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| Wed., 12/5 | 23.5-6 Effects of Radiation on Matter | RE30 HW23:RQ.13, 14, 17; P.22, 24, 25 due beginning of class |
| Thurs., 12/6 | Quiz Ch 23, Lab 11 Polarization (Perhaps move quiz to Friday) | |
| Fri., 12/7 | 23.5-.6 (continuation) | |
| Mon., 12/10 | Review | |

Handouts:

- Info for Quiz on Ch. 23 (Thursday!)

Equipment

- Radio transmitter (boom box, long banana wires to make antenna out of speaker output, amplifier, and speaker)
- Scattering in cloudy tank demo
- Polarizer sheets
- Resonance Demo

Things to collect

- Experiment Boxes

Demos: watch 2D_radiation and 3D_radiation simulations again to refocus us.

So, last time we saw that an accelerating charge generated “Electro-Magnetic Radiation,” i.e., tangential components of E and B. I think it’s important to pause and think about just what *is* and *isn’t* special about these components (relative to the familiar Coulbombic and Biot-Savart fields). Why do we put so much emphasis on these? First, these tangential components decay as $1/r$ rather than $1/r^2$. Second a neutral conglomerate (ex. the sun) would generally have all its radial components of electric (and velocity associated components of magnetic) field cancel (just as many positive as negative charge particles, the same spatial distribution, a symmetric velocity distribution). These two things mean that only the electric and magnetic fields associated with acceleration propagate with appreciable strength an appreciable distance.

Demo: 23_antenna.py

Imagine a charged particle oscillating up and down inside the antenna. The radiation then ripples out (in almost all directions, but this is just looking in the plane where it’s strongest), and you see it decaying with distance.

Energy

Basic: Long before we’d met the radiation terms, we’d learned that there was energy associated with setting up a charge configuration / an electric field and a current configuration / a magnetic field. It makes intuitive sense that energy shouldn’t be expended in simply *maintaining* such states, only in *changing* them. We can see how that plays out in terms of the energy in the fields. In the simplest case – changing a charge / current configuration means accelerating a charge; that, in turn, means radiative electric and magnetic fields.

A very important consequence of the $1/r$ drop-off of this “radiation” field is that it *can* remove energy from its source, whereas the regular Coulombic field, that drops off as $1/r^2$ can’t. It is for this reason that we call the $1/r$ field (that associated with acceleration) the “Radiation” field. Mind you, *either* can impart energy on other charges in their path, and thus remove energy from the source, but only the “radiation” field can take the energy and just run with it regardless of any recipients of that energy. In this sense, the ‘radiation’ fields really take on a life of their own.

Conceptual: This makes some sense if you consider the *whole* field – both terms, in that “kink” kind of picture. If a charge is stationary or even moving at a constant velocity – the field too is stationary or in steady state (recall that the field drifts with a moving charge). Just as it takes no effort to keep a particle in steady state (work-energy theorem // Newton’s 1st), it takes no effort to keep the field in steady state; however, when the charge is accelerated, the field gets realigned and that takes some doing – energy. The radiation term is the “kink” that ripples out through field to realign it, much like when you make your bed and you realign your sheet by sending little ripples running through it. So the energy that’s carried away by this kink is that invested in realigning the electric field to fit with the source charge’s new state of motion.

Mathematical: (don’t go through all this, just give the basics) To see this, consider the energy associated with these fields. Back when we met capacitors, we’d found that the energy invested in establishing an electric field was

$$\text{Energy}_E = \frac{1}{2} \epsilon_0 E^2 \text{Vol} .$$

It’s informative to imagine a shell of thickness d that radiates out from a point source, and to look at the energy associated with the field within that shell. $\text{Vol} \approx 4\pi r^2 d$ and

$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$ for a stationary point charge. So, the energy associated with the field in that shell is $\text{Energy}_E = \frac{1}{2} \epsilon_0 \left(\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \right)^2 (4\pi r^2 d) = \frac{1}{8\pi\epsilon_0} \frac{q^2}{r^2} d$. What’s important to note here is that the energy contained in the shell dies off like $1/r^2$. That means that, by the time such a shell of field radiates out to infinity, it’s got no energy in it – unless it encounters another charge to interact with, this field cannot truly remove energy from the source charge.

