times 10^{-5}$, so the estimate is the right order of magnitude.

Problem 6.20

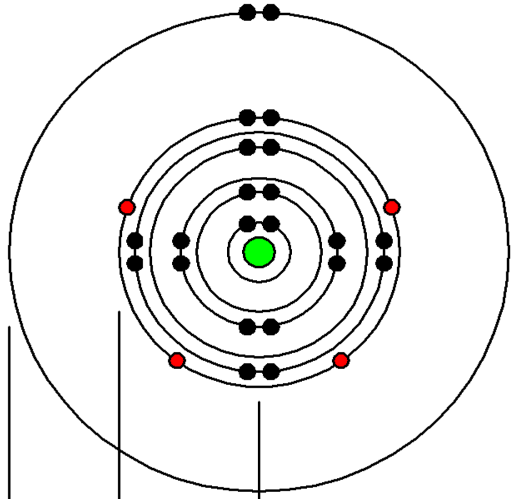
How would you demagnetize a "permanent" magnet (such as the wrench we have been discussing, at point *c* in the hysteresis loop)? That is, how could you restore it to its original state, with $M = 0$ at $I = 0$?

If the object is not delicate, place in a region with zero magnetic field and heat it above the Curie point. It could also be dropped repeatedly on a hard surface or pounded with a hammer.

If the object is delicate, place it between the poles of an electromagnet. Magnetize is back and forth many times, reducing the size of the maximum field each time the direction is reversed.

The Origin of Ferromagentism

Let's look at Iron, the poster child of ferromagnetism (and the origion of "ferro"). First, consider an individual iron atom:



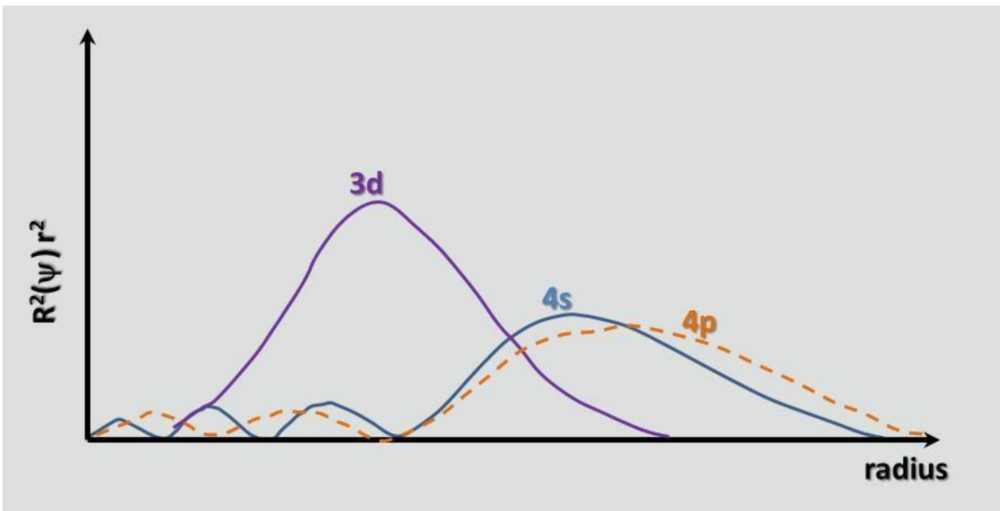
Iron (Fe)
Atomic Number 26
Electron configuration $1s^2 2s^2 2p^6 3s^2 3p^6 3d^6 4s^2$ (www.coolmagnetman.com/maghow.htm)

This is just a schematic, don't take it too literally, but it does communicate some important information. The green dot at the center represents the nucleus containing all the neutrons and protons; its size is *greatly* exaggerated. The different rings represent the average radii of the different electronic levels. The dots (both black and red) represent the electrons occupying those levels.

