Exam 1 (Ch 2, 3)

Ch 2.

2.1. The Electric Field

2.1.1 Intro

2.1.2 Coulomb's

$$\vec{F}_{Q\leftarrow q} = \frac{qQ}{4\pi\varepsilon_o}\frac{\vec{\mathscr{r}}}{(\mathscr{r})^3}$$

2.1.3 Electric Field

$$\vec{E}_q = \tfrac{1}{4\pi\varepsilon_o}\frac{q}{\mathscr{r}^2}\hat{\mathscr{r}}$$

$$\vec{E}(\vec{r}) = \sum_{i=1}^{n}\vec{E}_i(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^2}\hat{\mathscr{r}}_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^3}\vec{\mathscr{r}}_i$$

Prob. 2.2 (field of 2 point sources)

2.1.4 Continuous Charge Distributions

Example 2.1(field of a line charge)

Field of a Sheet

Prob. 2.5 (field of a ring)

2.2 Divergence and Curl of Electrostatic Fields

1.15 ab, 1.18ab (practice with Div & Curl)

2.2.1 Field Lines, Flux, and Gauss's Law

$$\Phi_E = \oint \vec{E}\bullet d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

2.2.2 The Divergence of E

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

2.2.3 Applications of Gauss's Law

Given piece-wise defined field due to cylinder, find charge density (cylindrical coordinates)

Given charge density of sphere, find field (spherical coordinates)

Given charge density of plane, find field (Cartesian coordinates)

2.2.4 The Curl of E

$$\vec{\nabla}\times\vec{E} = 0,\ \oint \vec{E}\cdot d\vec{\ell} = 0$$

2.3 Electric Potential

2.3.1 Intro to Potential

$$\Delta V_1 \equiv \frac{\Delta P.E._{1\to 2}}{q_2} = -\int_a^b \vec{E}_1(\vec{r}_2)\cdot d\vec{\ell}\ ,\quad \boxed{\vec{E} = -\vec{\nabla}V}$$

$$V(r) = \frac{q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]_r^{\infty} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Exam 1 (Ch 2, 3)

Given E, demonstrate it's curl-less, and find V.

2.3.2 Comments on Potential

2.3.3 Poisson's Equation and Laplace's Equation

$$\boxed{\nabla^2 V = -\rho/\varepsilon_0}$$

2.3.4 The Potential of a Localized Charge Distribution

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{\mathscr{r}_i}$$

Prob. 2.22 - Potential of a Uniform Line Charge given the field

2.3.5 Summary; Electrostatic Boundary Conditions

2.4 Work and Energy in Electrostatics

2.4.1 The Work Done to Move a Charge

$$W_{q_i} = \frac{1}{4\pi\varepsilon_0} q_i \sum_{j\neq i} \frac{q_j}{\mathscr{r}_{ij}} \quad W_{\to q\, a\to b} = q(V(b) - V(a))$$

Brought charge in from infinity to presence of others

2.4.2 The Energy of a Point Charge Distribution

$$W = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{j<i}\frac{q_i q_j}{\mathscr{r}_{ij}} = \frac{1}{2}\sum_{i=1}^{n} q_i V(P_i)$$

Assembled simple point-charge configurations

Assemble Capacitor

2.4.3 The Energy of a Continuous Charge Distribution

$$W = \frac{1}{2}\int \rho V\, d\tau, \quad W = \frac{\varepsilon_0}{2}\int_{\text{all space}} E^2\, d\tau$$

Energy of building a solid sphere (field piece-wise defined)

2.4.4 Comments on Electrostatic Energy

2.5 Conductors

2.5.1 Basic Properties

***E* = 0 inside a conductor**

ρ =0 inside a conductor

Any net charge resides on the surface(s) of a conductor

***V* is constant throughout a conductor**

$\vec{E}$ is perpendicular to the surface, just outside a conductor

2.5.2 Induced Charge

Sphere with off-center Cavity.

2.5.3 Surface Charge and the Force on a Conductor

$$\vec{F}_{ext} = (\sigma A_{patch})\vec{E}_{ext}$$

Exam 1 (Ch 2, 3)

$$P = \frac{\vec{F}_{ext}}{A_{patch}} = \frac{\sigma^2}{2\varepsilon_o} = \varepsilon_o \frac{E_{net}^2}{2}$$

2.5.4 Capacitors

$$C = \frac{Q}{V}, \quad V = \left|\vec{E}\right| d = Qd/A\varepsilon_0$$

Ch. 3

3.1 **Laplace's Equation** (region with no charge density)

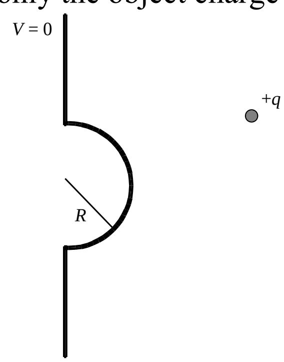
$$\nabla^2 V = \vec{\nabla} \cdot \left(\vec{\nabla} V\right) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

- Implies:
    - *no local maxima or minima*

      $$V(x,y) = \frac{1}{2\pi R} \oint_{\substack{\text{circle of} \\ \text{radius } R}} V \, d\ell$$

    -
    - Uniqueness th'm: If you found *a* solution that satisfies the boundary conditions, you've found *the* solution.