Now consider a shell of “radiation” field, $E_{\text{radiative}} = \frac{1}{4\pi\epsilon_0} \frac{qa \sin \theta}{c^2 r}$.

$$\text{Energy}_E = \int_0^{\pi} \frac{1}{2} \epsilon_0 \frac{1}{4\pi\epsilon_0} \left(\frac{qa \sin \theta}{c^2 r} \right)^2 2\pi r^2 d \sin \theta d\theta = \left(\frac{q^2 a^2 d}{6\pi\epsilon_0 c^4} \right).$$

Regardless of the numeric value, what’s important is that this amount of energy *doesn’t* depend on the radius of the shell. Then in our picture (Vpython 3-D radiation), as the shell radiates out to infinity, it *takes energy away with it*.

Added work to accelerate charge: It’s interesting to note that if you accelerate a *charge* for time Δt , then it will create a shell of radiation of thickness $d = c\Delta t$, and it will carry away the corresponding amount of energy, thus

$Work = \Delta K.E. + E_{rad}$ - to accelerate the charge / change its kinetic energy a desired amount, you actually have to do a little *extra* work since some of it will get drained off by the associated radiation. Returning to the sheet analogy, it's kind of like recognizing that you don't just roll over in bed – you rearrange your sheet in the process.

Magnetic: We were just considering the energy associated with the *electric* field to make the point that the “radiation” truly *radiates* energy. Actually, it's got both electric and magnetic fields associated with it. The same basic story is true for the magnetic field. So we'll skip that, but we should formulate the total energy density in radiation.

Recall that when we met inductors, we found that the energy invested in establishing a magnetic field was

$$Energy_B = \frac{1}{2} \frac{1}{\mu_0} B^2 Vol$$

Total: But then again, in the radiation, $E = cB$. So, combining this with the energy in the electric field and phrasing both in terms of electric field,

$$\frac{Energy_{rad}}{Vol} = \epsilon_0 E^2$$

Poynting vector

Here we have the density of something that is radiating, flowing – it's a great candidate for the concept of “flux.” In the way we've been using the term “flux”, energy “flux” should be

$\frac{Energy}{time} = \frac{energy}{Volume} Area \cdot speed$, but what is called “energy flux” in this context is actually the energy flux (per area) – you multiply by the area of interest to get the rate of energy transfer through it.

$$Energy\ flux\ (per\ area) = \frac{Energy_{rad}}{Vol} c = \epsilon_0 E^2 c = \frac{1}{\mu_0} EB$$

This can be rephrased as a vector pointing in the direction of propagation by recalling that $E \times B$ points in the direction of propagation, and they're perpendicular to each other in radiation, so the magnitude is just EB . The resulting vector is called the Poynting vector

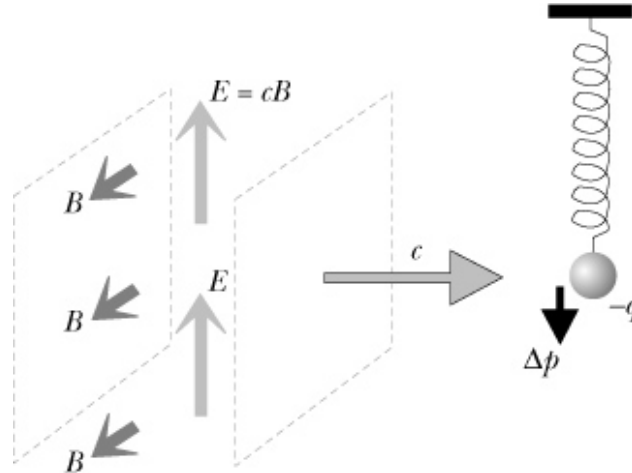
$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}.$$

Its use is that if you dot it into an area, you get the rate at which energy transmits through an area i.e., power radiated through that area.