Each level has a maximum allowed occupancy in accordance with the Pauli Exclusion Principle which says that no two electrons can be completely identical. What distinguishes electrons within the same atom are their spins, and the shapes/orientations & sizes of their orbitals – so, no two electrons can have the same orbits and spins. Naturally, the smaller radius orbits are generally more desirable, since they're closer to the nucleus and they have fewer repulsive electrons between them and the attractive nucleus. In the process of filling up a level, electrons take all compatible orientations and spins, to the effect that there is no *net* orbital or spinning motion – that is, however one electron may be orbiting, another has the opposite orbit. You'll notice that most of the dots are paired up, that represents there being two electrons with the same orbits but opposite spins. No net motion means no net magnetic field. Because it soon becomes important, I should note that electrons in the same level and the same orbit (but with opposite spin) are as close together for as much of the time as possible, and so they feel each other's electric repulsion more than do two electrons in the same level but different orbits. For this reason, it is preferable to be in *different* orbits, and this is ensured if electrons have the *same* spin – thus Hund's rule that a level is filled by first putting one electron in each possible orbital orientation, and with the same spin orientation, and only after that is done do opposite spin electrons get added into the different orientations.

Now look at the two outer most shells of the iron atom. The last shell is the 4s – 4 loosely refers its average radius and s refers to the orbit being spherically symmetric – with no net angular momentum. Only two electrons can occupy an s – spin up and spin down. The second to last shell is the 3d (d referrers to the magnitude of angular momentum, four different orientations are allowed with this particular magnitude; with two spins per orientation – the shell can hold 8 electrons). You'll notice two peculiar things, both of which are important for making iron Ferromagnetic.

First, the 3d is only partially filled; four of its electrons are *unpaired.* By Hund's rule, their spins are all aligned, so their contributions to the magnetic field add up. Yippee, an individual Fe atom produces a magnetic field! Oh, but so would any other atom (except for noble gasses), so maybe that's not such a big deal in and of itself. What is important is that, somehow, when iron atoms bond up to form a solid they a) retain this unparedness and b) the unpaired electrons not only pair with each other within the same atom, but also with each other in different atoms.



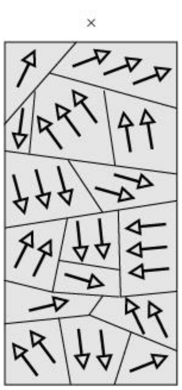
(http://www.theeestory.com/topics/9807)

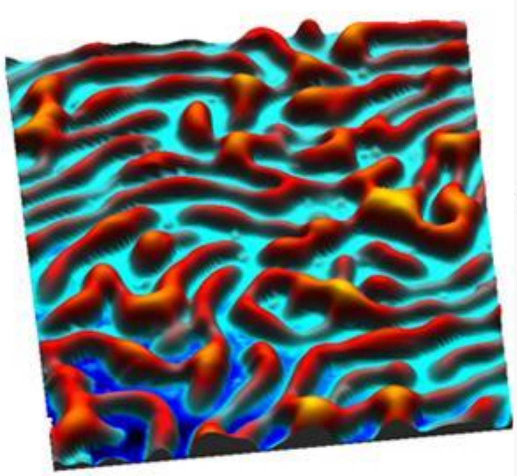
This relies on the other peculiar thing about iron atoms. Though the 4s orbit is on *average* larger than the 3d, the 4s is full and the 3d is not. This is because, while the 4s is *on average* larger than the 3d, that's just the average, and the radial distribution of the 3d orbital actually has some significant peaks well *inside* the 3d – giving it better visibility of and attraction to the nucleus than the 3d. Thus the 4s is actually the lower energy level, and so it gets occupied first. So, even though the iron atom has unpaired electrons, they *aren't* in the outer orbital, the electrons out there are paired, and it is the electrons in the *outer* orbital that dominate inter-atomic bonding. In truth, both 3d and 4s electrons do participate in the bonding, but since the 4s is *so* much larger than the 3d, it really dominates. Now, for *most* atoms, but not iron, it's the outer level that is only partially filled, which facilitates covalent bonding – that is, they essentially fill their vacancies by time-sharing outer electrons with their neighbors. Since these vacancies have the opposite spin orientations of the already occupied states, the bonding electrons are encouraged to anti-align, and thus there is no net spin, and no net field. The other extreme would be noble gases which have all their occupied shells full – their interactions are weak enough that they don't bond except at *very* low temperatures. However, iron has a full outer level *and* vacancies in its next level. The result is metallic bonding – that's where the electrons join up in an unlocalized electron sea (rather than pairing up in localized covalent bonds.) They *want* to do this because they get more room in the sea than when stuck to their own atoms, this spells a lowering of their energies (relative to being localized on their atoms). It's crucial to note that, the electron that occupies one of these extended ‘sea’ states, that stretches from atom to atom has got a particular spin and, locally, is still subject to Hund’s rule – it’s advantageous for it to have the same spin orientation as the other unpaired electrons in the sea!

