Thurs. 11/1	**O5** Instruments	**RE-O5; Lab** Abstract & Conclusions 2nd Draft
Mon. 11/5		**HW9:** O4:11, 12; O5: 1,3; **Lab** Notebook
Tues. 11/6	**T1** Temp, **T2** Ideal Gas	RE-T1&2
	L8: Lenses & Lab Report, **LR** 3 (**O4**)	PL8; Quiz 7: **O4, O5**
Thurs. 11/8	**T3** Gas Proc	RE-T3

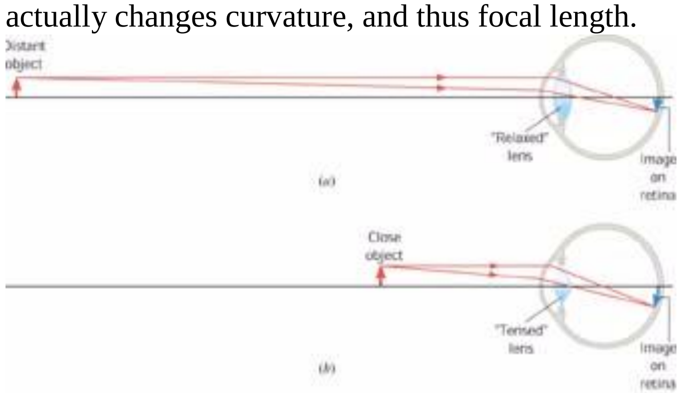
Should fix webassign problem so that “tube length” is $T = L - (f_e + f_o)$ rather than L.

- **Equipment**
  - Echo-Chamber Ray Tracing Program
  - Board Laser Optics set with extension cord
  - Table-top optics set and laser boxes to use on white boards
  - Eye model (need to fill with water to really use)
  - Microscope
  - Galileo telescopes
  - Rulers & graph paper
  - Lab 8 handout
  - Eye kits

O5 Optical Instruments
O5.1 Introduction
O5.2 The Simple Magnifier
O5.3 Simple Microscopes
O5.4 Refracting Telescopes

All of these instruments are used to help us see things, i.e., to augment the human eye.

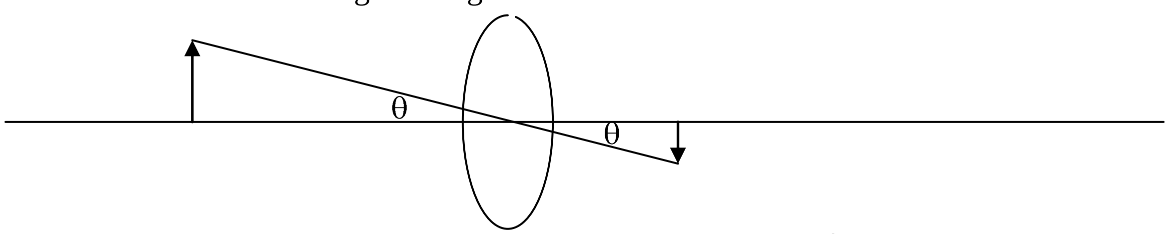
- **The Human Eye**
  - As you know from the lens equation and experienced in lab Tuesday, for a given lens, if you move the object, you simultaneously move the image – you had to move the frosted plastic sheet on which the image was resolved.
  - However, the eye has a fixed lens – to – image-plane (retina) distance, yet it can resolve objects at various distances. How does it do this?
  - One clue is that you can focus on only one distance at time – if you’re focusing on me, you can’t read a piece of paper just before you, and vice versa. So your lens actually changes curvature, and thus focal length.



