Mon., 3/23 Wed., 3/25 Thurs., 3/26 Fri., 3/27	**21.4-6** Using Gauss's & Intro to Ampere's **21.7-9** Maxwell's, Gauss's, and Ampere's **Quiz** Ch 21, **Lab 9** Ampere's Law **(*write up*)** **22.1-2,10** Intro to Faraday's & Lenz's	**RE24** **RE25** **RE26**
Mon., 3/30 Tues., 3/31	**22.3-4,.7** Faraday & Emf & Inductance	**RE27** **Lab 9 Write-up HW21:**RQ.12, 15,17; P.20, 23, 26

Make-up Tests

From Last Time

- **Electric Field Flux**
    - The Electric Field Flux through a bit of area is
        - $\Phi_E \equiv \vec{E} \bullet \vec{A}$
    - The Electric Field Flux *out* through a closed surface
        - $\Phi_E \equiv \oint \vec{E} \bullet d\vec{A}$ (where A points out of the enclosed volume)
- **Gauss's Law**
    - $\Phi_E = \oint \vec{E} \bullet d\vec{A} = \dfrac{Q_{enclosed}}{e_o}$

This Time

***Using* Gauss's Law to find the Electric Field:**

For some distributions of charge, Gauss's law to determine the magnitude of the electric field. In particular, distributions with very simple field geometries.

1. Use a symmetry argument to determine the direction of the electric field
2. Draw a Gaussian surface with each patch either perpendicular or parallel to the electric field
3. Apply Gauss's law to find the magnitude of the electric field

Example 1: Spherical shell with radius *R* and uniformly distributed charge *Q*

By symmetry, the electric field must point radially and its magnitude can only depend on the distance from the center of the shell. A sphere of radius r is a good Gaussian surface since it will be perpendicular to the electric field everywhere. The electric flux is:

$$\sum \vec{E} \cdot \hat{n}\, \Delta A = E\left(4pr^2\right)$$

$r < R$: no charge inside $\left(\sum q_{\text{inside}} = 0\right)$, so $E = 0$

$r > R$: all charge is inside $\left(\sum q_{\text{inside}} = Q\right)$, so

$$\sum \vec{E} \cdot \hat{n}\, \Delta A = E\left(4\pi r^2\right) = \frac{\sum q_{\text{inside}}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

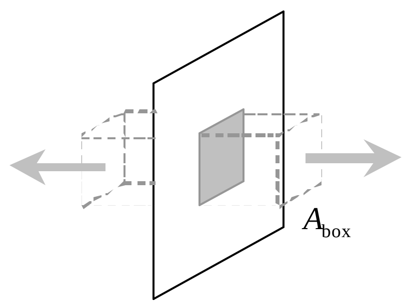
We've been using these results for quite a while now (the second part was just stated)!

Note: Gauss's law only helps get the magnitude of the electric field, not the direction.

Example 2: Large uniformly charged plate with charge per area of *Q*/*A*

By symmetry, the electric field near the center of the plate must point perpendicularly away from the plate. Its magnitude could depend on the distance from the plate.

A box that extends on each side of the plate is a good Gaussian surface since the electric field will be parallel or perpendicular to each side.



If the sides perpendicular to the plate each have an area A_{box}, the electric flux is:

$$\sum \vec{E} \cdot \hat{n}\, \Delta A = 2EA_{\text{box}}$$

The amount of charge inside the Gaussian surface is $\sum q_{\text{inside}} = \frac{Q}{A} A_{\text{box}}$, so

$$\sum \vec{E} \cdot \hat{n}\, \Delta A = 2EA_{\text{box}} = \frac{\sum q_{\text{inside}}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \frac{Q}{A} A_{\text{box}}$$

$$E = \frac{Q/A}{2\varepsilon_0}$$

Note: Need the Symmetry. Gauss's law cannot be simply used to determine the magnitude of the electric field for charge distributions that don't have the right kind of symmetry. For example, a cube with uniformly distributed charge. Step 1 fails in this case.

Charges on Metals:

We can make some quick, qualitative observations about fields and charges in metals using Gauss's Law. Use what we know about the electric field to determine properties of charge.

(1) **Static equilibrium** : $\vec{E} = 0$ inside metal

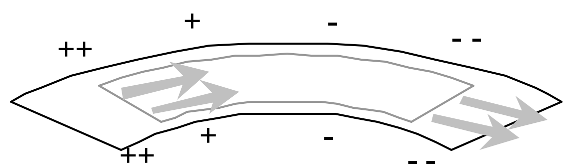
The electric flux through any shape of closed surface is zero, so there is no net charge inside.