Vocab note: Energy per area, per time is also called “intensity”

Momentum

Just to make things simple, imagine a charged ball on a spring. Maybe it's an ion in a lattice, maybe it's an electron around an atom. The electron is able to move a bit so it accelerates in the opposite direction of the electric field. This causes it to produce radiation, too!



$$\Delta p = F\Delta t = (qE)\left(\frac{d}{c}\right) \Rightarrow \Delta KE = KE_f \approx \frac{p^2}{2m} = \left(qE\frac{d}{c}\right)^2 \frac{1}{2m}$$

(note: one may ask what effect the magnetic field has on this. The answer is that while it may *redirect* the momentum, it doesn't change the *magnitude* of the momentum, and thus doesn't effect the amount of energy delivered to the charge.)

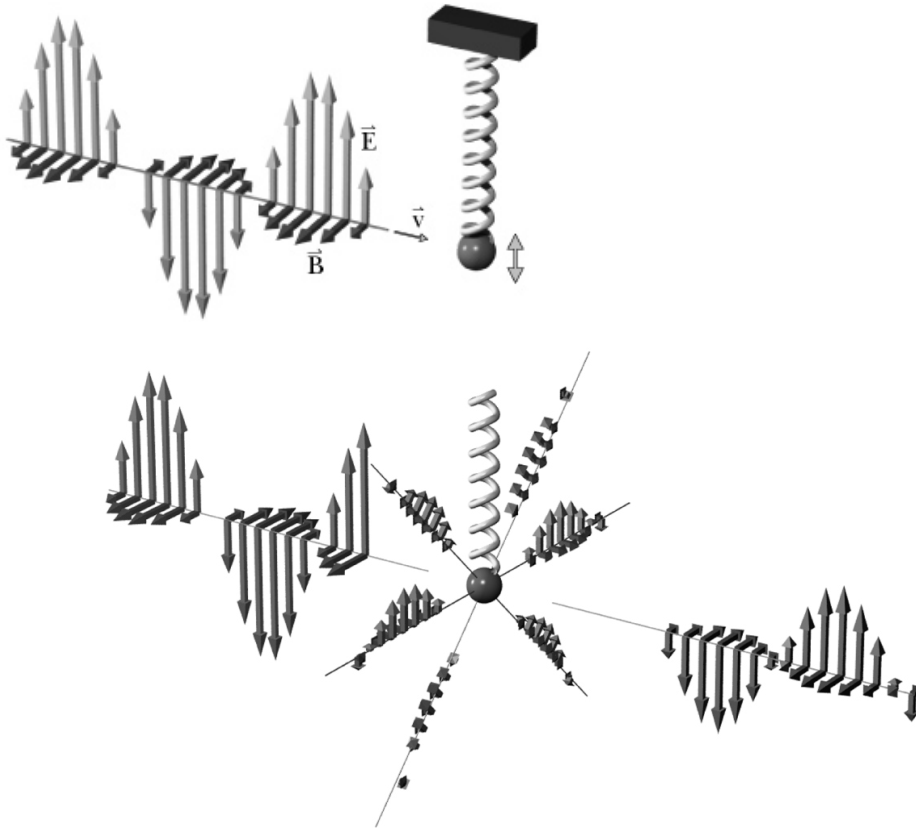
Effect of Radiation on a Neutral atom

Radiation Pressure / Momentum. Consider light radiating on a neutral material, say a piece of cardboard. The If, at some instant there's an upward electric field and an out of the page magnetic field, then the electric field pushes the positive charges up with a force of $F = QE$ and the negative charges down with a force of $F = -QE$, so there's no net force. Since the positive and negative charges are likely bound to each other, they don't go too far. But, that they move at all means that the accompanying magnetic field will interact with them: $q\vec{v} \times \vec{B}$ points forward for both the downward moving negative particles and the upward moving positive particles. So there's a net *forward* force on the charged particles.