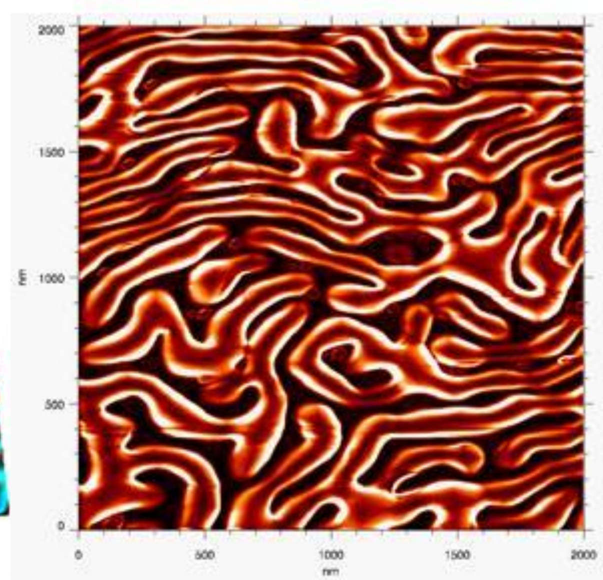
Note: If the 3d level were less full than it is for iron, it would be larger (fewer protons in the nucleus attracting its electrons) and so 3d’s unpaired electrons would play more prominent roles in the bonding, probably making the bonds more covalent like, and thus encouraging anti-alignment between bonding electrons. This is probably why K, Ca, Sc, Ti, and V aren’t famous ferromagnets. Conversely, if it were more filled, there’d be less unpaired electrons, so less field, and the 3d would be smaller, so weaker interaction between the electrons in one atom’s 3d and those in another’s.

Hook and Hall’s Solid State Physics has a useful discussion of Ferromagnetism in chapter 8. It discusses the origin of the “exchange” interaction, the existence of a conduction band, and suggests that the exchange interaction still holds for the electrons in that band. Fig. 8.4 gives the density of states for iron’s 4s and 3d in the solid. Some books online show more detailed densities of states, but the same idea holds – more states aligned than unaligned below the Fermi energy.

Here are two representations of a Magnetic Force Microscopy image of different domains on a Co/Pt surface (http://www.omicron.de/index2.html?/rom/mfm_image_taken_at_a_co_pt_multilayer_surface/index.html~Omicron)





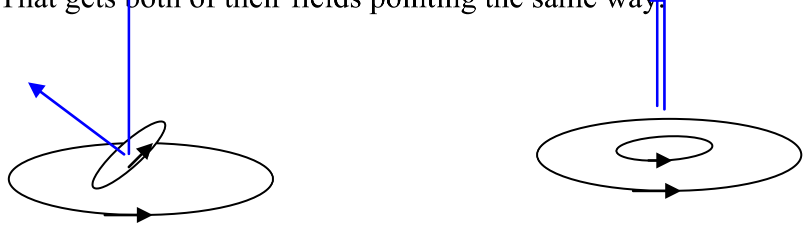


The different apparent heights reflect how high the MFM’s tip (cantilever with a microscopic magnet on the end) was pushed up or drawn down by the magnetic field of the surface – thus magnetic domains anti aligned with the tip push the tip away / are shown as red/yellow and high while domains aligned with the tip draw it down / are shown as blue/black and low.

