Fri., 11/30	**23.1,2,7** Ampere- Maxwell, E&M Pulse	RE28
Mon., 12/3 Wed., 12/5 Thurs., 12/6 Fri., 12/7	**23.3-4** Accelerating Charges Radiating **23.5-6** Effects of Radiation on Matter **Quiz** Ch 23, **Lab 11** Polarization (Perhaps move quiz to Friday) **23** Conceptual Maxwell's & Applications	RE29 RE30

Ampere-Maxwell Law:

In the last chapter, we saw that a time varying *magnetic* field was accompanied by (note: does not "cause") a curled electric field. Underlying that is the fact that a time varying magnetic field comes from a time varying current density which is comprised of accelerating charges (against a backdrop of stationary charges) which causes an anti-parallel electric field.

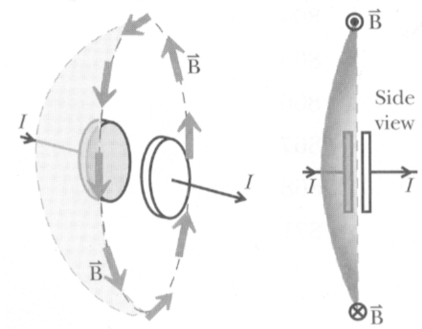
It's natural to ask if something like the reverse happens – does whatever causes a time varying electric field also cause a curled magnetic field? Well, what causes a time varying electric field? A time varying charge density which implies a varying current density. The argument's not as simple as in the previous case, but this gives rise to a different perceived current and thus a different magnetic field than if we had a spatially and temporally constant current (need to work on this – Griffiths p 428).

$$\vec{B}(\vec{r},t) = \frac{m_o}{4p}\int\left[\frac{\vec{J}(\vec{r}\,',t_r)}{\mathbf{r}^2} + \frac{\dot{\vec{J}}(\vec{r}\,',t_r)}{c\mathbf{r}}\right]\times\hat{\mathbf{r}}\,dt'$$

Taking the curl of this is no small feat, but it's been done by Heras in AJP 76 (6) June 2008, p. 592. The important point is that there's no *E* in it. As Griffith and Heald point out in AJP 59 (2), Feb 1991 p111, in the Ampere-Maxwell law, the dE/dt is really a "surrogate for ordinary currents at other locations."

We still have to "fix up" Ampere's Law to complete Maxwell's equations (the other 3 are complete). According to Faraday's Law, a changing magnetic flux is accompanied by a curly electric field. Similarly, we will see that a changing electric flux is accompanied by a curly magnetic field.

How do we know Ampere's Law is incomplete? Consider long, straight wires connected to a capacitor that is charging. Assume the conventional current is moving to the right, so the left plate becomes positive and the right plate becomes negative. Now apply Ampere's Law in the vicinity of the capacitor. First we'll do it the easy way.

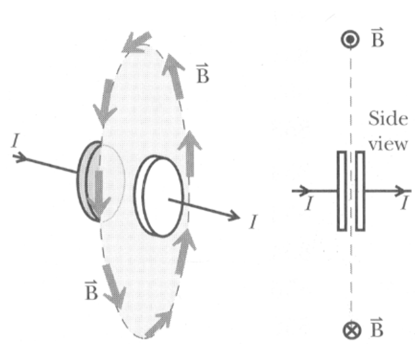


The magnetic field is perpendicular to the circular path. If the radius of the circle is *r*, Ampere's Law gives:

$$\oint \vec{B} \cdot d\vec{\ell} = B(2pr) = m_0 I,$$

$$B = \frac{m_0 I}{2pr} = \frac{m_0}{4p}\frac{2I}{r}.$$

Now, let's do it suprising way.



For a circular path in the plane through the middle of the capacitor:

$$\oint \vec{B} \cdot d\vec{\ell} \neq 0,$$

but there is no (zero) current passing through a flat surface inside the loop (e.g. – a soap film stretched flat over the circular path). Something is wrong with Ampere's Law.

