

Wed., 2/18	18.4-7 E. Field of Surface Charges, Transients, Feedback, Resistors	RE15
Thurs., 2/19	Quiz Ch 17, Lab 6: E. Field of Surface Charge	RE16
Fri., 2/20	18.8-11 Energy, Applications of the Theory, Detecting Surface Q	Exp 18,19,22-24
Spring Recess		
Mon., 3/2	19.1-5 Capacitor Circuits	RE17 , Exp 29,30
Tues., 3/3		HW18: RQ.38, 41, 44; P.49, 52, 56 (hint: consider V at spheres when $L \gg R, r$)
Wed., 3/4	19.6-.14 Capacitor & Resistor Circuits	RE18 , Exp 31-34
Thurs., 3/5	Quiz Ch 18, 19.15-17,19 Meters and RC Circuits	Exp 35-37

344 recant – there *is* an electric field inside a battery, and a corresponding voltage. But this is in the direction as to oppose current flow through it. It’s the chemical potential that drives the flow in spite of the opposing voltage.

Change reading and RE accordingly for next time

Load Old Vpython

Materials

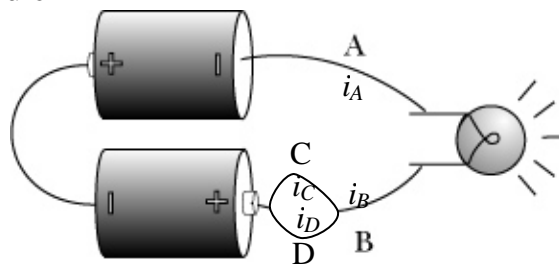
- Quiz ch 17 handout
- Lab 18 handout
- Batteries – anyone need new ones?

Equipment

- Alan’s charge on a wire demo (HV power supply, chain of Mega-Ohm resistors, and a pithball on a string)
- Curvy Circuits power point
- Electric Hockey (also found in PhET)

Last Time

- **Application of E & M fundamentals to understand circuits.**
- **Electron current: number of electrons passing a cross-section per time.**
 - $i = nAv_{ave}$
- **Current is constant in series**
- **Node rule**



- $i_A = i_B = i_C + i_D$

- **Drude Model**

- Electrons get accelerated forward, stop randomly due to collisions, and accelerate again.

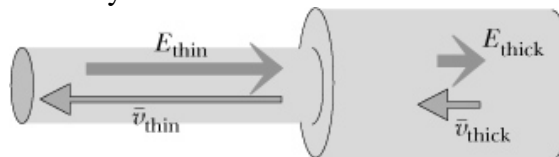
- $\bar{v}_{drift} = \frac{eE\overline{\Delta t_{half}}}{m_e}$
 - Where the time is half the average time between collisions and is material and temperature dependent.
- $\bar{v}_{drift} = \left(\frac{e\overline{\Delta t_{half}}}{m_e} \right) E = uE$
 - Where $u \equiv \left(\frac{e\overline{\Delta t_{half}}}{m_e} \right)$ is electron mobility.

○ $i = nA(uE)$

○ **Implications**

▪ **Varying E Strength.**

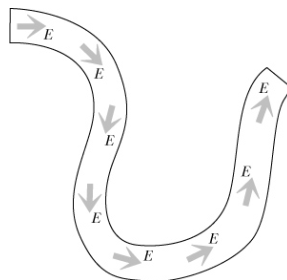
- Constant current and constant electron density requires E varies inversely with A.



- **General.** This is actually a very general result for any fluid flow: pushed through a narrow constriction, the fluid speeds up. This is used in needleless shots. The medicine gets pushed through such a fine opening that it goes so fast, the stream itself pushes through your skin.

▪ **Varying E direction.**

- v's direction varies to always point along the wire, so must E.



- **E is constant across a cross-section (not stronger or weaker near edges).**

So, we deduced that, *if* the current is constant, as we can measure and reason it to be, *then* the electric field *must* behave in these ways. We didn't say *what causes* the field to do this, just that we deduce that it must. This time, we explore the *cause*.