Reradiating / Scattering and the Conservations

So, the passing field imparts energy to the ball, and the amount is proportional to E^2 . Of course, the ball, being charged, is accelerated by the field, but being charged, it radiates when accelerated. Of course, it doesn't just radiate in the direction of the initial field's propagation, it radiates all around (albeit, with an amplitude that depends on $\sin\theta$, and so amplitudes 0 in the direction of acceleration.)

If an EM wave passes, the processes is continuous – reradiation is called *scattered* light.



This is at the heart of all scattering and reflection of light. We can also see in it the potential for conservation of momentum and energy. Just looking along the original direction of radiation (to make things simple):

$$\left. \frac{\text{Energy}}{\text{Volume}} \right|_i \propto E_i^2,$$

$$\vec{a} \propto \vec{E}_i,$$

$$\vec{E}_{re-rad} \propto -\vec{a} \propto -\vec{E}_i,$$

$$\text{so } \vec{E}_f = \vec{E}_i + \vec{E}_{re-rad} = \vec{E}_i - \text{const} \vec{E}_i = \vec{E}_i (1 - \text{const})$$

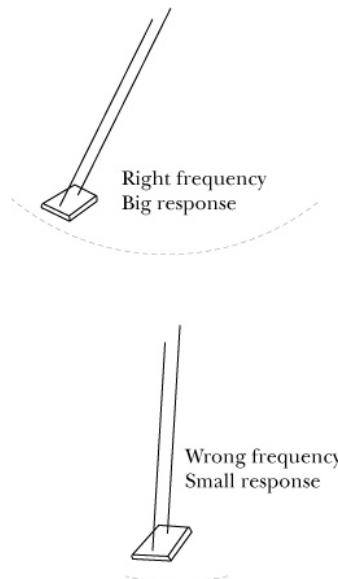
$$\left. \frac{\text{Energy}}{\text{Volume}} \right|_f \propto E_i^2 (1 - \text{const})^2$$

So, the reradiated field diminishes the net field from its original value – less field, less energy in the field. Energy that was in the field has been transferred to the ball.

Resonance. Oscillating electric and magnetic fields incident on a neutral atoms induces an oscillating polarization – say, moving the negative charges down and the positive charges up, and then the positive charges down and the negative charges up... Of course, the negative charges are a little freer to move, (being much lighter) so it looks more like the electron cloud

oscillating up and down about the relatively stationary nucleus – much like an electron dangling from a spring (in fact, the spring constant is $k_{sp} = \left| \frac{d^2U}{dx^2} \right|$).

Conceptual: Any oscillatory system has a natural frequency that, if just hit once and then left alone, it would oscillate at. For a mass on a spring, you may recall that that frequency is $\omega_o = \sqrt{\frac{k_{sp}}{m}}$ (think – bigger mass hanging from the spring – more sluggish response to force – moves slower, stiffer spring – bigger restoring force, moves faster). In the most extreme case of an on-off-on-off driving force, you can easily see how pushing a person on a swing every time he/she swings back gets the swing going further and further – that’s driving at the natural frequency. On the other hand, pushing the person, sometimes, and *pulling* the person other times, isn’t nearly so effective – that’s driving *off* the natural frequency.



We call this phenomenon of a system responding strongly to the right frequency “resonance”, or we say that the system “resonates” at its natural frequency ((or members of its harmonic family.))