If we trust our answer in the firs instance, then we *should* get the same answer in the second case. So, let's pencil in the result, and see if we can relate it to *something* that is piercing the second buble.

$$\oint \vec{B} \cdot d\vec{\ell} = m_0 I_{wire} = ?_{capacitor}$$

Bubble 1 Bubble 2

Let's see if we can relate what's going on in the wire to what's going on in the capacitor.

$$I_{wire} = \frac{dQ_{cap}}{dt}$$

Where, $E = \frac{Q/A}{e_0} \Rightarrow Q_{cap} = e_0 EA = Q_{cap} = e_0 \Phi_{E.(open.area)}$

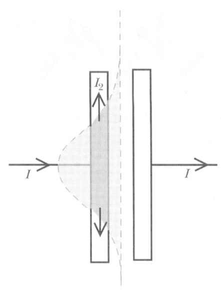
So, $I_{wire} = \frac{dQ_{cap}}{dt} = e_0 \frac{d\Phi_{E.(open.area)}}{dt}$

So,

$$\oint \vec{B} \cdot d\vec{\ell} = m_0 I_{wire} = m_0 e_0 \frac{d\Phi_{E.(open.area)}}{dt}$$

Bubble 1 Bubble 2

More generally, if we had some bubble that caught a little bit of both current and changing electric-field flux, we'd get



$$\oint \vec{B} \cdot d\vec{\ell} = m_0 \left[\sum I_{\substack{\text{inside}\\\text{path}}} + e_0 \frac{d\Phi_{\text{elec}}}{dt} \right],$$

holds for <u>any</u> surface defined by a closed path. It even works for the surface below, if all of the currents are taken into account (including the ones on the capacitor plate).

Demo: Maxwell-Ampere VPython

<u>Maxwell's Equations</u>: these are the complete versions!

Gauss's law:	$\oint \vec{E}\cdot\hat{n}\,dA = \frac{1}{e_0}\sum Q_{\substack{\text{inside}\\ \text{surface}}}$	$\vec{\nabla}\cdot\vec{E} = \frac{r}{e_0}$
Gauss's law for magnetism:	$\oint \vec{B}\cdot\hat{n}\,dA = 0$	$\vec{\nabla}\cdot\vec{B} = 0$
Faraday's law:	$\oint \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\left[\int \vec{B}\cdot\hat{n}\,dA\right]$	$\vec{\nabla}\times\vec{E} = -\frac{\P\vec{B}}{\P t}$
Ampere-Maxwell law:	$\oint \vec{B}\cdot d\vec{\ell} = m_0\left[\sum I_{\substack{\text{inside}\\ \text{path}}} + e_0\frac{d\Phi_{\text{elec}}}{dt}\right]$	$\vec{\nabla}\times\vec{B} = m_0\left[\vec{J} + e_0\frac{\P\vec{E}}{\P t}\right]$

These equations describe the electric and magnetic fields produced by charges and currents (moving charges). The other rule needed to summarize electromagnetism is the Lorentz force:

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B},$$

which describes how the fields affect charges.

Alternatively, one just needs Coulomb's Law and Special Relativity – but it's a pain to get anywhere with those two.

Maxwell's derivation of light waves: (no local sources, curl Faraday's and put Ampere's into it; note $\nabla\times\nabla\times E = \nabla(\nabla\bullet E) - \nabla^2 E$ but first term is zero in absence of sources.) So $E = E_o \sin\left(2p\left(\frac{t}{t} - \frac{x}{l}\right)\right)$ solves where we find that $\frac{l}{t} = v = \frac{1}{\sqrt{e_o m_0}}$

What's special about radiation term?