This Time

18.4 What Charges make the Electric Field in the Wires?

- It takes charges to generate electric fields. So, where are the charges that generate *this* electric field that keeps the current going with the right velocity (magnitude and direction) everywhere?
- **Wrong Answers.** The book addresses some plausible, but wrong answers.
 - **Are the excess charges inside the conductors?**
 - **Q.** Why does the book discount this?
 - Not *inside* a conductor. These charges would themselves up and move in response to the field.
 - **Are the excess charges on the battery?**
 - **Q.** Why does the book discount this?
 - If they were on the battery, then we'd have the field of a dipole – but that doesn't agree with observed currents. For example – moving a lightbulb closer and further from the battery-dipole would increase and decrease the field strength it experienced and thus the current –but that doesn't happen. Rotating the lightbulb perpendicular to the battery doesn't kill the current, though there would be no perpendicular component of a dipole's field.
- **On the wire surfaces.**
 - If the excess charge isn't on the battery and isn't in the wires, the only place it can be is *on* the wires' surfaces. To understand how it gets there, we can run through the dynamics that gets the charges there.

18.4.3 A mechanical battery

- The big motivator in a circuit is the battery. That's the thing that directly or indirectly gets the current going. So we'll start there. If we understand its operation, everything else's operation will follow logically.
- **Bottom Line.** A battery does what it can to maintain a given charge separation between its two electrodes. Thinking of them like capacitor plates, that means there's a corresponding electric field within and a corresponding voltage drop across the battery. When you wire a battery up to a circuit, allowing charges to flow from one electrode to the other (thus reducing the charge separation), the battery does what it can to counter that by continuously putting new – charges on the – electrode and + charges on the + electrode.

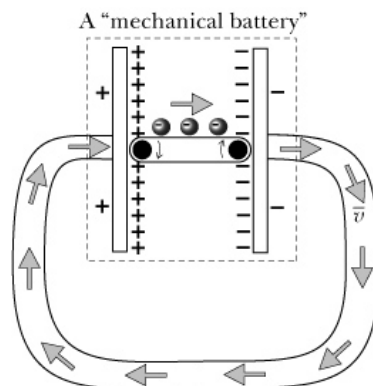
PowerPoint

- **Chemical view**
 - For those who have had chemistry, the reaction going on in a typical lead-acid battery is $Pb + PbO_2 + 2(2H^+ + SO_4^{2-}) \rightarrow 2PbSO_4 + 2H_2O$
 - This actually plays out as three separate reactions – one at one electrode, one in the solution, and another at the other electrode. In the process, 2 electrons are *deposited* on the negative electrode, and 2 protons *await* them at the positive electrode – thus a charge separation is established.
 - Over all, the process is chemically favorable. Like a rock rolling down a hill to reduce its gravitational potential energy, this process naturally happens because it reduces its chemical potential energy.

- Note that the rate at which the charges get ‘conveyed’ from one electrode to another depends upon the random walking of the ions as they diffuse from the electrode where they’re generated, the random chance of they’re meeting up to create intermediate products, and *their* subsequent random diffusion until they happen to bump into the other electrode. There is at no point really a specific force *pushing* the ions to the appropriate electrode – it’s just random walk diffusion.
- In fact, as the charges build up on the electrodes, a specific force *does* develop that *opposes* the continued flow of charged particles – the electric force due to the field generated by the charges separated onto the two electrodes.
- But, if a battery isn’t attached to anything, then a charge separation builds up on the two electrodes, and that means an electric field and an electric force / voltage difference and an electric potential energy which eventually gets great enough to completely oppose the diffusion of the charges.
- When you wire up the battery to a circuit, for every electron that leaves the – terminal and arrives at the + terminal, the internal electric field is reduced enough to allow the chemical process to move forward to put one more electron on the – terminal and take one more off the + terminal – maintaining charge separation.