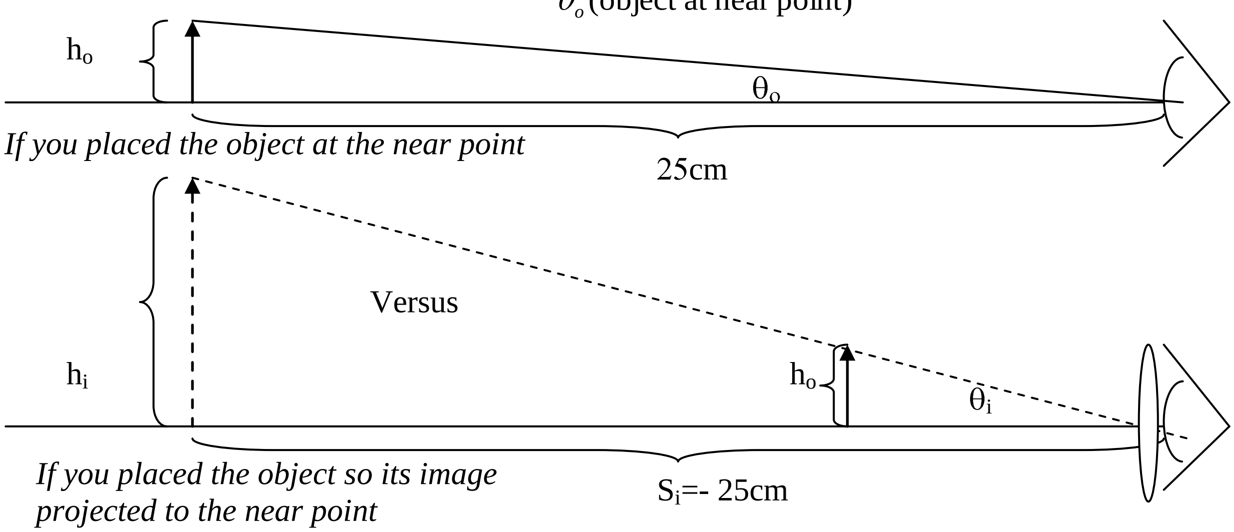
Note: this is something that even 3-D movies can’t simulate.

  - **Near & Far sighted.** A person is ‘nearsighted’ if they can’t bulge their lens to see very far or ‘farsighted’ if they can’t stretch it to see near.

- **Angular Magnification.**
  - Joe & Jane Consumer, don't know or care about a magnifying glass's "focal length", they care about how much bigger things appear when viewed through the lens than when viewed with the naked eye. Angular Magnification is defined to speak to that. Two different Angular Magnifications are defined; one to tell you the biggest useful magnification you can get with the lens (I'll call that m_{big}), the other is the most comfortable magnification you can get with the lens (I'll call that m_{comfy}.) In either case, these magnifications can be related back to the lens's focal length. Before we can do that, we need a little background.
  - **Angular Height.** The final image in an optical system involving the eye is that projected upon the retina at the back of the eye. Rather than phrasing its size in terms of how many milimeteres tall the image is, we can phrase it in terms of the 'angular height,' or the angle subtended by the image relative to the lens. Remembering that the ray from the object that passes through the center of a lens travels a straight path, you can appreciate that the angular height of the object is the same as that of the image – image and object can have different distances and heights, but same angular height.



  - **Maximum Angular height & Near Point.** If your sole goal was to maximize something's angular height, you'd simply put it right up against your eye. Unfortunately, we can't focus our eyes on things that close. The closest a person *can* focus is called the **'near point'**; that's generally about 25cm from the eye.
  - **Maximum Angular Magnification.** So, the m_{big} is defined as the ratio of the angular height of the object's image if *it* were projected to 25cm from the eye vs. the angular height of the object if *it* were placed 25 cm from the eye.
    - $m_{big} \equiv \dfrac{\theta_i(\text{image at near point})}{\theta_o(\text{object at near point})}$



- Though the ratio of angles looks only vaguely related to the regular magnification, *M*, we'll see that, assuming the magnifying glass is near the eye (think of a monocle) and that these angles are small, then it's approximately the same thing.