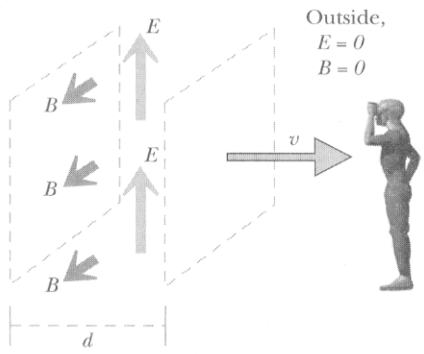
A steady state current (not against a neutralizing backdrop) *does* produce electric and magnetic fields, and they drop of like $1/r^2$. If this current *accelerates* there's another term in the electric and in the magnetic fields – this extra term drops off like 1/r. Thus an *accelerating* charge can be felt much stronger much further away.

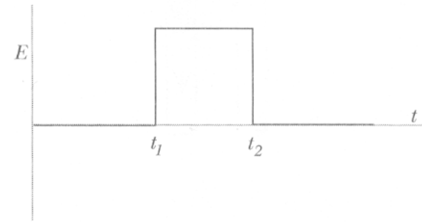
Traveling Electromagnetic Fields:

We want to show that there are configurations of time-varying electric and magnetic fields that can move through empty space and satisfy Maxwell's equations. Rather than starting with the equations, we will propose a field configuration, then show that they can satisfy Maxwell's equations (in empty space). In the process, we will find a relation between the sizes of the electric and magnetic field and the speed of propagation.

Consider a moving slab/pulse with the electric field in the +*y* direction (upward) and the magnetic field in the +*z* direction (out of the page) and moves in the +*x* direction (rightward) at a speed *v*. The fields are uniform within a slab and zero everywhere outside it (to be self-consistent, it would actually have to be infinite in the y-z plane). If you stand in one place on the *x* axis, the fields will vary with time.

Demo: Vpython 23-pulse_sq

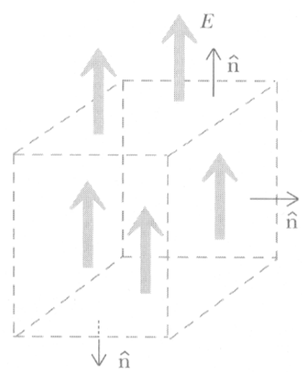




Let’s check each of Maxwell’s equations:

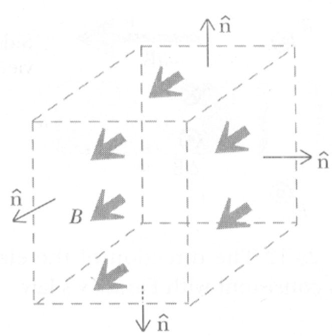
1. **Gauss’s law**

   Pick a closed box with sides perpendicular to the coordinate axes as the Gaussian surface. The electric flux on the on the top and bottom will be the same size, but opposite signs, so the net flux is zero. This is consistent with Gauss’s law, since there is no charge in empty space.



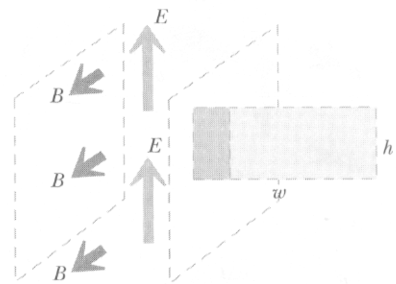
2. **Gauss’s law for magnetism**

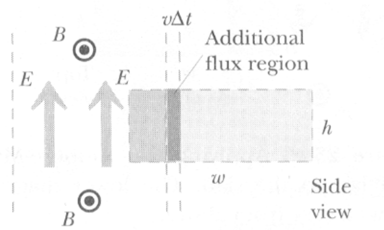
   Pick a closed box with sides perpendicular to the coordinate axes as the Gaussian surface. The magnetic flux on the on the front and back will be the same size, but opposite signs, so the net flux is zero. This is consistent with Gauss’s law for magnetism.



3. **Faraday’s law**

   Pick a closed, rectangular path in the *xy* plane with a height *h* and a width *w*.