$\vec{E} = 0$ thus $Q_{encl} = 0$

(2) **Steady state:**

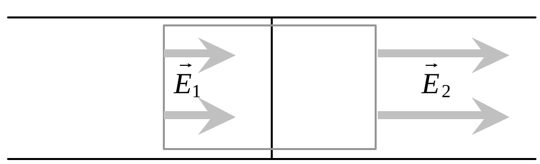
In a uniform, current-carrying wire with constant area, $\vec{E}$ has a constant magnitude and points along the wire. Draw a Gaussian surface just inside the wire (diagram below). The electric flux on the surface is zero, because the electric field points in one end and out of the other. Therefore, there can be no net charge inside the wire.

$\vec{E}$ uniform thus $Q_{encl} = 0$



Transition between two materials, Suppose there is a transition between wires with the same area A and mobile electron density n, but different mobilities $u_1 > u_2$. The electron currents must be the same (Node Rule), so $E_1 < E_2$. Draw a Gaussian surface just inside the wire that cross the transition (diagram below). The electric flux on the surface is positive, so there must be a net positive charge inside the surface. It is on the interface (see Fig. 18.37 on p. 642).

$\vec{E}$ *not* uniform thus Q_{encl} *not*= 0



Whiteboard Activity: Applying Gauss's Law

A. Uniformly-Charged Rod

A thin rod of length L has a positive charge Q distributed uniformly along its length.

- **Field Geometry**. Use a *symmetry* argument to determine the *direction* of the electric field near the center of the rod.

- **Choosing the Gaussian bubble.** What shape of Gaussian surface can you draw so that each part is either perpendicular or parallel to the electric field?

- **Doing the Math.** Use Gauss's law to find the magnitude of the electric field at a radial distance r from the rod near its center.

B. Solid Uniformly-Charged Sphere

A solid sphere of radius R has a positive charge Q distributed uniformly throughout its volume.

- **Field Geometry.** Use a symmetry argument to determine the direction of the electric field.

- **Choosing a Gaussian Bubble.** What shape of Gaussian surface can you draw so that each part is either perpendicular or parallel to the electric field?

- **Doing the Math.** Use Gauss's law to find the magnitude of the electric field at a distance r from the center of the sphere for:

  $r > R$

  $r < R$

"Gauss's Law" for Magnetism:

Flux is a very general mathematical idea. The flux of anything (water for example) through a closed surface depends on the sources / sinks of that something enclosed by the surface. If neither (or equal sources and sinks) are enclosed, then there's no net flux.

Applied to Magnetism. Another way to say that there are no magnetic dipoles is that there is no (zero) magnetic flux through any closed surface:

$$\oint \vec{B} \cdot \hat{n}\, dA = 0.$$

In other words, there are no points that the magnetic field radiates away from.

That's a boring, and not so useful relation. However, there is a relations, *similar* to Gauss's Law, that *is* similarly useful for magnetism.

Ampere's Law:

You can determine what is inside a *loop* by knowing the magnetic field around the loop. Sketch some examples:

- **B-field tangent to a loop & counterclockwise – current out of page**
- **B-field tangent to a loop & clockwise – current into page**

We want to show that the path integral of the magnetic field around a closed *loop* is proportional to the amount of current piercing through the loop:

$$\oint \vec{B} \cdot d\vec{\ell} = m_0 \sum I_{\text{inside path}}.$$

1. Find the proportionality constant– use a single, straight wire carrying current I in the center of a circle of radius r. Integrate counterclockwise (direction of $d\vec{\ell}$).

   The magnetic field is the same size everywhere on the circle:

   $$B_{\text{wire}} \approx \frac{m_0}{4p} \frac{2I}{r},$$

   so the path integral is approximately:

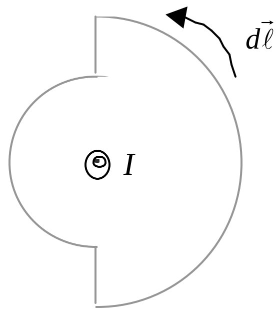
   $$\oint \vec{B} \cdot d\vec{\ell} = B_{\text{wire}} 2pr = m_0 I$$

   Argument for Generality. Though not proven here, it turns out that this is better than an approximation, it's an equality. You may accept that the approximation becomes an equality in the limit that r goes to zero; then, having established that it is true for the differentially thin loop around a differentially small current, you can argue that it must scale up to be true for a big loop around the same differentially small current, and then, the supper position principle can be applied to add up the effects of not so differentially small *I*.

This is what we want. It is true regardless of the radius of the loop (B varies as $1/r$ and the circumference varies as r).

