Mon., 3/30 Tues., 3/31 Wed., 4/1 Thurs., 4/2 Fri., 4/3	**22.3-4,.7** Faraday & Emf & Inductance **22.8-9,.6** Energy, Diff. form, Superconductors **Quiz** Ch 22, **Lab 10** Faraday's Law **23.1,2,7** Ampere- Maxwell, E&M Pulse	**RE27** **HW21:**RQ.12, 15,17; P.20, 23, 26 **RE28**

Equipment: Cow magnet and long copper tube and long aluminum tube, or "cow magnet slug"

Faraday's law: The induced emf for a closed loop is equal to the rate of change of the magnetic flux on the area enclosed by the loop,

$$\text{emf}_{\Delta B} = -\left.\frac{\partial \Phi_{\text{mag}}}{\partial t}\right|_A .$$

Where

$$\text{emf}_{\Delta B} = \oint \vec{E}_{NC} \cdot d\vec{\ell} \quad \text{and} \quad \Phi_{\text{mag}} = \int \vec{B} \cdot \hat{n}\, dA .$$

$$(\text{Tm}^2 = \text{Weber})$$

The book points out that, since $0 = \oint \vec{E}_C \cdot d\vec{\ell}$, we could just as well write $\text{emf}_{\Delta B} = \oint \vec{E} \cdot d\vec{\ell}$

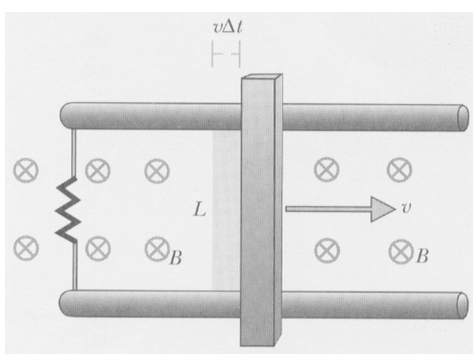
Review: julie's Lec W12_3, all questions; W13_1 Questions 1-3

Different books dub slightly different things "Faraday's Law", but they are often sloppy about specifying exactly what they mean. Our book seems to be restricting the definition to just the *emf* that arises because of a change in magnetic field.

Look at Clicker questions 2 & 1.

Relationship between Faraday's law and motional emf:

Let's look again at the circuit in a uniform magnetic field with a moving bar.



Recall that we defined *emf* as (non-coulombic) work per unit charge, so $emf = \dfrac{\vec{F}\cdot\vec{L}}{q}$.

Previously, we found the "motional *emf*" is vBL by considering the magnetic force $q\vec{v}\times\vec{B}$ on the electrons in the moving bar.

Faraday's Law can be generalized to include motional *emf* as well as that due to changing *B*.

For my own amusement (after Purcell):

$$\frac{d\Phi_B}{dt} = \left.\frac{\partial\Phi_B}{\partial t}\right|_A + \left.\frac{\partial\Phi_B}{\partial t}\right|_B$$

$$\frac{d\Phi_B}{dt} = \int\frac{d\vec{B}}{dt}\cdot d\vec{A} + \frac{\int_{A_f}\vec{B}\cdot d\vec{A} - \int_{A_i}\vec{B}\cdot d\vec{A}}{dt} = \int\frac{d\vec{B}}{dt}\cdot d\vec{A} + \frac{\int_{\Delta A}\vec{B}\cdot d\vec{A}}{dt} = \int\frac{d\vec{B}}{dt}\cdot d\vec{A} + \frac{\int\vec{B}\cdot\left(\Delta\vec{l}\times d\vec{l}\right)}{dt}$$

$$\frac{d\Phi_B}{dt} = \int\frac{d\vec{B}}{dt}\cdot d\vec{A} + \frac{\int\vec{B}\cdot\left(\vec{v}dt\times d\vec{l}\right)}{dt} = \int\frac{d\vec{B}}{dt}\cdot d\vec{A} + \oint\vec{B}\cdot\left(\vec{v}\times d\vec{l}\right) = \int\frac{d\vec{B}}{dt}\cdot d\vec{A} - \oint\left(\vec{v}\times\vec{B}\right)\cdot d\vec{l}$$

$$\frac{d\Phi_B}{dt} = \int\frac{d\vec{B}}{dt}\cdot d\vec{A} - \int\vec{\nabla}\times\left(\vec{v}\times\vec{B}\right)\cdot d\vec{A}$$

$$\frac{d\Phi_B}{dt} = \int\left(\frac{d\vec{B}}{dt} - \vec{\nabla}\times\left(\vec{v}\times\vec{B}\right)\right)\cdot d\vec{A}$$

(where Dl is how much the border has moved during dt.)