- **Mechanical Model**

- **Equilibrium.** The book offers a mechanical analog. The chemical process, like a conveyor belt, drives charges from one side of the battery to the other. In equilibrium (the battery not attached to anything) the electric field generated by the charge separation completely opposes the force of the belt, and the charges stop moving across.
- **Wired up.** When the battery is wired up to a circuit, charges can leak off the electrodes and into the circuit and thus the internal electric field reduces and allows the conveyor belt to ‘win’ and put more charges back on the electrodes.



18.5 Connecting a Circuit: The Initial Transient

- But I’m getting ahead of myself. Now let’s say that we attach two wires to the two electrodes of the battery – but we don’t attach them to each other yet – they’re just

dangling. Well, you can consider the – electrode & its wire as one odd shaped conductor. Very quickly, charge on the electrode will now distribute itself over the wire fairly uniformly on the surface - in such a way as to make 0 net Electric field inside. Ditto for the + electrode and its wire. As charge leaves the electrodes to coat the wires, the ‘conveyor belt’ in the battery runs to replenish the charges on the electrodes until we have a new equilibrium.

- Next, we connect the two dangling ends of the wires. When you connect the two wires, the + and – charges right at the connection recombine. That means there’s a non-uniformity in the charge density there, so charges flow, and so some – charges from further downstream flow further upstream to recombine with + charges there... this continues until you get electrons from the battery swimming all the way to the batteries + terminal to recombine – steady-state continuous flow with a continuous charge gradient.

18.5.1 Why a light comes on right away

- It may take an *extremely* long time for an individual electron to make it from the switch to the lightbulb, traveling with a drift velocity around 5×10^{-5} m/s; however, when the charge density changes at the switch, that generates a change in the electric field (which propagates at c) which in turn changes charge densities everywhere else. So the electrons in the light bulb start moving.

18.6 Feedback

- Both times we talked about setting up a current in the wire, we used the idea of feedback – that a change in charge density changes the field which in turn changes the charge density again until we get to a new steady state situation.

18.6.1 Feedback and current magnitude: leads to current equalization

- Ask them – give a picture like 18.29, ask how things would play out.
 - *If* the current flowing out of a section of wire ever exceeded the current flowing in, that would result in a charge depletion in that region which would serve to make the region more attractive to charge - draw charge in more quickly and pass charge out more slowly...until the rates equalized and current *became* constant across.

18.6.2 Feedback and current direction: makes current follow the wire

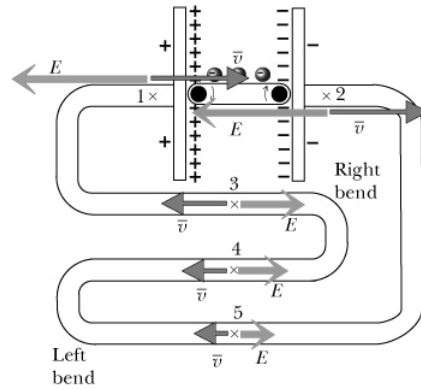
- Ask them – give a picture like 18.31, bending the wire (without initially changing charge density), ask how things would play out.
 - Charges would run into the bend of the wall. There, they would deflect subsequent charges from doing the same. Voila, current follows the bend.

18.6.2 Summary of Feedback

- **Charges arrange themselves on the surface as to maintain constant current down the wire.**
 - The equilibrium special case of this is that that current is 0.

18.4.4 Field due to the battery

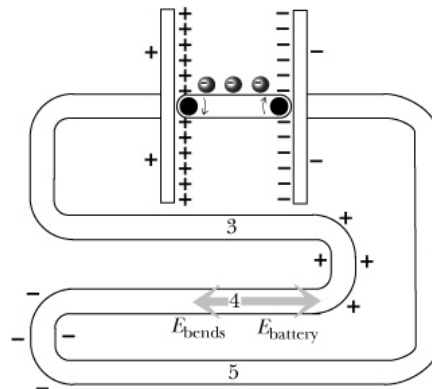
Just to demonstrate how robust the feedback process is – we can start from *any* initial condition, and see that the charge distribution will evolve into the same steady-state.



