Tues. 12/6 Wed., 12/7 Thurs. 12/8	**T9** Heat Engines *S2*, *S7* **L12:** Heat Engine (**T9**) Review	**RE-T9; Lab** Notebook & Full Report 11 (no revisions) **PL 12; Quiz 10:** T7, T8 **Lab:** 2nd Draft L10
Mon., 12/12		**HW14:** T9: S.2,8,9; **Lab** Notebook (no report)
Exam 2 Wed Dec 14th @ 9a.m., Fri Dec 16th @ 6pm, or Sat Dec 17th @ 9a.m.; let's decide		

- **Equipment**
  - Ppt.
  - Stirling Heat Engine
    - my tea mug
  - Lab equipment
    - Piston, hot plate and water, cold water

Announcements

For next time – look over old HW and quizzes; bring questions

Also look over your schedule of exams for your other courses and let me know which of our three times is preferred.

Heat Engines

T9.1 Perfect Engines Are Impossible
T9.2 Real Heat Engines
T9.3 The Efficiency of a Heat Engine
T9.4 Consequences
T9.5 Refrigerators
T9.6 The Carnot Cycle

Intro.

When we began Unit T, I noted that thermodynamics and the industrial revolution played out together; without our understanding of thermodynamics, there'd be no industrialization – there'd be no modern world. Now we'll see that connection explicitly in the thermodynamics of what makes things run – engines (and refrigerators).

Demo: Stirling Engine.

- **Simple Explanation:** The engine sits between hot water and cool room air. The water heats the air in the engine, thus energy flows in, and air cools the air in the engine, thus energy flows out. In the process, some of the energy brought in is put to work, turning the propeller. This is a simple example of a heat engine.
- **Relevance:** Ever since the industrial revolution, heat engines have been at the heart of our industrial & technological society. The vast majority of work that our society does is powered by some variation of a heat engine.
- **Connection to previous material:** With the 1st law, we relate heat, work, and internal energy. With the 2nd law, we relate entropy and the evolution of systems. With the thermodynamic relation, we relate entropy and internal energy. Together, these give us the tools to describe how heat engines work.
- **More to come:** In the advanced Statistical Mechanics & Themodynamics course next year, you'll consider a wider variety of heat engines and more aspects of them.

T9.1 Perfect Engines Are Impossible

T9T.1 Classify the following hypothetical perpetual motion machines as 1st kind – violators of the conservation of energy, or the 2nd kind – decreasers of entropy.

a. An electric car runs off a battery which drives the front wheels. The car's rear wheels drive a generator which recharges the battery.
b. An electric car runs off a battery. When the driver wants to slow down the car, instead of applying the brakes he or she throws a switch that connects the wheels to a generator that converts the car's kinetic energy back to energy in the battery.
c. An engine's tank is filled with water. When the engine operates, it slowly freezes the water in the tank, converting the energy released to mechanical energy.
d. Compressed air from a tank blows on a windmill. The windmill is connected to a generator that produces electrical energy. Part of that electrical energy is used to compress more air into the tank.
e. A normal heat engine is used to drive an electric generator. Part of the power from this generator runs a refrigerator that absorbs the waste heat from the engine and pumps it back into the hot reservoir.

T9.2 Real Heat Engines

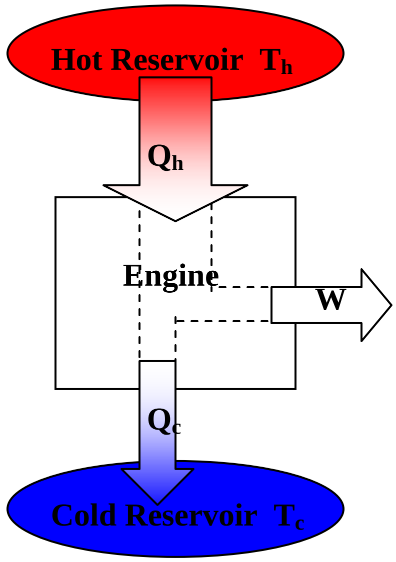
Perhaps the best way to start thinking about a real heat engine is to think about a *specific* heat engine, and then abstract from there; so let's walk through the steps of the Stirling engine.