Demo: Resonance (show that, given same driving force, different balls oscillate more or less, depending on how close the driving frequency is to the resonance frequency)

Quantitative: Let’s apply Newton’s 2nd // the Momentum Principle to our ‘charge on a spring’ system that’s getting driven by an oscillating electric field.

$$m \frac{d^2x}{dt^2} = -k_{sp}x + qE_i \cos(\omega t)$$

This is another one of those darned differential equations (like we’d seen for an RC circuit and for an LR circuit.) The solution (which you’re free to verify by plugging back in) is

$$x(t) = \frac{qE_i}{|\omega_o^2 - \omega^2|} \cos(\omega t).$$

As you can see, the closer the driving frequency is to the resonance frequency, the bigger the amplitude of the oscillation. In fact, in this model, ignoring any drag forces, when the two frequencies are equal the system would fly apart! There's a famous movie of a bridge doing just that.

Of course, if a charged particle is being oscillated, it should itself be radiating.

Recall that the electric field it produces is $\vec{E}_{rad} = \frac{1}{4\pi\epsilon_o} \frac{-q\vec{a} \times \hat{r}}{c^4 r}$.

Color Vision

Blue Sky

The 0th-order effect:

Sun light, i.e., oscillating electric and magnetic fields, incident on a neutral air molecule oscillates the electron cloud about the relatively stationary nuclei.

These, now accelerated, charges reradiate in all directions. The effect appears to be that they preferentially reradiate in the Bluer end of the spectrum (shining blue light all around) & leave the Redder end of the spectrum relatively unchanged.

Why is that?

Let's put together what we know about all these steps.

As we've already seen, as a function of time, the center of the electron cloud is

$$x = X \cos \omega t \text{ where } X = \frac{qE_i}{|\omega_o^2 - \omega^2|}.$$

But it's the *acceleration* that directly figures into the reradiation, so let's find that:

$$a = \frac{d^2 x}{dt^2} = -\omega^2 X \cos \omega t$$

We *could* plug this into our equation for radiation, but it suffices to note that the field is proportional to a and therefore proportional to ω^2 . But that's not the end of it. Our eyes are sensitive to the *energy* not the amplitude of a field, and the energy content goes like

$$\frac{\text{Energy}_{rad}}{\text{Vol}} = \epsilon_o E^2$$

So the energy in the field goes like

$$\text{Energy} \propto E^2 \propto \omega^4.$$

Here is the source of the frequency, and thus color, dependence. To make that connection, I need to correlate color to frequency:

| | | | | | |
|-----------------|--------|--------|------------------|------|--------|
| Red | Orange | Yellow | Green | Blue | Violet |
| Long wavelength | | | Short wavelength | | |
| Low frequency | | | High frequency | | |

What it tells us is that *even if* light of all frequencies were uniformly incident on our charged oscillator, it would accelerate, and therefore reradiate / scatter, *more* for the higher frequency light. Furthermore, since it's the energy, not the field, that matters in the end, we notice the high frequencies still more!

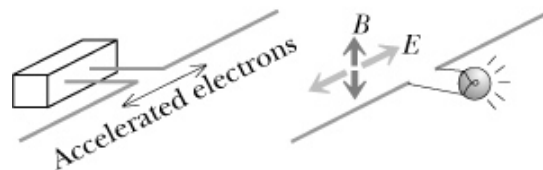
2nd order effect: notice that there's also a frequency dependence in X ; as long as the frequencies you're comparing are all much less than the resonance frequency, then this effect has little impact.

Now, in truth, the sun *doesn't* radiate light evenly across all frequencies, and as you already saw, there may be some resonance effects for light near an atom's electronic resonances. Both of these are secondary effects. The actual spectrum of the sun's light is fairly broad and centered around yellow; that may effect the specific hue of blue – but not that it's blue to begin with. The atomic electronic resonances are in the UV (when you hit one, you do indeed 'break' the atom), so that's outside the visible and probably has negligible effect on the color we perceive.

