Physics 310
2b – Circuit Transients and Oscilloscopes

Mon. 1/25 Wed. 1/27 Thurs. 1/28 Fri. 1/29	**Ch 2.5-.6, & 6.6-7, App B-2:** Capacitors, Inductors, & Oscilloscopes **Quiz** Ch 2 & 6, **Lab 2:** Oscilloscopes & Circuit Transients more of the same **Ch 3, 4.5, 6.5:** AC Circuits	**HW2**: Ch2 Pr 1,2,3,6* ,8,10 **Lab 2 Notebook**

Materials:
- Optional: Supplement about capacitors' horrid labels (from pp. 51-53 of H&H Studentmanual) – be careful about polarities!
- Set up computer for http://www.falstad.com/circuit/ - from circuit menu, select basics / capacitor, Inductor, LCR
- 5Spice set up to analyze problem Ch1 #25 to demonstrate its use.

Announcements
- **Lab –** Change of guidelines:
  - not need to be in pen
  - no uncertainty work unless explicitly requested
  - In general, you'll make comparisons between theory and experiment by % differences.
  - Do you Pre-Lab work on separate paper so you can easily tape it into your notebook at the appropriate place.
  - Save to thumbdrive / email yourself 5Spice code so you can use it in class for generating theoretical values to compare.

Study List for Quiz #2:
1. Capacitors (series/parallel combinations).
2. Inductors
3. RC Circuit transients (charging and discharging)
4. RL Circuit transients

Equation List: [units are in square brackets]

$$1/C_S = 1/C_1 + 1/C_2 + 1/C_3 + \ldots \qquad C_p = C_1 + C_2 + C_3 + \ldots$$

$$v_C = Q/\mathrm{C}\ [1\ \mathrm{V}=1\ \mathrm{C/F}] \qquad dv_C/dt = i/C \qquad \tau_{RC} = RC\,[1\ \mathrm{s} = 1\ \Omega\cdot F]$$

$$v_L = L(di/dt)\ [1\ \mathrm{V} = 1\ \mathrm{H\cdot A/s}] \qquad \tau_{RL} = L/R\ [1\ \mathrm{s}=1\ \mathrm{H}/\Omega]$$

<u>Topics</u>:
- Symbols for capacitors (also polarized) and inductors.
- Voltage relation for capacitors and series & parallel combinations
- Voltage relation for inductors
- RC circuits (transient behavior – turning voltage on or off)
- RL circuits (transient behavior – turning current on or off)

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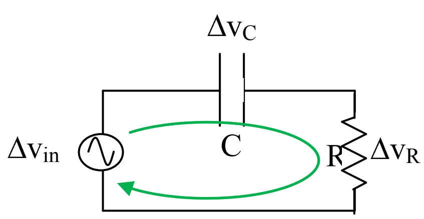
- RC combinations as differentiators or integrators (see pp. 25-7 of H&H for math)
- RLC transient behavior - “ringing”

1. Do all inductors have ferrite in them or just certain ones?
2. I get how to calculate the inductance, but what exactly is the significance of inductance?
3. Could you please go over the graphs that go along with the resistor-inductor circuits?
4. 1. What exactly is it that makes an RC circuit an RC circuit? What are the important qualities that make then have the derivative and proportionality rules?
5. Lissajous figures?
6. How to graph Vout or Vin vs. time.

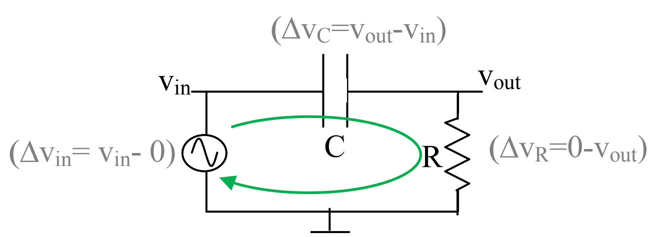
Chapter 2: Capacitors and Inductors

Alternative Representations

- You will encounter three different schematic representations of circuits; it’s worth getting familiar with them and recognizing their equivalence. The representation we’ve used thus far is particularly useful for helping us think about what the *current* is doing, since we explicitly draw the path the current takes and the corresponding voltage changes across different elements. However, as we go on in this course, our focus will shift to looking more at the voltage values at specific locations in the circuit; also, we’ll look at more and more complex/cluttered circuits. For both of these reasons, its nice to eliminate from our schematics representations of the rather boring lines along which the current travels, unimpeded, back to the source’s negative terminal.
- As for switching from focusing so much on the voltage *across* an element to the voltage *at* a point, the representation shifts like this