When the moving "slab" to partially overlaps the rectangle, in a time Δt the area that the magnetic field passes through increases by $\Delta A = h(v\Delta t)$. The magnetic flux increases by $\Delta\Phi_{\text{mag}} = B\Delta A = Bhv\Delta t$, so the rate of change of the magnetic flux is:

$$\frac{d\Phi_{\text{mag}}}{dt} \approx \frac{\Delta\Phi_{\text{mag}}}{\Delta t} = Bhv\,.$$

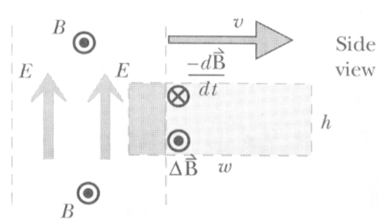
The integral of the electric field along the path is:

$$\oint \vec{E}\cdot d\ell = Eh,$$

so Faraday's law (only worrying about absolute values) is satisfied if $Eh = Bhv$ or:

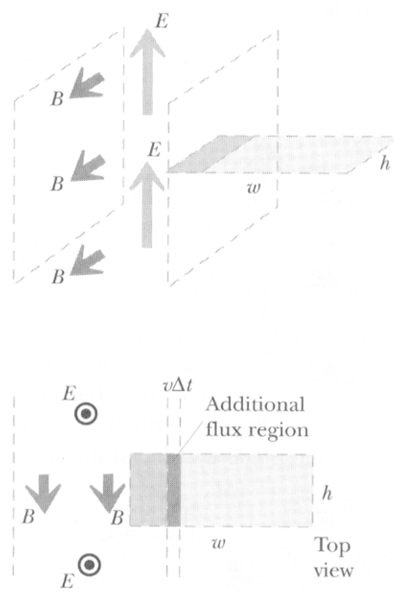
$$E = Bv\,.$$

The direction of the electric field is also consistent with Faraday's law as shown below. Since $-d\vec{B}/dt$ is in the –y direction (into page), $\vec{E}$ must be along the path in the CW direction.



4. **Ampere-Maxwell law**

Pick a closed, rectangular path in the *xz* plane with a height *h* and a width *w*.



When the moving "slab" to partially overlaps the rectangle, in a time Δt the area that the electric field passes through increases by $\Delta A = h(v\Delta t)$.

The electric flux increases by $\Delta\Phi_{\mathrm{elec}} = E\Delta A = Ehv\Delta t$, so the rate of change of the magnetic flux is:

$$\frac{d\Phi_{\mathrm{elec}}}{dt} \approx \frac{\Delta\Phi_{\mathrm{elec}}}{\Delta t} = Ehv\,.$$

The integral of the magnetic field along the path is:

$$\oint \vec{B}\cdot d\ell = Bh\,,$$

so the Ampere-Maxwell law (only worrying about absolute values) is satisfied if (there is no current in empty space):

$$\oint \vec{B}\cdot d\vec{\ell} = m_0\left[\sum I_{\substack{\text{inside}\\ \text{path}}} + e_0\frac{d\Phi_{\mathrm{elec}}}{dt}\right]$$

$$Bh = m_0 e_0 (Ehv)$$

$$B = m_0 e_0 v E$$

We have shown that this pulse can propagate in empty space. Once the pulse is started, no source (charges or currents) are needed to keep it going!

The two conditions together can be solved for the speed of propagation:

$$B = m_0 e_0 v(vB),$$

so:

$$v^2 = \frac{1}{m_0 e_0}$$

$$v=\sqrt{\frac{1}{\mu_0\varepsilon_0}}=\sqrt{\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{4\pi}{\mu_0}\right)}=\sqrt{\left(9\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(10^{7}\ \mathrm{A/T\cdot m}\right)}=3\times10^{8}\ \mathrm{m/s}=c$$

which is the speed of light in vacuum! That was measured long before Maxwell calculated how fast an electromagnetic pulse or wave would propagate.

We can rewrite the first condition as:

$$E=cB.$$

The direction of $\vec{v}$ is the same as the direction of $\vec{E}\times\vec{B}$

Time for HW 22 questions

Monday: other types of electromagnetic radiation and its source