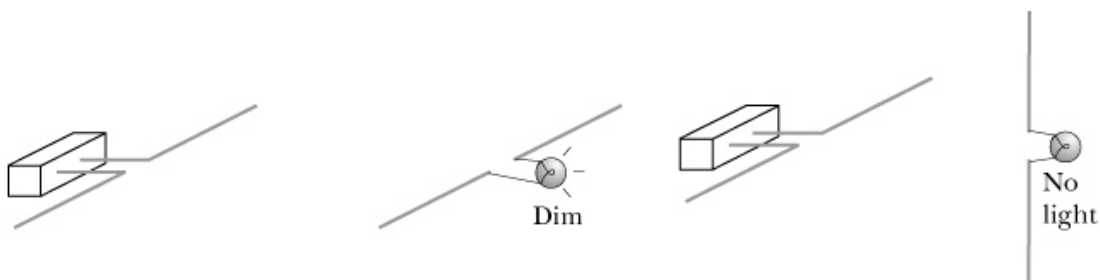
Polarization

Transmitters and Antenas

If the electric field for EM radiation is always in a particular direction, it is *polarized*. The antenna shown below (on left) produces a horizontal electric wave as the charges on the opposite ends are alternated. The direction of E is the direction of the polarization.



The antenna on the right can detect EM radiation if the electric field wave can cause electrons to move across the bulb. The first orientation works, but the second does not.

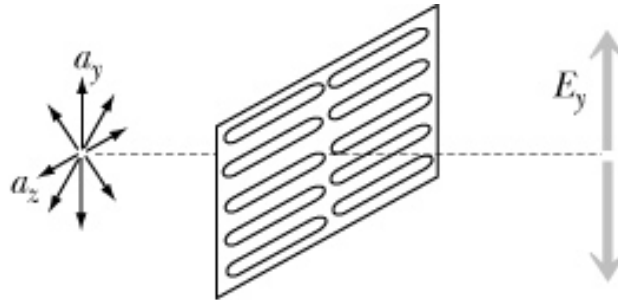


Polarizers

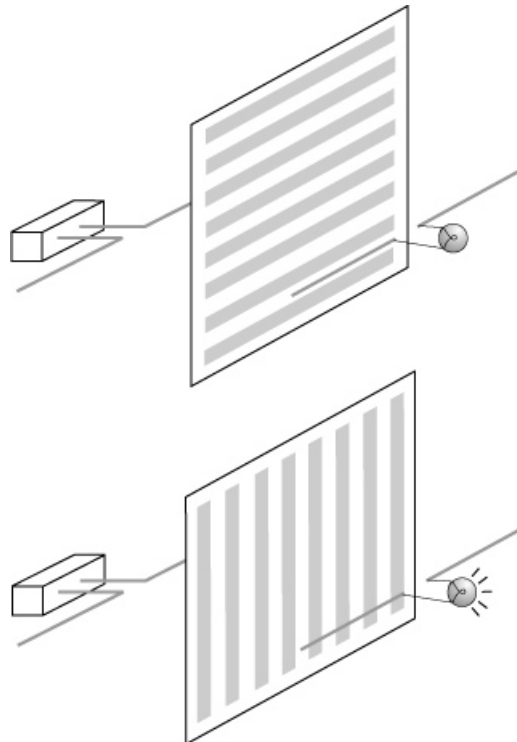
If the light is produced by randomly oriented emitters (e.g. thermal motion of electrons in a light bulb), the light produced is *unpolarized*.



After passing through a polarizer, light is polarized. The polarizer is made of long molecules oriented as shown below. Why does this happen?