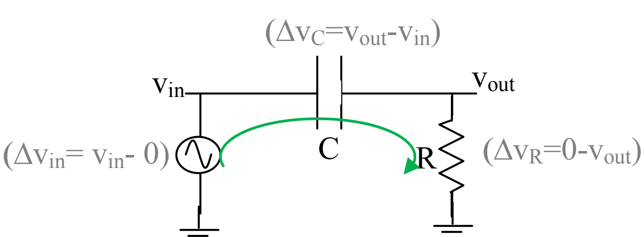


To



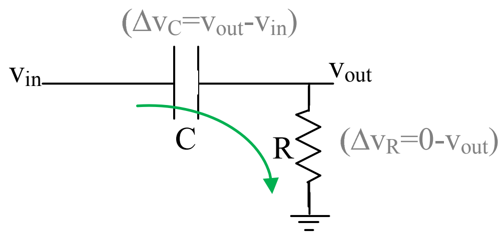
Now the voltage values being labeled are those at particular locations, and are implicitly measured relative to “ground.” The part of the circuit that’s at the reference value has a tied to it a symbolic spade that would be stuck in the ground. I’ve put in parentheses how the voltages *across* the circuit elements are related to the voltages at the different points.

The next step in the evolution is to ditch the really boring wire along the bottom, and individually label the important “grounded” points in the circuit



Sure, if both the resistor and the supply have an end stabbed in the ground, then they are electrically connected, so we just aren't explicitly drawing that connection

Now, a very slight variation on this theme is that the schematic might not bother representing the voltage source with a component; you don't need to know what physical entity generated v_{in} to know how v_{out} will be related to v_{in}.

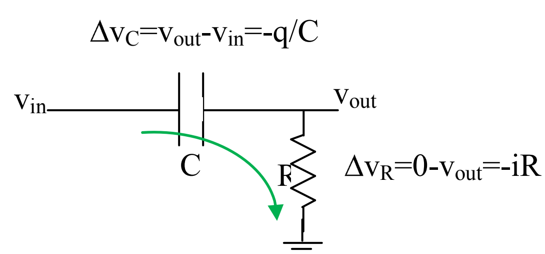


Evolving our Circuitry Concepts toward Signal Processing. Remember that the first day of class I said modern electronics is mostly about first encoding information (say, the words you speak) in an electronics signal, usually the voltage, and then operating on that information. This kind of schematic representation lends itself to considering a circuit that way. For example, say v_{in} comes from a microphone and v_{out} goes to a loudspeaker. So, the information is what you're saying; that's written into time variations in a voltage (rather than spatial variations ink) and then manipulated by this little circuit before being read back out. How does this little circuit 'process' your voice? As we'll see next chapter, it filters out the lower frequency components, so maybe it's selecting which part of the 'signal' is appropriate to send to a tweeter.

Differentiators

This bit of circuitry actually performs a useful function in the context of a larger, more complex circuit. The output voltage is *approximately* the derivative of the input voltage.

$$\Delta V_C = v_{out} - v_{in}$$
$$-q/C = v_{out} - v_{in}$$
$$-i/C = \tfrac{d}{dt}V_{out} - \tfrac{d}{dt}V_{in}$$
$$-\frac{V_{out}}{R}/C = \tfrac{d}{dt}V_{out} - \tfrac{d}{dt}V_{in}$$
$$V_{out} = RC\left(\tfrac{d}{dt}V_{in} - \tfrac{d}{dt}V_{out}\right)$$

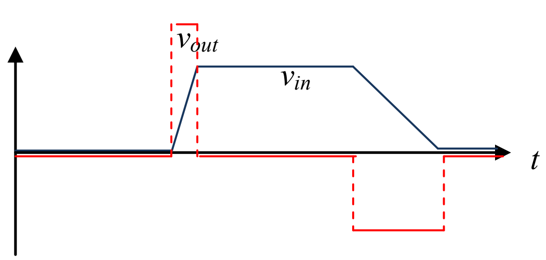


If V_{out} changes much more slowly than V_{in}, (say, RC is quite large) then

$$V_{out} = RC\tfrac{d}{dt}\left(V_{in} - V_{out}\right) \approx RC\tfrac{d}{dt}\left(V_{in}\right)$$

In a few chapters, we'll see how incorporating an operational amplifier (op-amp, for short) into this kind of circuit makes that approximation, for most practical purposes, an equality.