So, translating a lens's focal length into its angular magnification,

From the two sketches,

$$m_{big} = \frac{\theta_i}{\theta_o} = \frac{\tan^{-1}(h_i/25cm)}{\tan^{-1}(h_o/25cm)} \approx \frac{h_i}{h_o} = M = -\frac{S_i}{S_o}$$

Considering just the second sketch,

$$\frac{1}{S_o} = \frac{1}{f} - \frac{1}{S_i}$$

Substituting this in for the $1/S_o$ above,

$$m_{big} = \ldots \approx M = -\frac{S_i}{S_o} = -\left(\frac{S_i}{f} - \frac{S_i}{S_i}\right) = 1 - \frac{S_i}{f}$$

Since we're interested in when the image is projected to the near point, that is 25cm from the eye, S_i = – 25cm (negative since it's a virtual image)

So,

$$m_{big} = M = 1 + \frac{25cm}{f}$$

- **Most Comfortable magnification.** While the near point is the nearest distance for focusing on something, and thus where it should be if you want the largest angular height, focusing here for long periods of time can be straining; it's easier to relax the eye and focus off into space, that is, at infinity. So, I'll define *another* angular magnification comparing the angular height of the image *at infinity* to the angular height of the image sitting at the near point.

$$m_{comfy} \equiv \frac{\theta_i(\text{image at } \infty)}{\theta_o(\text{object at near point})}$$

- Again, when the object's actually 25 cm out,

$$\theta_o = \tan^{-1}(h_o/25cm) \approx \frac{h_o}{25cm}.$$

- And

$$\theta_i = \tan^{-1}(h_i/S_i)$$

- While both h_i and S_i may be infinity, their *ratio*, and thus the angle isn't. In fact, looking at the bottom figure on page 2, you can see the familiar fact that the triangle defined by hi and Si is similar to that defined by h_o and S_o.

$$\theta_i = \tan^{-1}(h_i/S_i) = \tan^{-1}(-h_o/S_o) \approx -h_o/S_o$$

(I've slipped in the negative sign simply because the Si shown would have a negative value.)

- So,

$$m_{comfy} \equiv \frac{\theta_i(\text{image at }\infty)}{\theta_o(\text{object at near point})} \approx \frac{-h_o / S_o}{h_o / 25cm} = -\frac{25cm}{S_o}$$

- Now, remember, we're thinking about when the image is projected to infinity; what *is* the corresponding object distance? Looking at the lens equation,

  $$\frac{1}{S_o} = \frac{1}{f} - \frac{1}{S_i},$$

  Having the image at infinity, $S_i = \infty$ corresponds to having the object at…

  $$\frac{1}{S_o} = \frac{1}{f} - \frac{1}{\infty} = \frac{1}{f} - 0 = \frac{1}{f}$$

  The focal point, $S_o = f$.

So,

$$\theta_i = \tan^{-1}(h_i / S_i) \approx \frac{h_i}{S_i} = -\frac{h_o}{S_o} = -\frac{h_o}{f}$$ (last step since $S_o = f$ in this setup)

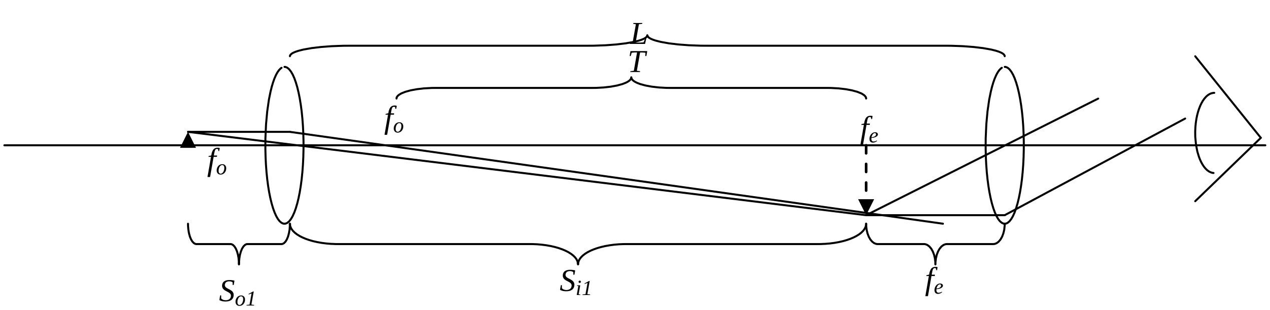
So,

$$m_{comfy} \approx -\frac{25cm}{f}$$

Exercise O5.2.1: What is the maximum angular magnification of a magnifying glass of focal length 11 cm? What is the angular magnification for relaxed eye viewing?