→ **Ppt.**

Some general Vocab:

- **Heat Engine =** a system that takes in energy via heating and expels energy by doing work (and a little cooling.)
  - **Working substance** = the material that is getting heated, doing the work. This can be water in a steam engine, air in the pistons we'll experiment with.
    - **Relate to our demo.** The working substance is air that is heating and cooling as it moves around inside the fancy chamber.
  - **Cycle.** To be a useful engine, the system must ultimately return to its initial (macro) state. No practical object can withstand its temperature forever increasing, its volume forever increasing,… So our engine must "cycle" through states as it operates.
  - **Strokes.** The cycle is often broken down into a hand full of discrete steps, such as an adiabatic compression or an isothermal expansion. These steps are referred to as strokes.
- **Reservoir =** For heat to flow, one needs at least two bodies at different temperatures. So aside from the working substance, the heat engine features one object kept hot, which supplies the heat to the working substance. It also features one object kept cold, which draws any excess heat from the working substance. These are the **Hot** and **Cold** **Reservoirs.**
  - **Relate to our demo.** In the demo, what's the hot reservoir? What's the cold reservoir?
    - Hot reservoir = Cup of boiling water & hot air
    - Cold reservoir = Room of cool air

- Without getting into the details of how a specific engine works, it is sure to have the common features: hot reservoir, cold reservoir, heat flow in, heat flow out, and work. These can be represented in an abstract schematic.



  - **Energy in cycles.** An implication of going through cycles is that all the state variables are the same at the beginning and at the end; among them is the total energy. So, the first law leads to $\Delta U = Q_{h\to s} - Q_{c\to s} - W_{s\to} = 0 \Rightarrow |Q_h| = |Q_c| + W$, note, the terms have been defined so their signs are explicit: Energy flows in from the hot reservoir and out as work and to the cold reservoir.
  - **Work in and out note:** The W here is the *net* work done. In practice, there is often a little bit of work done *on* the engine to keep it working, but that is less than the work done *by* the engine, so the net work is out.

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- **Efficiency = benefit/cost** $eff = \frac{W}{Q_h}$.
  - In terms of just the heat flows, that's $eff = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|}$.
  - According to this relation, efficiency can't be > 1 since heat doesn't flow backwards. Efficiency approaches 1 only in the limit that $Q_h >> Q_c$

T9T.3 - A heat engine produces 300 W of mechanical power while discarding 1200 W into the environment (its cold reservoir). What is the efficiency of this engine?

A. 0.20
B. 0.25
C. 0.33
D. Other (specify)

T9.3 The Efficiency of a Heat Engine

T9T.2 - In a maximally efficient heat engine, the amount of entropy that the hot reservoir loses as heat flows out of it is exactly balanced by the entropy that the cold reservoir gains as the waste heat flows into it (True or False).

- **Entropy and Heat Engines' theoretical limit**
  - In the quasistatic limit, ΔS = Q/T. Looking at it from the perspectives of the heat *reservoirs*, the *hot reservoir's* entropy decreases with cooling (heat flow out), $\Delta S_{hot} = -\frac{|Q_h|}{T_h}$, while the entropy of the *cold reservoir* increases with heating (heat flow in) $\Delta S_{cold} = \frac{|Q_c|}{T_c}$. Also, through one full cycle, the engine itself must return to its initial state, so, among other things, its entropy must not change $\Delta S_{engine} = 0$. The total change in entropy of the universe is then $\Delta S_{total} = \Delta S_{hot} + \Delta S_{cold} + \Delta S_{engine} = -\frac{|Q_h|}{T_h} + \frac{|Q_c|}{T_c} + 0$.
  - However, the 2nd law says that *universal* entropy won't be reduced through a process, so $\Delta S_{total} = \frac{|Q_c|}{T_c} - \frac{|Q_h|}{T_h} \geq 0 \Rightarrow \frac{|Q_c|}{T_c} \geq \frac{|Q_h|}{T_h} \Rightarrow \frac{|Q_c|}{|Q_h|} \geq \frac{T_c}{T_h}$
  - Finally, returning to the expression for a heat engine's efficiency, $eff \leq 1 - \frac{T_c}{T_h}$.
    - For one thing, the reduced efficiency happens for non-quasistatic processes.