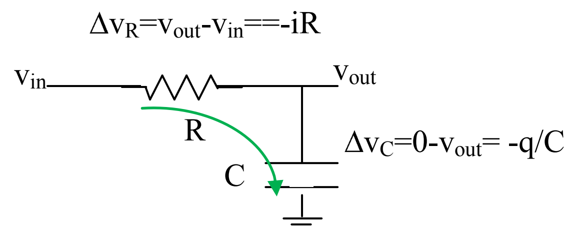
For an *ideal* differentiator,



Q: Why differentiate? In isolation, little snippets of circuitry like a 'voltage divider' or a 'differentiator' look a little pointless. They're usually small segments of larger circuitry, of a 'device' that's actually supposed to accomplish something. Let's imagine one, like a resistive position sensor that produces a voltage which is proportional to something's position. Then if you're interested in the *velocity,* you pass that signal to a bit of 'differentiator' circuitry. A virtue of doing this is that the circuitry doesn't care how 'non-analytic' the signal is, it just differentiates.

Integration

Of course, the inverse of differentiation is integration, and flipped around a little, this circuit's output voltage is *approximately* the time integral of the input voltage.



$$\Delta v_R = -iR = v_{out} - v_{in}$$
$$iR = v_{in} - v_{out}$$
$$\int (iR)\,dt = \int (v_{in} - v_{out})\,dt$$
$$qR = \int (v_{in})\,dt - \int (v_{out})\,dt$$
$$(-C\Delta v_C)R = \int (v_{in})\,dt - \int (v_{out})\,dt$$
$$RCv_{out} = \int (v_{in})\,dt - \int (v_{out})\,dt$$
$$v_{out} = \frac{1}{RC}\left(\int (v_{in})\,dt - \int (v_{out})\,dt\right)$$

If the integral of V_{out} is much less than that of V_{in}, then

$$V_{out} \approx \frac{1}{RC}\int V_{in}\,dt$$

Again, in a later chapter we'll use an operational amplifier to make both the Differentiator and Integrator much more exact.

Ch 2a (capacitor) Group Problems (different problem for each group)

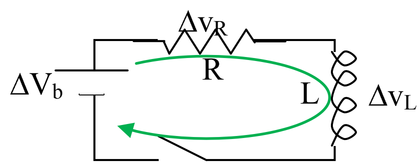
2-4 Inductance

$$\Delta V = -L\frac{dI}{dt}$$

- You can look back to Phys 232 to see where this relationship comes from. Actually, the derivation there, though traditional, is a little misleading. Just to jog your memory, the traditional derivation is to go through Faraday's law that correlates change in magnetic flux to non-coulombic electric field. Since a magnetic field is proportional to the current that produces it, Faraday's law can be rephrased in terms of the changing current rather than the accompanying field. Meanwhile, the electric field can be related to an accompanying voltage across a distance.

- Thinking physically, fundamentally, time varying current means accelerating charges and accelerating charges generate an electric field that is not canceled by that of the stationary ion cores in the wire. Qualitatively, this field opposes the acceleration, opposes the change in current. Whether the current's increasing (as when a circuit is turned on) or decreasing (as when a circuit is turned off). For more detail, look at Griffith's equation 10.29 to see that a time-varying current is responsible for a field, naturally the charges within that stream itself are subject to the fields of their neighbors. Alternatively, look at Griffith's equation 10.65 to see that accelerating charges are responsible for fields (technically, not at locations in line with the acceleration, but a charge ever-so-slightly off axis will feel something.)

2-5.2 Resistor-Inductor Circuits



Qualitatively

- When the switch closes, the current grows quickly; however, that's exactly what an inductor responds the strongest to – a quickly changing current, and the voltage it generates opposes the change as if the inductor were a huge resistor, so the current starts out small. Eventually, the current will settle down to a constant value; at that point, there will be no more change in current, so the inductor will no longer actively oppose the flow, i.e., it will be little more than a stretch of wire.