Faraday's Law relates the first term in the integral to the non-coulombic electric field:

$$\int\frac{d\vec{B}}{dt}\cdot d\vec{A} = -\oint\vec{E}\cdot d\vec{l} = -emf_{\Delta B}$$

whereas

$$\int\vec{\nabla}\times\left(\vec{v}\times\vec{B}\right)\cdot d\vec{A} = \oint\left(\vec{v}\times\vec{B}\right)\cdot d\vec{l} = emf_{motional}$$

so,

$$-\frac{d\Phi_B}{dt} = emf_{\Delta B} + emf_{motional} = emf_{\Delta\Phi}$$

For more general consumption:

While this can be done in terms of the integral for magnetic flux, the same result can be more easily obtained by consider the change in flux through a differentially small patch of area, over which B is relatively constant,

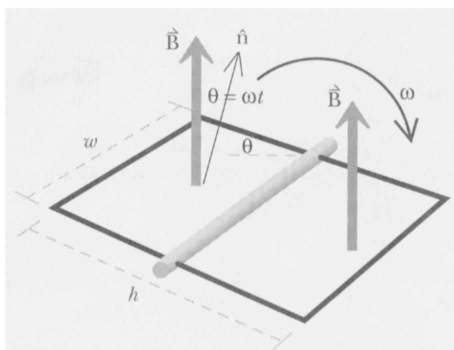
$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(B_\perp A) = A\frac{d}{dt}(B_\perp) + B_\perp\frac{d}{dt}(A) = -emf_{\Delta B} + B_\perp L\frac{dx}{dt} = -emf_{\Delta B} + B_\perp Lv = -emf_{\Delta B} - emf_{\Delta A}$$

Some texts call *this* Faraday's Law. Whatever we call it, the fact of the matter is that, *either way* your magnetic flux changes, *emf* is induced:

$$\boxed{\frac{d\Phi_B}{dt} = -emf_{\Delta\Phi}.}$$

While we derived this just considering a differential patch of area (over which B is uniform and along which v is constant) the final result holds for a much larger area (over which B is not uniform and along which v is not constant.) The justification is that a general area can be constructed out of a quilt of such differential patches.

Rotation changes A$_{\text{perpendicular}}$, and flux – induces emf. The general result even holds when the area or field is changing direction (say, via rotation). Recall the rotating coil that we'd considered in a previous chapter. Suppose the coil starts perpendicular to the magnetic field and rotates at a rate *w*. The angle of the coil is $q = wt$.



The magnetic flux through the loop is:

$$\Phi(t)_{\text{mag}} = BA(t)_{\perp} = Bhw\cos(wt),$$

where $A_{\perp}$ is the area perpendicular to magnetic field. The induced emf is:

$$\text{emf} = -\frac{d\Phi_{\text{mag}}}{dt} = wBhw\sin(wt).$$

Summary:

An emf around a loop can result from a changing magnetic flux in two ways:

- the magnetic field can change
- the area of the loop can change (in magnitude *or* orientation)

The magnetic flux is $\Phi_{\text{mag}} = B_{\perp}A$, so the emf is:

$$\text{emf} = \frac{d\Phi_{\text{mag}}}{dt} = \frac{d}{dt}(B_{\perp}A) = \frac{dB_{\perp}}{dt}A + B_{\perp}\frac{dA}{dt}.$$

There are some subtle differences between the two terms.

- D**B.** If the magnetic field is changing, there is a non-Coulomb *electric field* $\vec{E}_{\text{NC}}$ that curls in the region of changing flux. That means that $\text{emf} = \oint \vec{E}_{\text{NC}} \cdot d\vec{\ell} \neq 0$. So the force driving a current around a loop is *electric*.
- D**A.** If the area is changing because part of the loop is moving through a magnetic field, there is a magnetic force which drives electrons around the loop. The non-Coulomb force in this case is the *magnetic* force.