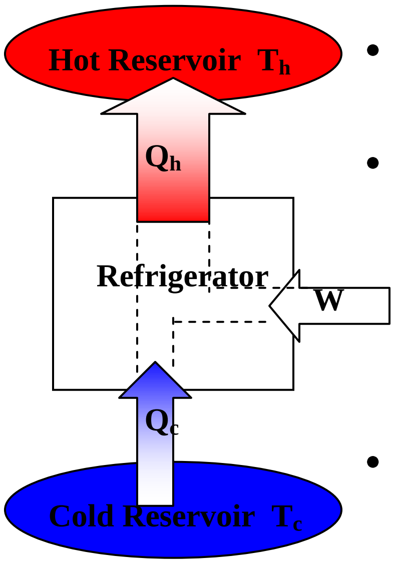
T9T.5 - Imagine that you are trying to design a personal fan that you wear on your head and operates between your body temperature (37° C) and room temperature (22° C). What is the maximum possible efficiency of this device?

A. 170%
B. 95%
C. 54%
D. 46%
E. 5%
F. Other (specify)

T9.4 Consequences

- **Qualitative Conclusions**:
  - Keep the hot side hot and the cold side cold to maximize efficiency.
    - Any temperature difference will do
    - The greater the difference, the greater the efficiency
  - The first law says 'you can't win' – you can't get out more work than heat put in.
  - The second law says 'you can't break even' – you can't avoid dumping waste heat to the cold reservoir.

T9.5 Refrigerators
Ppt.



- The refrigerator has the opposite effect as the heat engine. Work is done on the working substance so the heat can be drawn out of the cold reservoir and deposited in the hot reservoir.
- **Question:** For heat to flow from the cold reservoir into the working substance, how must the substance's temperature compare with that of the reservoir?
  - Cooler

  Question: For heat to flow from the working substance to the hot reservoir, how must the substance's temperature compare with that of the reservoir?
  - Hotter.
- **Conclusion:** The work that is done on the working substance must cool it before touching the cold reservoir and heat it before touching the hot reservoir.
- **Compressors:** This can be achieved by compressing the substance to raise its temperature, letting cool in contact with the hot reservoir, then decompressing it so that it cools even more, and finally it draws heat from the cold reservoir.
- **Coefficient of performance**
  - As with the engine, we're interested in what we get vs. what it costs. In this case what we pay for is work and what we get is cooling of the cold reservoir, so that's $\frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c} = \frac{1}{\frac{Q_h}{Q_c} - 1} = CoP$ This is referred to as the Coefficient of Performance.

T9T.7 - A refrigerator uses 100 W of electrical power and discards 600 W of thermal power into the kitchen. What is its coefficient of performance?
A. 0.17
B. 0.20
C. 5
D. 6
E. Other (specify)
F. Impossible because it violates conservation of energy

Theoretical Limit
- Again, the Second Law has something to say about how good this can be.

  $0 \le \Delta S_{total} = \Delta S_c + \Delta S_h = -\frac{Q_c}{T_c} + \frac{Q_h}{T_h} \Rightarrow \frac{T_h}{T_c} \le \frac{Q_h}{Q_c}$, the sign reversal reflects that the heat is now flowing *from* the cold reservoir and *into* the hot one.

- $\dfrac{1}{\frac{T_h}{T_c}-1} \geq CoP$ is the theoretical limit
    - For an engine where the point is maximizing the *work* and minimizing waste *heat flow into the cold reservoir*; it was best when Th was much larger than Tc. Now, to maximize *heat flow out of the cold reservoir*, and minimize *work*. So it is best when Th is very near Tc.

- What's the maximum CoP of a household air conditioner for some reasonable temperatures?
    - Let's say that the room's held at 76°F = 298 K
    - Let's say outside is a horrid 106 °F = 314 K
    - $CoP \leq \dfrac{1}{\frac{T_h}{T_c}-1} = \dfrac{1}{\frac{314\,K}{298\,K}-1} = 18$, so for every Joule of work done, 18 J of heat are removed.

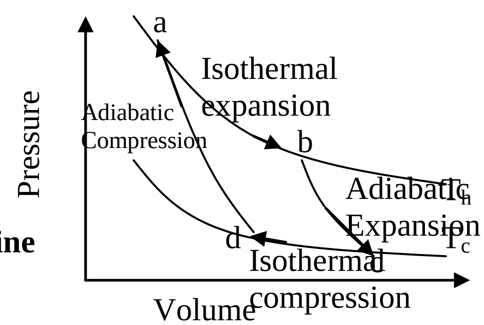
- **T9T.8 -** Someone comes to your house selling a device that draws heat from the groundwater and supplies that heat to your house. The salesperson claims that the amount of heat entering the house will far exceed the electrical energy supplied to the device. What the salesperson claims is physically impossible (True or False).