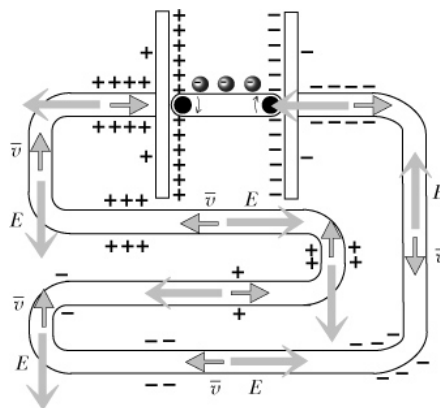
- Say we instantly connected a curvy wire to both ends of a battery. In the initial instant, this *is* the only field we've got, so it's got to be what gets charge motion started things started. But what happens next?

18.4.5 Charge buildup on the surface of the wire

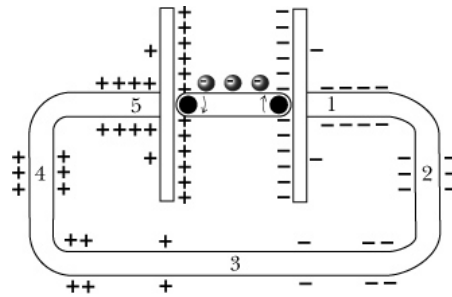
- In a curvy circuit of wire, above is a picture of what the field would initially look like, and what velocities charges would initially have.
- **Charge build up.** This quickly pushes charges into the vertical walls of the wire. *That* charge build up generates field of its own which redirects subsequent charges. Looking particularly at bend 4, One bend becomes positive and the other negative as shown below.



- The charges continue to redistribute on the surface of the wire until the electric field has a uniform size and direction parallel to the wire as shown below. The surface charge is only shown in some places, but it changes smoothly in between.

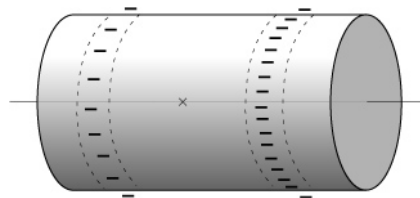


- Viola! This is the charge distribution that supports *steady-state* current flow.
- The surface charges for a simpler circuit are shown below. It is easier to understand how this produces the right electric field.



18.4.6 The electric field of a simple surface-charge gradient

- You'll notice that these pictures all show surface-charge gradients. Aside from being necessary to smoothly transition from the large - charge at the - terminal and + charge at the + terminal, it is necessary for driving a current. You may recall from a problem a few chapters ago, on the inside of a uniform cylinder of charge, its field is 0. It's easy to imagine that this is true on axis (for every charge, there is an equidistant one in the opposite direction, so no net field), but it pans out being true throughout. Along a straight wire, a uniform charge density gives you a uniform field. You'll see this in lab.



- At point x, the field due to the left ring is higher than that due to the right ring, so there's a net field to the right.

Demo: Erings.py

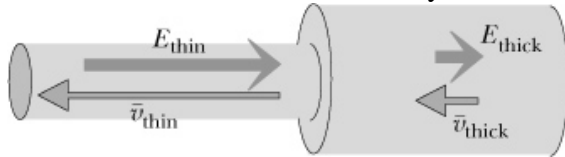
18.6.7 Amount of surface charge

- If you have a 3 Volt battery and a 0.1 m wire connecting its two terminals, then there must be a 3V drop over 0.1 m of length, i.e., a field of 30 V/m. This corresponds to a charge density of about 10^6 - 10^7 electrons / cm. That's not much compared to what you get on a piece of tape.
- But if you have 6,000V drop over 0.1 m, now we're talking observable charge density!

DEMO: (Use 6-kV supply or show Sherwood's video) The size of the surface charge is typically very small for circuits with batteries, so it is not possible to see the effect on nearby charged objects. It takes a power supply that maintains about a 10,000-volt potential difference to see the effect of the surface charges.