Demo: http://www.falstad.com/circuit/

Quantitatively

- Applying the Loop Rule,

  $$\Delta V_b + \Delta v_R + \Delta v_L = 0$$

  - $$\Delta V_b - iR - L\frac{di}{dt} = 0$$

  $$\Rightarrow \frac{di}{dt} = \frac{1}{L}\left(V_b - iR\right)$$

-
- Guess exponential with a constant: $i = A + Be^{-t/\tau}$
- Plug in and find that, if the final case is that it's as if the inductor weren't there at all, $A = -B = -V_b / R$, and $\tau = L/R$
- $i = -\left(V_b/R\right)\left(1 - e^{-tR/L}\right)$

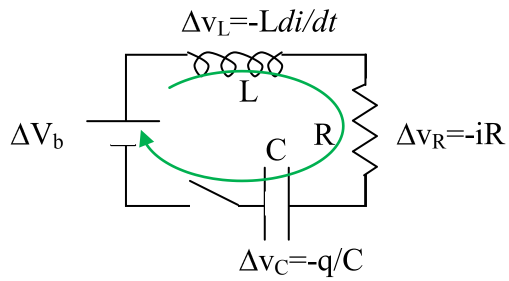
Qualitative and Quantitative

- So, the current starts at 0, fully opposed by the induced voltage across the inductor, and grows up to V_b/R, as if the inductor were just a straight wire.
- **Analog**
  - Here we go again, with another pair of 'electronic' parameters determining a time. Here's a physical analog (perhaps a little more of a stretch than the hour glass for

an RC circuit): A mass pulled through a viscous fluid. The time is how long it takes to come to terminal velocity / constant speed. Aside from the force you exert, two things determine that time: the object's mass and the drag coefficient. On the one hand, the drag grows with the object's velocity – more resistance the faster it goes; on the other hand, the more massive the object is, the slower its velocity *changes* in response to your applied force. For a given applied force, these two things determine how long it takes for the object to get going fast enough that the drag balances your applied force. In the circuit, it's the charges' rate, Current, that we're watching; the resistor impedes flow with a V=IR term (proportional to the rate, like the drag) while the inductor slows the *change* in flow with a V=L dI/dt term (like the mass).

Appendix B

B-2 Transient LRC Circuit Response

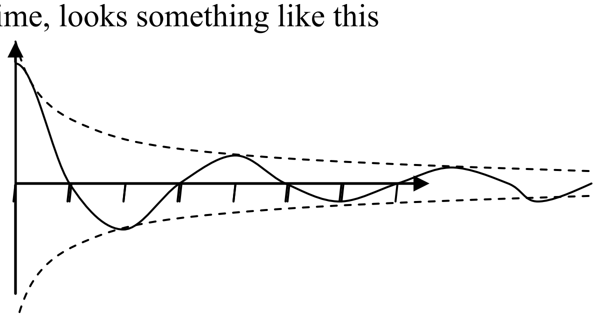


$$\Delta V_b + \Delta V_L + \Delta V_R + \Delta V_C = 0$$

- $$\Delta V_b - L\frac{di}{dt} - Ri - qC = 0$$

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + iC = 0$$

- (the last equation results from differentiating and multiplying by -1)
    - **Analogy**
        - You may have seen a second-order differential equation like this before, in Advanced Classical Mechanics. That would have been to describe the Damped Harmonic Oscillator. The *resistance* introduces *damping* (a drag like term), the *inductor* introduces *inertia* (a term that resists change), and the *capacitor* introduces *resilience* (a term that stores charge and then releases it again).
        - This is mathematically of the same form as
        - $0 = m\frac{d^2 y}{dt^2} + c\frac{dy}{dt} + k_{sp} y$ for a damped harmonic oscillator.
    - Since these equations have the exact same mathematical form, we know that the solutions must as well, both quantitatively and qualitatively. So let's first transfer over our qualitative understanding of a mass on a spring, then we'll have an idea of what we're shooting for with the math.
    - Qualitatively, you hang a mass from a spring and pluck it, the mass bobs up and down, up and down,… and if you wait long enough the oscillation dies away. (If the mass-spring system is just hanging there in air, it'll take a while; if it's hanging in water, it'll die off much sooner.) A plot of the mass's position, x, vs. time, looks something like this



- (For more about this system, see p 64-69 of Fowles 4th Ed.)
-

Returning to the system we're actually interested in – current flowing through an LRC circuit, the current should oscillate and dampen in the exact same way. Let's find the mathematical expresson

- **Guess a solution to** $L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + i/C = 0$**:**
  - $i = I_o e^{-\gamma t}$
- **Plug it in:**

$$0 = L\frac{d^2}{dt^2} I_o e^{-\gamma t} + R\frac{d}{dt} I_o e^{-\gamma t} + I_o e^{-\gamma t}/C$$

  - $0 = L\gamma^2 I_o e^{-\gamma t} - R\gamma I_o e^{-\gamma t} + I_o e^{-\gamma t}/C$