05.3 Simple Microscopes

The simple microscope adds another lens between the actual object and the magnifying glass. Often the lens near the object is known as the "objective" and that near the eye is known as the "eyepiece." We're still going with the comfy (image at infinity) magnification for the last / eyepiece stage, so the "object" that the eyepiece lens looks at, i.e. the image produced by the objective lens, is at the eyepiece's focal point.



As usual, the magnification from the original object to the final image that's viewed by the eye is the product of the two lens's magnifications.

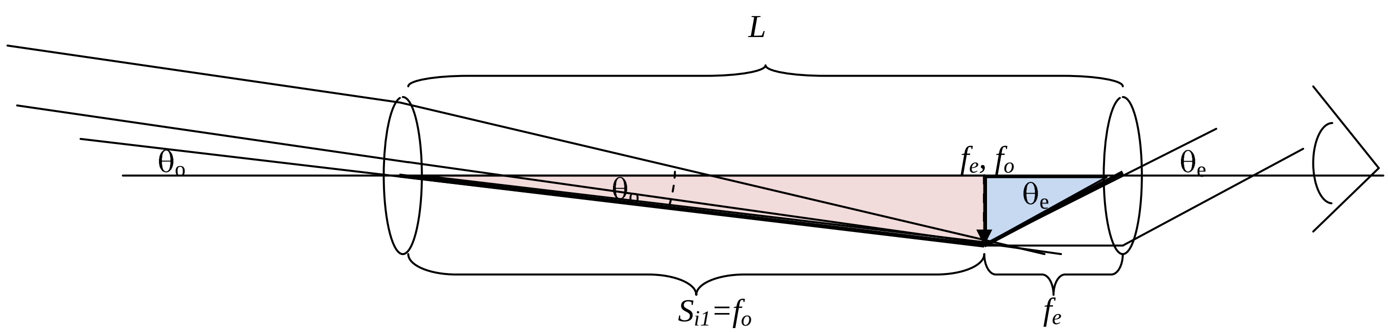
$$m_{micro.comfy} = M_o m_{ecomfy} = \left(-\frac{S_{i1}}{S_{o1}}\right)\left(\frac{25cm}{f_e}\right) = -\left(\frac{S_{i1}}{f_o} - \frac{S_{i1}}{S_{i1}}\right)\left(\frac{25cm}{f_e}\right) = -\left(\frac{S_{i1}}{f_o} - 1\right)\left(\frac{25cm}{f_e}\right) = -\left(\frac{S_{i1} - f_o}{f_o}\right)\left(\frac{25cm}{f_e}\right)$$
$$= \left(\frac{-T}{f_o}\right)\left(\frac{25cm}{f_e}\right)$$

(unless I'm missing something, the book's mention of T having being mentioned before and the 'approximation' being made here are off base.)

Exercise O5.3.1 A microscope has a tube length, T, of 20 cm and an eyepiece with a 1 cm focal length. What must be the focal length of the objective lens to get a magnification of -200?

O5.4 Refracting telescopes.

The telescope works just like a microscope except that it is used to look at very distant objects rather than near ones. In the astronomical limit, that means the rays are coming in parallel, and the image formed at the objective's focal point.
Then again, the eyepiece acts as a simple magnifier placed so that that image is at *its* focal point. The situation looks something like this:



With a telescope, there's no point in comparing how big the image appears to how big the object would appear *if brought in to 25cm* since the whole point of using a telescope is that the object is *quite far away* and you've got no control over that. So, we define the telescope's angular magnification simply in terms of the object's angular height and the image's angular height. If we approximate the object as being *infinitely* far away, then it's quite easy to phrase this ratio in terms of the focal lengths.
I've highlighted the two triangles that help us to think about this. Putting angles in ratio (and throwing in a negative sign to indicate that the ray direction is flipped / the object is flipped), we have a magnification of:

$$m_{tele} = -\frac{\theta_e(\text{image at infinity})}{\theta_o(\text{object at infinity})} = -\frac{\tan^{-1}(h_i / f_e)}{\tan^{-1}(h_i / f_o)} \approx -\frac{h_i / f_e}{h_i / f_o} = -\frac{f_o}{f_e}$$

Exercise O5.4.1 A telescope has an angular magnification of -100 and is 1.00 m long. What are the focal lengths of the lenses?