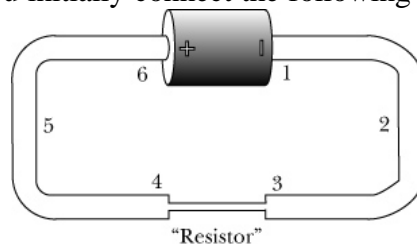
18.7 Surface Charge and Resistors

- Now that we understand the causal relationship between surface charge and Electric field (a charge gradient generates a field), we can look back at a case where we've already deduced that the field *must* vary and understand *why* that happens. Specifically, last time we said that if a wire gets thin, the field must get strong to increase the drift velocity and maintain the current. To refresh your memory, our reasoning went like this.

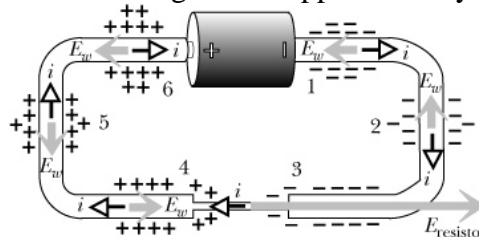


- Ask them to reason through it:
 - $i = nAv_{drift} = nAuE$
 - $i_{thick} = nA_{thick}uE_{thick} = nA_{thin}uE_{thin} = i_{thin}$
 - $E_{thin} = \frac{A_{thick}}{A_{thin}} E_{thick}$

- Now we know that the charge gradient must be greater (the charge density must change faster) over the thin wire than over the thick wire.
- Dynamic Picture.** This is pretty easy to picture happening.
 - Say you initially connect the following circuit to the battery.



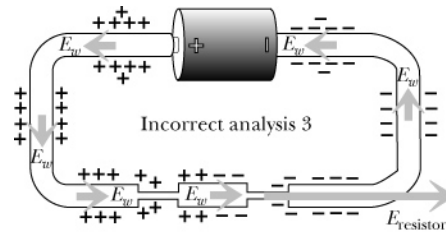
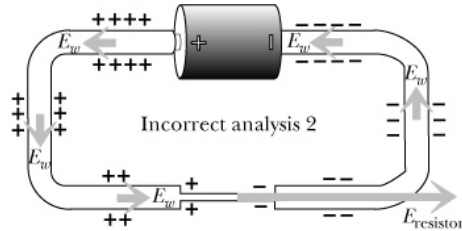
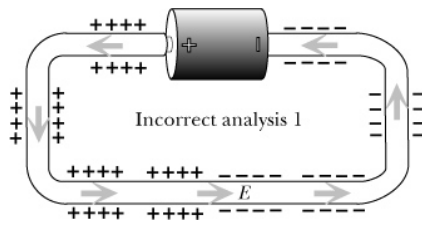
- A broad flow of electrons goes rushing down the wire at a high speed. When they hit point three, the shoulders of that flow smack into the wall – charge accumulates there, and current backs up, slowing back down the line. Pretty soon, current's going slowly at point 2 and slowly at point 1. Meanwhile, *within* the thin wire, the electrons keep shooting along at their high speed. When they come out at the other side, at point 4, you get a shadow of the opening cast onto the nearby wider walls – a charge depletion. In fact, you get the + mirror image on the + side of the resistor (+ terminal readily draws electrons from wire, and from the vertical walls at point 4.) Across the “resistor”, from point 3 to 4 you then have a significant charge gradient.
- Surface Charge.** The surface charges look approximately like the following.



Not that the change (or gradient) of the surface charge is larger between 3 and 4, which is what creates a larger electric field.

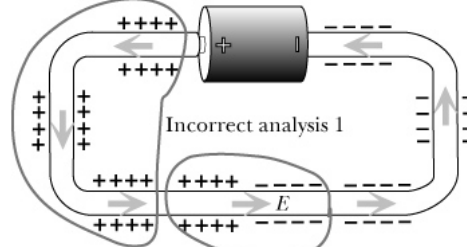
Hint: It is best to use $i = nAue$ to figure out the electric field first, then determine the surface charge.