More schematically, we will represent this with several magnetic dipole moments (arrows) in the same direction. If there is no external magnetic field, the domains of iron are in random directions as shown below, so it produces no net magnetic field.

Multiplying Effect

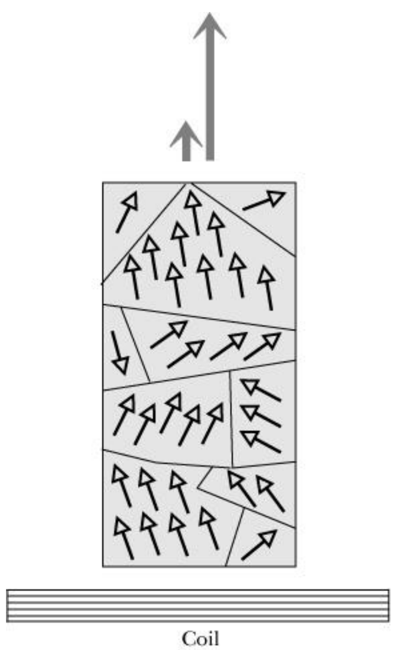
When an un-polarized chunk of ferromagnetic material is subjected to an external field, it induces polarization (ether whole-sale flipping unaligned domains, or more subtly expanding aligned domains.) we've yet to see how this works, since we've not really dealt with the magnetic force yet, but if you recall the demo I did the first day of this chapter, two parallel currents attract each other, so, say you've got a big, stationary current loop and a small current loop that's free to rotate, you can see that the one that can rotate would rotate to get its current as parallel as possible relative to the big loop. That gets both of their fields pointing the same way.



Demo: need a big current loop and a small rotating one (perhaps we have one in our magnet-wire pendulum kit.

Now, if we think of each magnetic domain as a current loop, we can see that it will rotate to align with the external current. In terms of field, that means that its aligns so that its field is parallel to the externally applied one. So, an externally applied field reorients the microscopic current loops (that already exist) in a Ferromagnet so as to increase the net field.

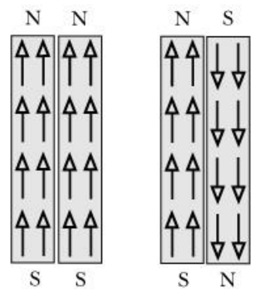
If an external magnetic field is applied (e.g. with a coil in the figure below), the domains will somewhat in the same direction. This will produce a magnetic field in the same direction as the external magnetic field (the "magnetic multiplier effect" of Exp. 17.11).



When the external field is removed, the domains mostly return to the disordered state for some materials (e.g. iron), but remain more aligned in other materials (e.g. Alnico).

The atoms interact strongly at short range to cause neighboring atomic magnetic dipole moments line up with each other. This is the source of the magnetic domains.

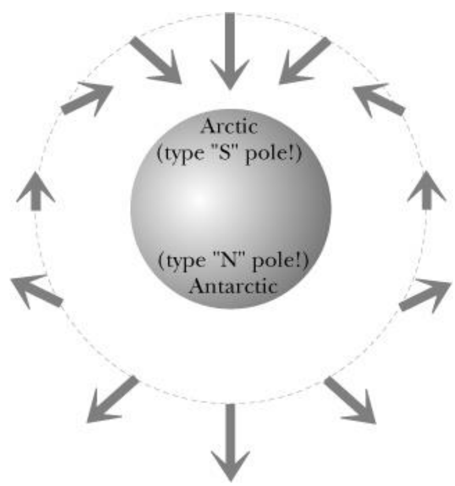
Why don't all of the magnetic domains oriented in the same direction to form a single domain? That would be unstable. You can model the situation with two bar magnets. A magnet (or domain) will have tendency to line up with the field of a magnet next to it. If the magnets start side-by-side as shown below on the left, one will flip. Of course, the magnets will line up if they are end-to-end.