    $0 = L\gamma^2 - R\gamma + 1/C$
  - So, our guess works al long as γ satisfies this quadratic relation. Appealing to the standard solution for a quadratic, there are two possible values for γ:
    - $\gamma_{\pm} = \frac{R \pm \sqrt{R^2 - 4L/C}}{2L} = \frac{R}{2L} \pm \frac{\sqrt{R^2 - 4L/C}}{2L}$
  - The general solution should be a linear combination of the two particular solutions:
    - $i = A_+ e^{-\gamma_+ t} + A_- e^{-\gamma_- t}$
  - Looking at the square root, there are three qualitatively different cases,
    - **Critically Damped:** $R^2 = 4L/C$
      - $\frac{1}{\tau} \equiv \gamma_{\pm} = \frac{R}{2L}$ so,
      - $i = I_o e^{-t/\tau}$
    - **Over Damped:** $R^2 > 4L/C$
      - $\gamma_{\pm} = \frac{R}{2L} \pm \frac{\sqrt{R^2 - 4L/C}}{2L}$
        - The exact values of A_+ and A_- are determined to meet the initial conditions of $i = I_o$ and, assuming the capacitor is initially uncharged,

- Imposing the Charge Condition goes like this:
  $q\big|_{t=0} = 0$
  $\int idt\Big|_{t=0} = 0$
  $\left(-\frac{A_+}{\gamma_+}e^{-\gamma_+ t} - \frac{A_-}{\gamma_-}e^{-\gamma_- t}\right)\Bigg|_{t=0} = 0$
  $-\frac{A_+}{\gamma_+} - \frac{A_-}{\gamma_-} = 0$
  $A_- = -A_+\frac{\gamma_-}{\gamma_+}$
- So, $i = A_+\left(e^{-\gamma_+ t} - \frac{\gamma_-}{\gamma_+}e^{-\gamma_- t}\right)$

  - Then imposing the current condition goes like this:
    - $I_o = i\big|_{t=0} = A_+\left(e^{-\gamma_+ t} - \frac{\gamma_-}{\gamma_+}e^{-\gamma_- t}\right)\Bigg|_{t=o} = A_+\left(1 - \frac{\gamma_-}{\gamma_+}\right)$
    - So, $A_+ = \frac{I_o}{\left(1 - \frac{\gamma_-}{\gamma_+}\right)}$ and
    - $i = \frac{I_o}{\left(1 - \frac{\gamma_-}{\gamma_+}\right)}\left(e^{-\gamma_+ t} - \frac{\gamma_-}{\gamma_+}e^{-\gamma_- t}\right)$

- **Under Damped:** $R^2 < 4LC$ For reasons that will become obvious shortly, I'll flip the order of the terms under the root and factor out the associated -1, which, when taken outside the root gives us an *i* (as in "the square root of -1", not as in "current".)
  - $\gamma_\pm = \frac{R}{2L} \pm i\frac{\sqrt{4L/C - R^2}}{2L}$
    - Then defining $\frac{1}{\tau} \equiv \frac{R}{2L}$ and $\omega \equiv \frac{\sqrt{4L/C - R^2}}{2L}$ so that
  - $\gamma_\pm = \frac{1}{\tau} \pm i\omega$
  - Using this, and using some new names for our constant factors,
  - $i = Ae^{-t/\tau}\left(B_+e^{i\omega t} + B_-e^{-i\omega t}\right)$
  - For that matter, Another way to write this is
  - $i = Ae^{-t/\tau}\left(B\cos\omega t + D\sin\omega t\right) = A'e^{-t/\tau}\cos(\omega t - \varphi)$
    - The values of the phase shift and leading constant are chosen to match the initial current and capacitor charge (say, 0 charge).
      - $i=I_0$

- If you feel like it, you can see where imposing these conditions gets you, similar to what I've done for over-damped.

Group Problem 2

Prob 10 hint: Approximate derivative as ratio of finite differences, then the Δt is the 1ms that's given.

Chapter 6: Test Equipment and Measurement

6-6 The Oscilloscope

Over the last 5 or so years, digital o'scopes have really supplanted the analog ones that the text describes, so it's not so important to get the details of how the older ones work, but the basic capacities are the same.

Q: Lisajou figures: For most applications, I think these are more confusing than they are helpful, especially since it's become easier for o'scopes to read frequencies, however, the basic idea is this:

Say you're a robot with a pen in your hand and you have one sinusoidal signal controlling the horizontal position of the pen and another controlling the vertical position. Now, if the two signals have frequencies that are harmonics of the same fundamental, then you'll draw closed shapes.

Demonstrate.

6-7 The Signal Generator and Frequency Counter