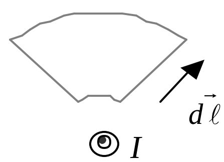
2. **Loop Geometry doesn't matter.** Show that the shape of the loop doesn't matter for a single wire.

   Any shape of loop can be made with radial segments and arcs centered on the wire.



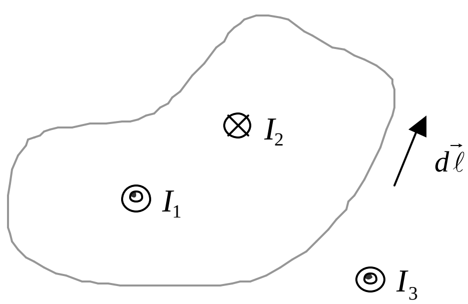
3. **Only depends on *en-looped* current.** The path integral of the magnetic field around a closed loop is zero if a wire is outside.

   Once again, any shape of loop can be made with radial segments and arcs centered on the wire. However, for every segment with a positive path integral, there will be one that is negative and the same size. Therefore, the total path integral is zero.



4. **Multiple Currents.** Ampere's law holds for multiple currents

   Ex: three currents – one into the loop, one out of the loop, and one outside of the loop



$$\oint \vec{B}_1 \cdot d\vec{\ell} = m_0 I_1$$

$$\oint \vec{B}_2 \cdot d\vec{\ell} = -m_0 I_2$$

$$\oint \vec{B}_3 \cdot d\vec{\ell} = 0$$

Add these equations together:

$$\oint \left(\vec{B}_1 + \vec{B}_2 + \vec{B}_3\right) \cdot d\vec{\ell} = m_0 \left(I_1 - I_2\right)$$

$$\oint \vec{B} \cdot d\vec{\ell} = m_0 \sum I_{\text{inside path}}$$

The direction of the currents inside the loop must be taken into account. If you curl the fingers of your right hand in the direction of the path around the loop, your thumb will point in the direction that is positive for current. Currents pointing in the opposite direction are negative.

What Ampere's Law Does *not* say (applies for whole loop, not segments)

Note that Ampere's law says it is the *complete* loop integral that depends exclusively on the piercing current – it does *not* say that the field or even $\vec{B} \cdot d\vec{l}$ over a given segment must depend only on the piercing current. If you imagine three parallel wires and draw an Amperian loop around just one of them, certainly the field at every point on the loop is influenced by all *three* currents; *however,* Ampere's law *does* say that, when you integrate over the whole loop – the contributions of external currents canc el out of the sum. We noted something similar for Gauss's Law last time – you get the simple result only when you sum flux through the whole surface.

Also, in this form, it only speaks for continuous *currents* – that is, it cannot handle a single moving point charge.

Using Ampere's Law to find the Magnetic Field:

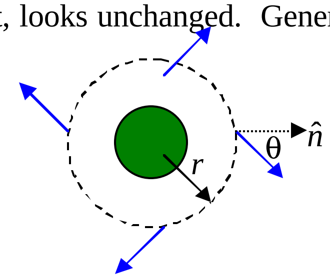
Like Gauss's Law, Ampere's Law is of particular *use* when the field geometries are simple. For some distributions of charge, Ampere's law to determine the magnitude of the magnetic field.

1. **Field Geometry.** Use a symmetry argument to determine the direction of the magnetic field
2. **Amperian Loop.** Draw a Amperean loop that is either perpendicular or parallel to the magnetic field
3. **Math.** Apply Ampere's law to find the magnitude of the magnetic field

C. Thick Current-Carrying Wire

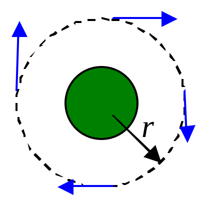
A long, thick wire of radius R carries a current I.

- **Field Geometry.** Use a symmetry argument to determine the direction of the magnetic field near the center of the wire's length.
    - We have cylindrical symmetry in the current flow, so we must have cylindrical symmetry in the magnetic field. That necessitates that, at any point a distance r from the center of the wire, the field must have the same strength and the same direction (in terms of r and q), so that if the wire is rotated, the field, just like the current, looks unchanged. Generally, this says the field looks like:



- **No Radial Component by Gauss's Law**
    - Note: as illustrated, symmetry arguments alone don't *force* the field to be *tangential* to the surface. That comes from Gauss's Law for Magnetism:
        - $\oint \vec{B} \cdot \hat{n}\, dA = 0$
    - Imagine for a moment that we enwrap the wire in a Gaussian shell, our Amperian ring is just a cross-section of that. Applying symmetry, we still have that B is constant and of constant orientation relative to the area everywhere on the surface, so $B \sin q \oint dA = B \sin q L p r = 0$. The only way to make this equal zero as we know it must is for sinθ =0, or θ = 90°.