In both cases, the size of the emf is equal to the rate of change of the magnetic flux. Some situations can be seen as one or the other effect depending on the reference frame – but there is not always one frame that works for all parts of the circuit, so that more generally only works locally.

Coils:

Suppose there is a coil with N turns, instead of a single loop. If the turns are all close together, the induced electric field $\vec{E}_{\text{NC}}$ is approximately the same for each one. The emf between the ends of the coil is the integral of $\vec{E}_{\text{NC}}$ along the entire length:

$$\text{emf} = \int_{N\,\text{turns}} \vec{E}_{\text{NC}} \cdot d\vec{\ell} = N\left(\int_{\text{one turn}} \vec{E}_{\text{NC}} \cdot d\vec{\ell} \right) = N\left(\text{emf}_{\text{one turn}}\right).$$

If the magnetic flux through <u>one</u> loop is Φ_{mag}, the emf can also be written as:

$$\text{emf} = -N\frac{d\Phi_{\text{mag}}}{dt}.$$

Several ways to change the Magnetic Flux:

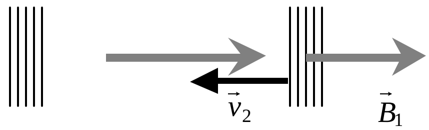
Exercise – Come up with ways to change the magnetic flux through a coil using either a second coil or a permanent magnet

All of the following will result in an induced emf in the coil 2 on the right.

1. Change the current in coil 1

2. Move coil 1 (with current through it)

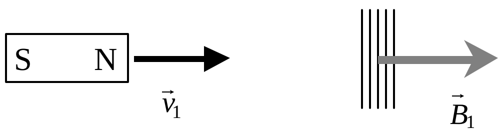
3. Move coil 2 (with current through coil 1)



4. Rotate coil 1

Rotate coil 2

5. Move the magnet relative to the coil (includes moving coil toward magnet)



6. Rotate the magnet

7. Rotate the coil

Clicker Questions 22.2 (about magnet dropped down tube) This is one of the situations described – moving the magnet relative to the coils / tube walls.

Demo: drop a magnet down a copper tube (not ferromagnetic) –very slow compared to free fall!

Each cross section of the pipe can be considered a loop. There will be induced currents around the pipe. These in turn produce magnetic fields, so it's like having two magnets interact. You will explain the slowing in terms of <u>forces</u> in Prob. 22.1 (c).

Inductance:

Faraday's Law tells us how an electric field and associated emf are related to a changing magnetic field. Of course, a changing magnetic field is caused by a changing current, so we should be able to directly relate the *emf* to its root cause – the changing current. We'll do that for the simple case of a solenoid and then generalize.

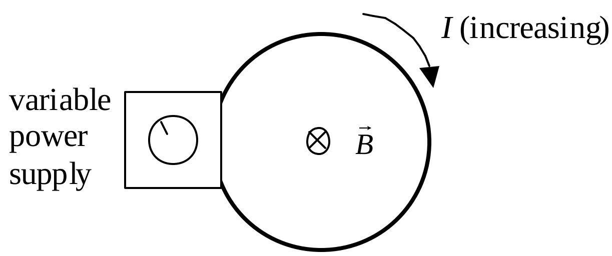
Suppose you have a circuit with a loop or loops of wire (even the simplest circuit makes one loop!). When current flows, there is a magnetic field produced that passes through the loop(s) which is proportional to the current. Therefore, there is a magnetic flux that is proportional to the current. Call the proportionality the "inductance" (or "self-inductance") L, so:

$$\Phi_{\text{mag}} = LI \ .$$

When the current *changes* there will be an *induced emf* according to Faraday's law:

$$\text{emf}_{\text{ind}} = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt} \ .$$

The negative sign indicates that the induced emf will point *against* the changing current (upstream if the current tries to increase, downstream if it tries to decrease), Consider the example of a single loop of wire with resistance R connected to a variable power supply. What direction is the emf?



If the power supply is turned up and the current flowing CW increases, the change in the magnetic field $\Delta\vec{B}$ is downward. Since $-d\vec{B}/dt$ is upward, the change in magnetic flux will induce an emf that will act like a "backward" battery which opposes the change in current. Recall the simulation I showed you last time – this comes from the electric field that points opposite to the charges' accelerations.