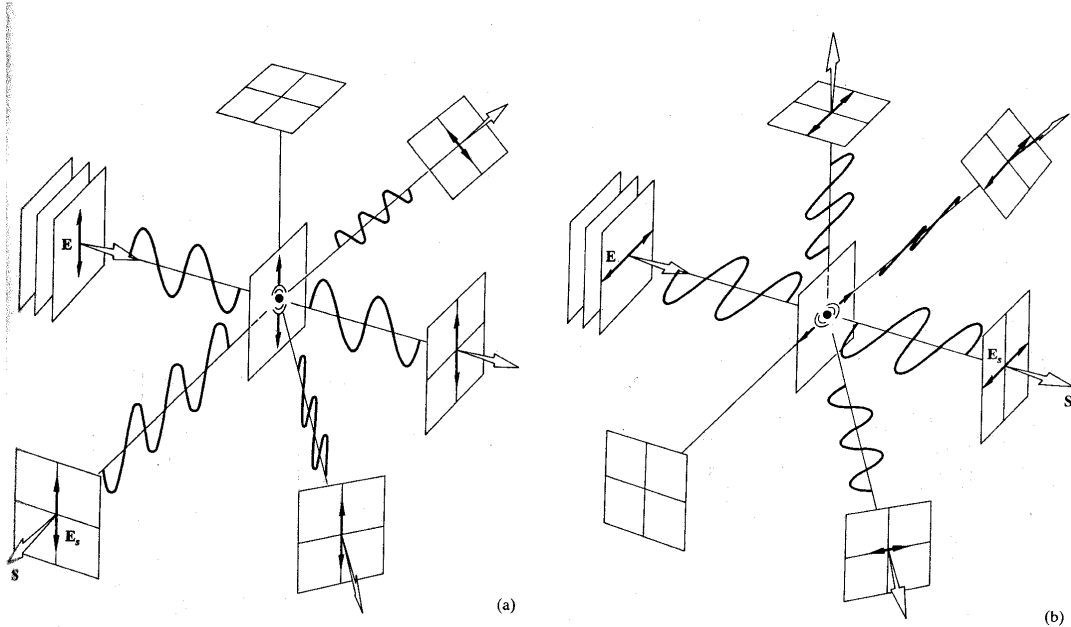
If the conductor is lined up with the electric field, the electrons can move and reradiate to cancel the incoming wave (first diagram below). If the conductor is rotated 90°, the electrons are not accelerated, so the EM wave passes through (second diagram).



DEMO: Show the blocking of microwaves in this way.

Polarization by Scattering

Suppose a polarized EM wave is scattered by an atom or molecule. What is the polarization of light in different directions?



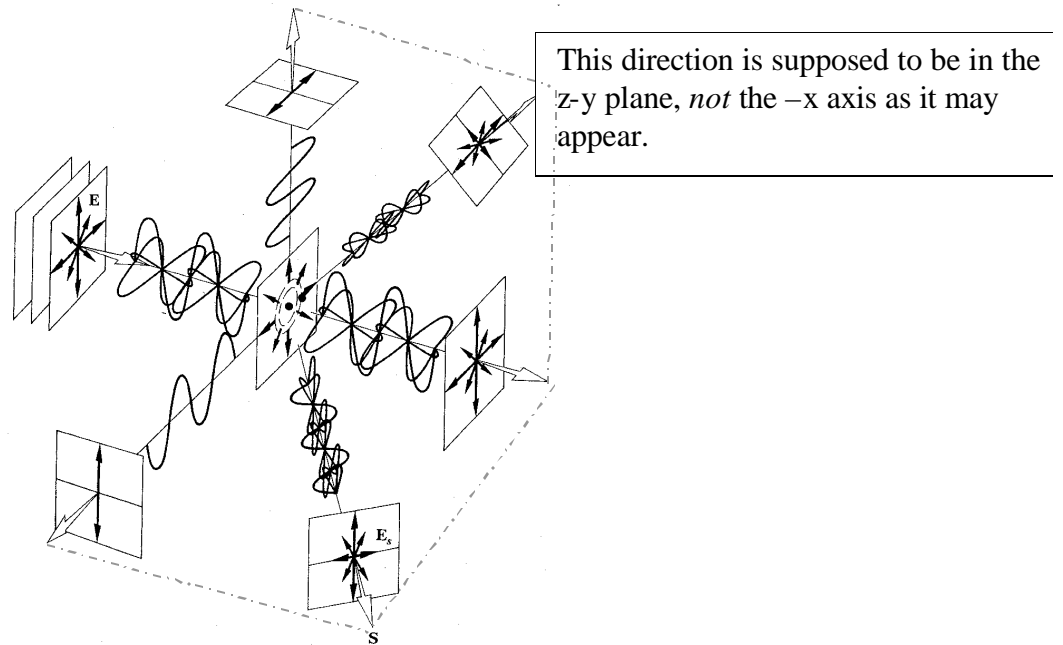
(a) Vertical Polarization

(b) Horizontal Polarization

From certain perspectives, the electron have no projected (perpendicular) acceleration, so there is no reradiation in those directions.