3.2 The Method of Images

- *Method of Images* – replace a problem with a simpler equivalent one (based on corollary of the first uniqueness theorem)
    - More specifically, usually if a boundary is an equipotential, then dream up a charge configuration *outside* the boundary that would help make it so.
- **Example:** Plane 'mirror'
    - $$\frac{q_o}{r_o} + \frac{q_i}{r_i} = 4\pi\varepsilon_o V(z=0) = const$$
    - (find q_i and r_i that satisfy for given q_o and r_o.)
- **Example:** things build of planes or planes and spheres
    - Warning: Like regular images and mirrors – the conductors reflect not only the object charge but also each other's image charges.
    - 

- **Things you can find once you have V:**

Exam 1 (Ch 2, 3)

- **Surface charge density**
  - $\sigma = -\varepsilon_o \frac{\partial V}{\partial n}$
- **Induced surface charge**
  - $q_{surf} = \int_{surf} \sigma da$
- **Force**
  - $\vec{F}_{cond \to q_o} = q_o \vec{E}_{cond}(\vec{r}_o) = q_o \vec{E}_{q_i}(\vec{r}_o)$
- **Work**
  - $W = \frac{\varepsilon_o}{2} \int_{all.space} E_{total}^2 d\tau = \frac{\varepsilon_o}{2}\left(\int_{outside} E_{total}^2 d\tau + \int_{inside} E_{total}^2 d\tau \right) = \frac{\varepsilon_o}{2}\left(\int_{outside} E_{total}^2 d\tau + 0 \right)$

3.4 Multipole Expansion

- $V(\vec{r}) = V_{\text{mon}}(\vec{r}) + V_{\text{dip}}(\vec{r}) + V_{\text{quad}}(\vec{r}) + \ldots$

  $V_{\text{mon}}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{1}{r}\sum_i q_i$

  .4 $V_{\text{dip}}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\sum_i q_i r_i' \cos\theta' = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$

  .5 $\vec{p} \equiv \sum_i q_i \vec{r}_i'$

  .6

  $\vec{E}_{dip} = -\vec{\nabla} V_{\text{dip}} = -\left(\frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi} \right)$

  $= \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$

- **Problem**: Discrete Points on axes
- **Problem**: Continuous charge distributions – line, sphere with different charge densities

Instructions and Equations You'll be given with Exam 1

Instructions

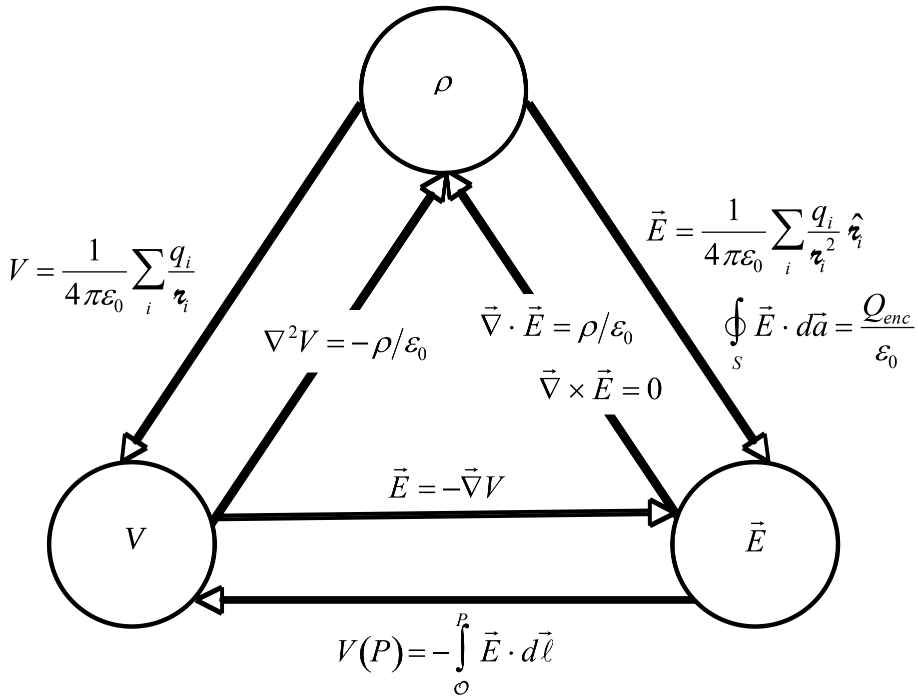
- Start each problem on a new page,. Clearly label all of your solutions.
- Show *all* of your work. You will only get credit for what you wrote, not what you meant.

Exam 1 (Ch 2, 3)

- In order to receive full credit, you must explain your physical reasoning and show your mathematical work in full. Getting the "right answer" in itself will only earn you a small fraction of the possible credit on a problem
- Be sure to include correct units with all numerical quantities, not just with your final answer.
- Use proper notation.

Potential Useful Information

Electrostatics:



$\varepsilon_0 = 8.85\times 10^{-12}\ \mathrm{C^2/(N\cdot m^2)}$ $\qquad$ $\vec{F}_{elec} = Q\vec{E}$

$E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = \frac{1}{\varepsilon_0}\sigma$ $\qquad$ $\vec{E}^{\parallel}_{\text{above}} = \vec{E}^{\parallel}_{\text{below}}$ $\qquad$ $V_{\text{above}} = V_{\text{below}}$

$W_{a\to b} = Q[V(b) - V(a)]$ $\qquad$ $C = Q/V$

$V_{mon}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ $\qquad$ $V_{dip}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$ $\qquad$ $\vec{p} = \sum q_i\vec{r}_i'$

In addition, you will get a copy of the inside of the front cover of the textbook.