- **Amperian Loop.** What shape of Amperean loop can you draw so that the magnetic field is tangent or perpendicular to each segment?



- **Math.** Use Ampere's law to find the magnitude of the magnetic field at a radial distance r from the center of the wire for:

$r > R$

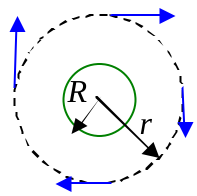
Now, applying **Ampere's Law** gives

$$\oint \vec{B} \cdot d\vec{\ell} = m_0 I_{\text{insidepath}}$$
$$B2pr = m_0 I_{\text{insidepath}}$$
$$B = \frac{m_0 I_{\text{insidepath}}}{2pr}$$

Exactly what we have for an infinitesimally thin wire. In fact, notice that the only assumption that we made about the current density was that it was cylindrically symmetric: that covers a line current (no width), a hollow shell of current, and everything in between – say, a current that drops of across the radius of the wire. As long as it's radially symmetric, the field looks the same.

$r < R$ if we assume uniform current density, I/A, then…

Hollow wire. Look at the case of a hollow shell of current:



The math woks out just the same:

$$\oint \vec{B} \cdot d\vec{\ell} = m_0 I_{\text{insidepath}}$$
$$B2pr = m_0 I_{\text{insidepath}} \quad \text{for r>R}$$
$$B = \frac{m_0 I_{\text{insidepath}}}{2pr}$$

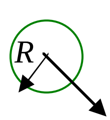
What about *inside* this hollow tube of current?

The symmetry and Gaussian arguments are the same, but I_{inside} is 0, so inside

B = 0.

Coaxial Cable

Now, what about a Coaxial cable? That's got a thin wire in the middle, carrying current one way, and a hollow tube encircling it and carrying equal and opposite current.



The field inside the tube is purely due to the inner wire, $B = \frac{m_0 I_{wire}}{2pr}$, r<R

While the field outside is due to both of the wire and tube of current,

$$B = \frac{m_0 I_{wire}}{2pr} + \frac{m_0 I_{tube}}{2pr}, \text{ r>R},$$

but if they have equal and opposite currents, then the two terms cancel so

B = 0 , r>R.

That, in fact, is one of the appeals of coaxial wires.

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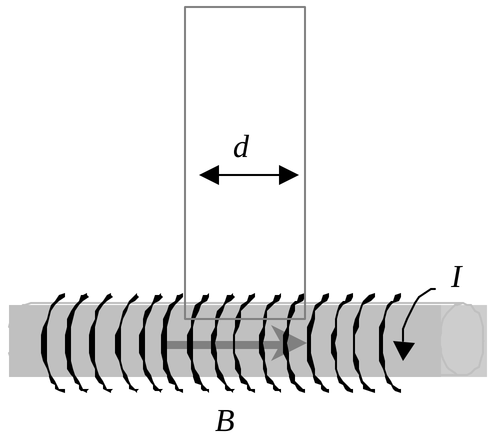
Solenoid. Now for something a little more complicated: the solenoid.

Symmetry: demands that the field is cylindrically symmetric. If we approximate the solenoid as infinitely long, i.e., we want an approximate answer good near the middle of its length, we can argue for linear symmetry too – that is, anywhere we look along the solenoid's length, the field must be pointing in the same direction.

Considering the fields due two or three consecutive rings, we can also argue that they cancel each other's radial components of the field (or a Gaussian surface could be employed to rule this out.) So we're left with only axial components.

Example 3: solenoid with N/L wraps per length carrying current I

The magnetic field inside the solenoid points in the direction shown below (RHR). Draw a rectangular loop that has a side parallel to the magnetic field. The loop can extend very far away from the solenoid so that the contribution of the top end must be zero.



The path integral is Bd and the amount of current in the loop is $d(N/L)I$, so:

$$Bd = m_0 \left[d(N/L)I \right]$$

$$B = \frac{m_0 NI}{L}$$

This is much more difficult to do using the Biot-Savart Law (see Ch. 17).

Note: it is not trivial to reason that the field is purely axial. Applying Ampere's Law in the plane of a current loop gives 0 (the current loop doesn't *pierce* the Amperian loop) so there's no angular component; applying Gauss's law tells us there's no radial component.

Apply to a Torus

II. Applying Ampere's Law

Friday: Practice with Gauss's law and Ampere's law