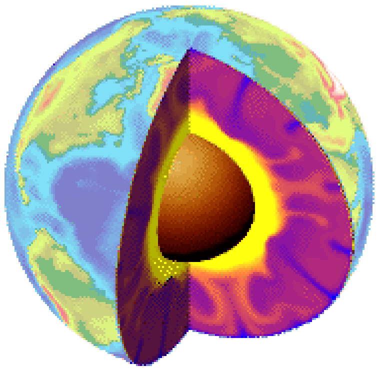
The interaction which causes the magnetic dipole moments to flip is a weaker, but has a longer range.

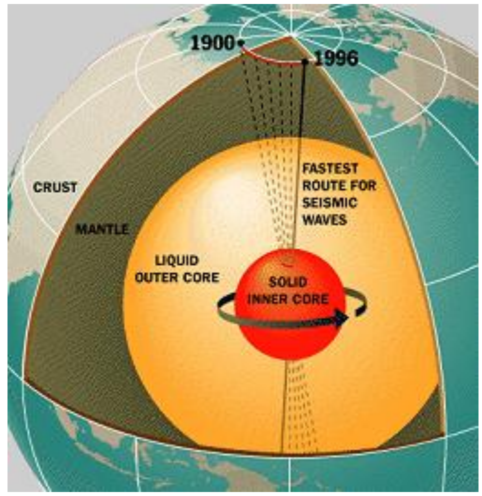
The Earth's Magnetic Field

The Earth field is that of a dipole as shown below. Unfortunately, the <u>magnetic</u> S pole is in the Artic at the <u>geographic</u> north pole. The N pole of a compass needle is attracted to the S pole in the artic.



Both with macroscopic currents in wires and microscopic ones in atoms of magnets, we can see that they are the sources of magnetic fields. What's the other big thing that has a field? The Earth. So, where's its current?

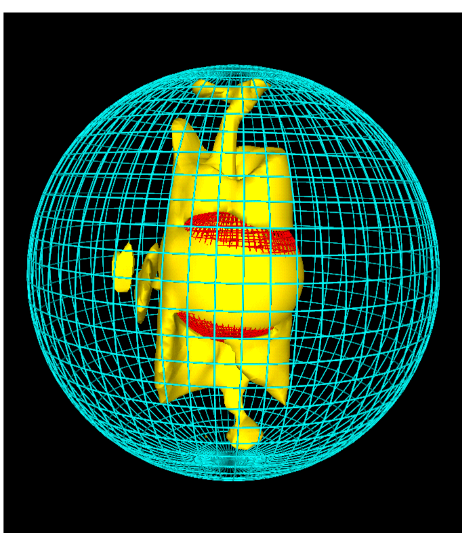


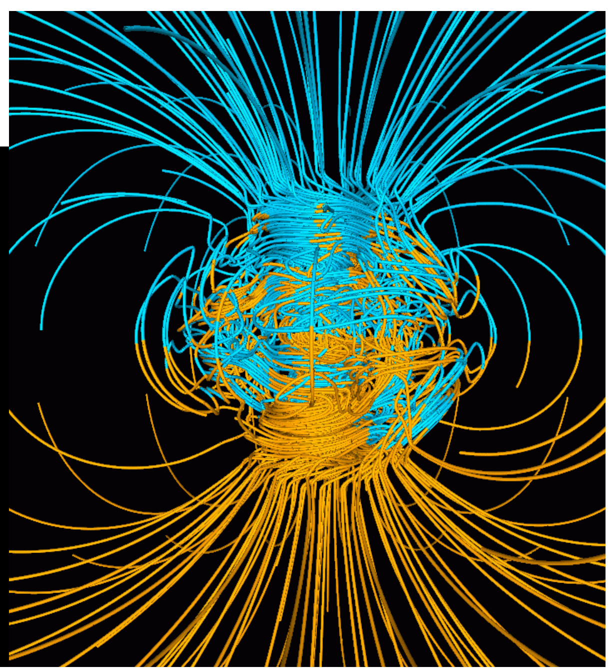


(http://science.nasa.gov/headlines/y2003/29dec_magneticfield.htm)