- **Heat Pump.** The point of a heat pump is not to cool the cool side but to heat the hot side (though it does both).
    - **A.** What would be the appropriate definition of the CoP for this device?
        - $CoP = \dfrac{What.you.want}{What.it.costs} = \dfrac{Q_h}{W}$
    - **B.** How would the 1st law allow us to rewrite the CoP in terms of the heats alone?
        - $\dfrac{Q_h}{W} = \dfrac{Q_h}{Q_h - Q_c} = \dfrac{1}{1-\frac{Q_c}{Q_h}} = CoP$
    - **C.** How would the 2nd law allow us to determine a theoretical limit to the CoP?
        - $\dfrac{T_c}{T_h} \geq \dfrac{Q_c}{Q_h}$ so $1-\dfrac{T_c}{T_h} \leq 1-\dfrac{Q_c}{Q_h}$ and $CoP = \dfrac{1}{1-\frac{Q_c}{Q_h}} \leq \dfrac{1}{1-\frac{T_c}{T_h}}$
    - **D.** Why is this better than an electric furnace?
        - The furnace converts electric work into heat. The best that can be hoped for is $Q_h = W$, or a CoP of 1.
        - The heat pump uses electrical work to transfer heat from a cold reservoir to a hot one. The best that can be hoped for is Qh = W + Qc which gives the above theoretical limit to the CoP.

T9.6 The Carnot Cycle ppt

- **Ideal Engine**

- Unfortunately, the Stirling Engine Cycle has mechanical limitations which prevent it from approaching the ideal efficiency of eff = 1 – T_c/T_h; assuming an ideal gas, you'll find a less favorable efficiency.
- The ideal can be approached by an engine executing the Carnot Cycle.
- **The Cycle**
  - Imagine a piston that, while being compressed and expanded, is also being slid back and forth between touching a hot reservoir and a cold reservoir.
  - **Isothermal Expansion.** We start with the piston compressed and hot, we touch it to the hot reservoir and allow it to isothermally expand.
  - **Adiabatic Expansion.** Now we pull it away from the reservoir and continue to expand it forcibly, doing some work on it. Now it adiabatically cools.
  - **Isothermal Compression.** When it's arrived at the same temperature as the cold reservoir, we place it in contact with the cold reservoir and allow it to shrink, istothermally.
  - **Adiabatic Compression.** Finally, we pull the piston away from the cold reservoir and continue to compress it, in so doing we warm it adiabatically. When it reaches the temperature of the hot reservoir, we place the small, hot piston in contact with it and begin again.



- **Efficiency of a Carnot Engine**
- a-b: Isothermal expansion

$$\Delta U = 0 = Q_{in} - W_s$$

- $$Q_h = W_s = \int_{V_a}^{V_b} P\,dV = \int_{V_a}^{V_b} NkT_h \frac{dV}{V}$$

$$Q_h = W_s = NkT_h \ln\left(\frac{V_b}{V_a}\right)$$

- b-c: Adiabatic cooling

$$\Delta U = -W_s$$

- $$\Delta U = -W_s = \frac{f}{2} Nk(T_c - T_h) \rightarrow V_b T_h^{\frac{f}{2}} = V_c T_c^{\frac{f}{2}}$$

- c-d: Isothermal compression

$$\Delta U = 0 = -Q_{out} - W_s$$

- $$Q_c = -W_s = -\int_{V_c}^{V_d} P\,dV = -\int_{V_c}^{V_d} NkT_c \frac{dV}{V}$$

$$Q_c = W_s = NkT_c \ln\left(\frac{V_c}{V_d}\right)$$

- d-a: Adiabatic heating

- $\Delta U = -W_s$
  $\Delta U = -W_s = \frac{f}{2} Nk(T_h - T_c)$, $V_a T_h^{\frac{f}{2}} = V_d T_c^{\frac{f}{2}}$

- $eff = 1 - \frac{Q_c}{Q_h} = 1 - \frac{NkT_c \ln(V_c / V_d)}{NkT_h \ln(V_b / V_a)} = 1 - \frac{NkT_c \ln\left(V_c T_c^{\frac{f}{2}} / V_a T_h^{\frac{f}{2}}\right)}{NkT_h \ln\left(V_c T_c^{\frac{f}{2}} / V_a T_h^{\frac{f}{2}}\right)} = 1 - \frac{T_c}{T_h}$

The ideal!