Exercise: 18.10 – have a pair of students look at each diagram

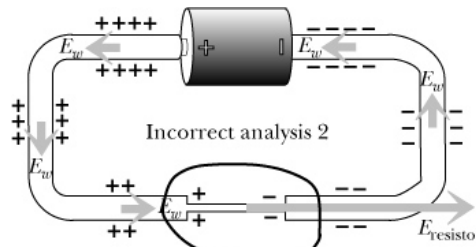


Solutions:

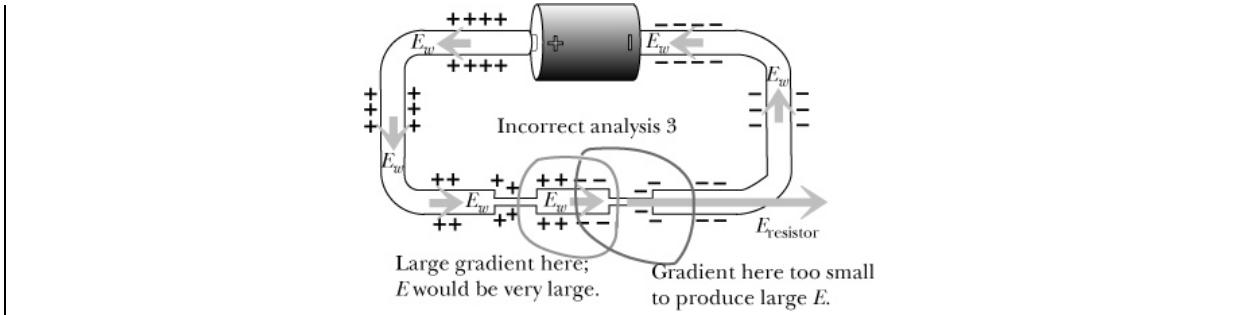
No charge gradient in this region, so E would be *very* small.



Huge gradient here, would make *huge* E .



Charge gradient here too small to create a large E .



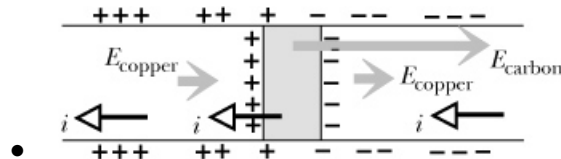
○ **Mobility Resistor.**

- In practice, a resistor isn't so much *thinner* than the rest of the wire, it is made of a lower mobility, or lower carrier density material, carbon is popular. Making things simple by assuming equal thickness,

$$i_{Cu} = n_{Cu} A u_{Cu} E_{Cu} = n_C A u_C E_C = i_C$$

- $$E_C = E_{Cu} \left(\frac{n_{Cu} u_{Cu}}{n_C u_C} \right)$$

- A resistor can also be just as wide as the wire, but made of a different material with a lower mobility u , such as carbon. In this case, there will be a buildup of charges on the surfaces between the materials.



Review questions from Chapter 18: Basic aspects of circuits

18.RQ.25 What is the most important general difference between a system in “steady state” and a system in “equilibrium”?

18.RQ.26 Describe the following attributes of a *metal wire* in steady state vs. equilibrium:

	Metal wire in steady state	Metal wire in equilibrium
Location of excess charge		
Motion of mobile electrons		
E inside the metal wire		

18.RQ.27 How can there be a nonzero electric field inside a wire in a circuit? Isn't the electric field inside a metal always zero?

18.RQ.28 Electron current $i=nAv=naueE$:

What are the units of electron current?

What is n ? What are its units?

What is A ? What are its units?

What is v ? What are its units?

What is u ? What are its units?

Handouts:

- Quiz for Ch. 17
- Handout on Electric Field of Rings

Activity:

Electric Field of Rings – students will write a VPython program

Friday: Resistors and Energy