Suppose an unpolarized EM wave is scattered by an atom or molecule. What is the polarization of light in different directions? In the plane perpendicular to the incoming wave, the light is polarized!

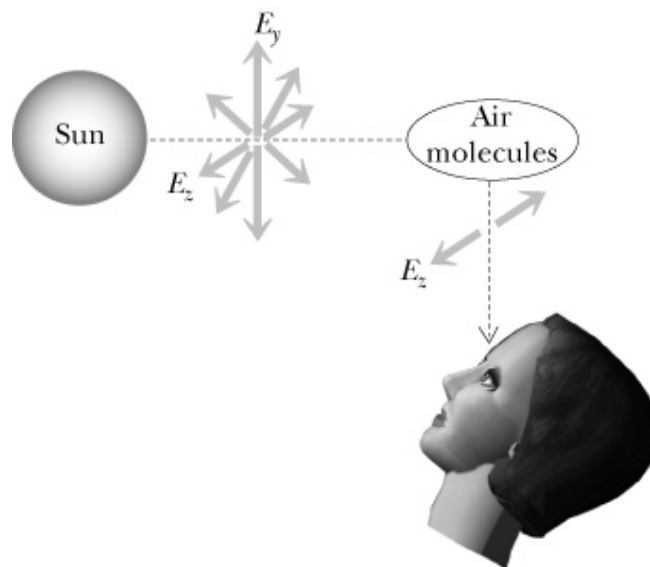
Project this image for them to see; it's worth more than a thousand words.



For the 90° view, it's easy to say "there will be no field oscillation in the original direction of propagation, and there will be no field oscillation in the direction of the new propagation – with those two perpendicular to each other, you've excluded two out of three coordinate axes – there's only one direction left."

Note that you have a less pronounced effect in the other directions: for example, the light propagating in the x-y plane has stronger z polarization while the light propagating in the y-z plane has stronger x polarization.

DEMO: Use polarizers to look at the sky. Light scattered by 90° is polarized so it can be blocked by rotating the polarizer! (see diagram)



DEMO: Blue skies and red sunsets with tank of water. Also, the light scattered by 90° is polarized.

The color of things

So, when we “see” an object, unless it generates light in the visible range of the spectrum (like a light bulb), we’re seeing re-radiated light. It’s “color” is a function of which frequencies of incident light it most strongly re-radiates. When you’re talking about insulators, like wood, the electrons are considerably less free to respond than in conductors (where there are quite free electrons), so you tend to get much more re-radiation from (clean) conductors than insulators (when metals oxidize – you’re back to a more insulating surface, and less re-radiation.) The relatively “free” electrons in a metal can oscillate in synch with most any impinging electric field, and so re-radiate a fairly unbiased spectrum. I don’t recall so well why metals have any color bias at all (say copper or gold.) I think it may be that these tend to step down higher energy light: one higher energy photon adsorbed, two lower energy photons later emitted. This would have to do with the hole-creation and annihilation process: a hole is created when an electron is excited above the Fermi level by adsorbing light, but then that hole is quickly filled by another, medium energy electron dropping into it, and another still higher energy electron drops into that hole – in that way the hole rises to the Fermi surface like a bubble and low energy photons are emitted for each step it takes up.

Friday: Wrap Up

Monday: HW 23 due, Review, and Quiz 23

Thursday: Final Exam at 9 a.m.