<u>Example</u>: inductance of a solenoid with a radius R, length d, and N turns (tightly wound)

The magnetic field inside the solenoid is:

$$B = \frac{m_0 NI}{d} \ ,$$

along the axis of the solenoid. The magnetic flux is (approximately) the same for each turn of the solenoid, so:

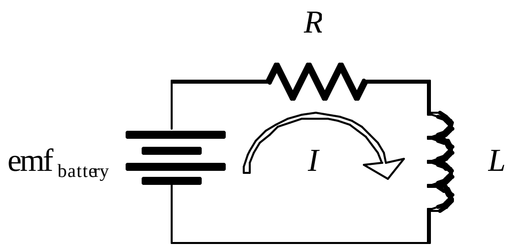
$$\Phi_{\substack{\text{mag}\\ \text{solenoid}}} = N\Phi_{\substack{\text{mag}\\ \text{one turn}}} = N[BA] = N\left[\left(\frac{m_0 NI}{d}\right)\left(pR^2\right)\right] = \left(\frac{m_0 N^2 pR}{d}\right) I \ .$$

Therefore, self-inductance of the solenoid is:

$$L=\frac{m_0 N^2}{d}pR^2 .$$

We’ll look at two examples of inductors in circuits. The symbol for an inductor in a schematic circuit diagram is a coil (because that’s what they usually are).

RL Circuit: a battery is connected to a resistor and inductor in series at time $t = 0$



The loop rule in the CW direction (with the conventional current *I*) gives:

$$\Delta V_{\text{battery}} + \Delta V_{\text{resistor}} + \Delta V_{\text{inductor}} = 0$$

$$\text{emf}_{\text{battery}} - IR - L\frac{dI}{dt} = 0 .$$

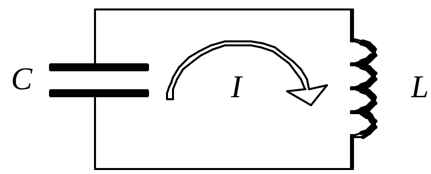
The second term is negative because the electric potential drops across a resistor in the direction the current moves. The third term is negative because the emf of the inductor is in the opposite direction as that of the battery. The solution with the initial condition that $I(t=0)=0$ is (check by substituting it back into the diff. eq.):

$$I(t)=\frac{\text{emf}_{\text{battery}}}{R}\left[1-e^{-\left(\frac{R}{L}\right)t}\right].$$

Features of the solution:

- The inductor is most important at early times when the current is changing rapidly.
- As the current changes more slowly, the inductor becomes less important. After a large enough time, the current is essentially constant and depends only on the battery and resistor: $I \to \text{emf}_{\text{battery}}/R$.
- The “time constant” for the circuit is L/R, which determines the length of the transient behavior. All circuits have some inductance, but it is often very small so the current comes to its “final” value quickly.

LC Circuit: a circuit with just a capacitor and an inductor (idealization because there is <u>always</u> some resistance in a real circuit)



Suppose the upper plate of the capacitor has an initial charge of $+Q_i$ at $t = 0$.

The loop rule in the CW direction (with the initial conventional current I) gives:

$$\Delta V_{\text{capacitor}} + \Delta V_{\text{inductor}} = 0$$

$$\frac{1}{C}Q - L\frac{dI}{dt} = 0.$$

The current is related to the charge by:

$$I = -\frac{dQ}{dt}, \text{ (taking Q to measure the charge on the capacitor)}$$

because the current is larger in the direction shown as the capacitor *dis*charges (as Q decreases). This gives:

$$\frac{1}{C}Q + L\frac{d^2Q}{dt^2} = 0$$

$$\frac{d^2Q}{dt^2} = -\left(\frac{1}{LC}\right)Q.$$

The solution (confirm by substituting it back in the diff. eq.) is:

$$Q(t) = Q_i \cos\left(\frac{1}{\sqrt{LC}}t\right)$$

so:

$$I(t) = -\frac{dQ}{dt} = \frac{Q_i}{\sqrt{LC}}\sin\left(\frac{1}{\sqrt{LC}}t\right),$$

The current oscillates back and forth with a period: $T = 2p\sqrt{LC}$. In a real circuit with resistance, the current will also decrease over time because energy is dissipated as heat.