The planet's liquid outer core is rich in iron. Smooth rotation of neutral metal liquid would not constitute a current. Instead, we need differently charged particles to be moving in different directions. Here's how that arises. First, it's quite hot in the liquid outer core – the inner core is about as hot as the surface of the sun – that's hot enough to ionize atoms, and thus allow positive ions and negative electrons to circulate sepeartely. Second there's a lot of turbulence, which locally gives some circulation as well as additional heating. This arises because the solid inner core and the planet surface rotate at different rates, because heat generated with in the core (inner and outer, due to friction as well as radioactive decay) convects out toward the surface - drawing hot materials up (and cooling in the process). So we end up with quite a seething mess, not too unlike our atmospheric weather. It is these 'hurricanes' and such that generate the strongest magnetic fields.

Here's a simulation highlighting the regions where the circulation would be expected to be greatest and the resulting magnetic field



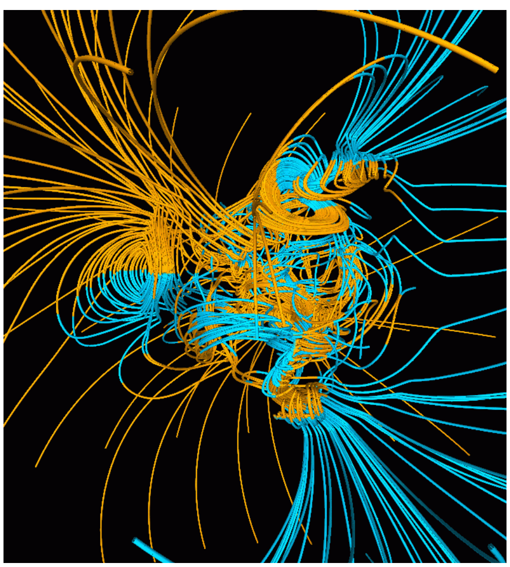


http://www.es.ucsc.edu/%7Eglatz/geodynamo.html

But since this depends on something chaotic like storms, maybe it's not completely surprising that its unstable. Not only can it migrate

(http://science.nasa.gov/headlines/y2003/29dec_magneticfield.htm)

But it can migrate all the way upside down. In the process, the Earth's magnetic field can get pretty gnarly.



The book points our that the earth's field isn't just parallel to the Earth's surface, it's that of a dipole, so it has vertical as well as horizontal components.

“controlling” free current: In describing equation 6.20, he says H allows us to express Amperes Law in terms of free current alone and "free current is what we control directly", can you explain what he means by this? He means we control if we choose to find free current or not?

$I_{f.enc}$ from M: How can we find I_f(enc)- free enclosed current, when only given magnetization?

I, too, would like to see this gone over Same! Seconded.

More info would have to be given. If it’s a linear medium, you can back it out; or perhaps the implication is that there is none.

Not explicitly Linear: If [the problem doesn't say that the material is linear or give a susceptibility constant or magnetization], must we assume that it's not linear? I know that a ferromagnet is absolutely not going to be linear, but if the material is diamagnetic or paramagnetic and when [], how do we know that it's not linear?

Right – at least, we mustn’t assume that it *is* linear.

B from free currents, bound currents: Could we do some examples of finding the B field due to the free current(s)? Also talk about when to use free current, bound current or both.

Yes this would be helpful. i agree, how do we know it a bound current or not. Agreed

B is from *all* the current; part of our task in a given problem is to determine what all the current is.

Permanent Magnet: Could we go over an example in figuring out how we see torque shift its weight to one side of an objects domain and how it then becomes saturated? Essentially understanding how we can calculate that an object has become a permanent magnet?

We don’t get quantitative with Ferrimagnets in this course

6.16: Can we do problem 6.16? Seconded.