Fri., 2/13	**17.10-11** Permanent Magnets	**Exp** 23-29 (work together for 2 magnets)
Mon., 2/16 Tues., 2/17 Wed., 2/18 Thurs., 2/19 Fri., 2/20	**18.1-3** Micro. View of Electric Circuits **18.4-6** E. Field of Surface Charges, Transients, Feedback **Quiz** Ch 17, **18.7-9**, **Lab 6:** E. Field of Surface Charge **18.10-11** Applications of the Theory, Detecting Surface Q	**RE14 , Lab Notebook** **HW17:**RQ.31, 32, 34; P.49, 51, 52 **RE15** **RE16** **Exp** 18,19,22-24

Modify Lab 17 to have them define a list of locations for hoop and for solenoid. That way, they don't have to as significantly alter the structure of their code when they switch from hoop to solenoid. See Bsolenoid.py

Equipment

- **Solenoid, 2Amp power supply, wires, field probe & labpro plugged into instructor computer**
- **Cow magnet**
- **Compasses**
- **Iron filings**

Announcements

- **Physics Candidate Visit**
  - Monday class
  - Monday talk: 4pm

Last Time

- Last class, we talked about analytically summing over different current configurations: the straight wire and the loop.
- **General Process**
  - Predict
  - Divide up current into pieces and draw $\Delta\vec{B}$ for a representative piece
  - Write an expression for $\Delta\vec{B}$ due to one piece
  - Add (integrate) up the contributions for all pieces
  - Check the result
- **Wire**
  - $\left|\vec{B}_{\text{wire}}\right| = \frac{m_0}{4p}\frac{LI}{r\sqrt{r^2+(L/2)^2}}$
  - If $L >> r$, then:
    - $\left|\vec{B}_{\text{wire}}\right| \approx \frac{m_0}{4p}\frac{2I}{r}$
- **Loop**
  - **On axis** $B_z = \frac{m_o}{4p}\frac{IR^2}{\left(r_o^2+R^2\right)^{\frac{3}{2}}}2p$
    - **In Lab**

- **Experiment.** Yesterday, you experimentally measured the loop's magnetic field as a function of distance along the axis. Your data *very* well fit this function.
- **Computation.** Though it's challenging to find an analytical expression for the off-axis magnetic field of a loop, it's not so hard solving for the field computationally. All of you quite successfully found that.
- **Demo:** Bloop_lab.py
    - **Talk through steps**
        - Drew a "hoop" for visualization
        - For a given observation location
            - Described each *dl* in terms of R, theta and d_theta
            - Found the field that each generated
            - Kept a running sum of the each segment's contribution to the field to find total field at the observation location.
        - Moved on to the next observation location

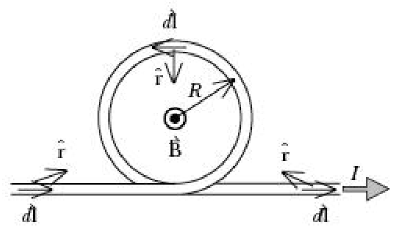
- **Back to our analytical solution. r >> R**
    - $\left|\vec{B}_{\text{loop}}\right| \approx \frac{m_0}{4p}\frac{2IpR^2}{z^3}$
- **Magnetic Dipole**
    - This lead us to the defining the magnetic dipole moment
        - $m = IA$
    - Then the magnetic field of the magnetic dipole is quite similar to the electric field of the electric dipole; indeed, the fields look quite similar too.
        - $\left|\vec{B}_{\text{dipole}}\right| \approx \frac{m_0}{4p}\frac{2m}{r^3}$
- **Solenoid**
    - The solenoid is a coil of current carrying wire. What's nice about it is that the field inside is pretty nearly constant. So, like the Capacitor is a convenient source of uniform electric field, the Solenoid is a convenient source of uniform magnetic field.
    - **Computation.** In lab, you very nearly tackled the solenoid.
        - **Structure:**
            - Create a series of current loops, or "hoops", and sum over all *their* contributions to the field at the observation location.
            - Step through multiple observation locations.
            - **Demo:** Bsolenoid.py
                - Look at structure
    - Note the near constant field inside.
    - **Demo:** power-up a solenoid and us field probe to show nearly constant.
    - **Demo**: 17_solenoid_drag (big field).py

- Click around outside the solenoid to see its field out there – what you'd expect of a stretched-out loop. (What we'll see for a bar magnet)

Exercises

1. A very long wire carrying a conventional current I in the direction shown is straight except for a circular loop of radius R. Calculate the magnitude and direction of the magnetic field at the center of the loop.

The magnetic field at the center of the ring is the sum of the magnetic field of the straight wire and the magnetic field of the loop. Both the wire and the loop make B out of the page, so the net field points out of the page.



$$\vec{B}_{\text{loop}} = \int \frac{\mu_0}{4\pi}\frac{I d\vec{l} \times \hat{r}}{r^2} \text{ and } d\vec{l} \perp \hat{r} \text{ everywhere on the loop, so}$$

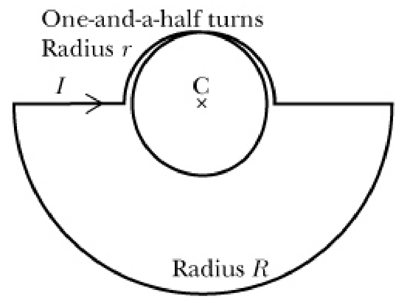
$$B_{\text{loop}} = \frac{\mu_0}{4\pi}\frac{I}{r^2}\int dl \sin 90° = \frac{\mu_0}{4\pi}\frac{I}{r^2}(2\pi r) = \frac{\mu_0}{4\pi}\frac{2\pi I}{r} \text{ out of the page}$$

(special case of on-axis field of loop)

$$B_{\text{straight wire}} = \frac{\mu_0}{4\pi}\frac{2I}{r} \text{ out of the page}$$

$$B_{\text{net}} = \frac{\mu_0}{4\pi}\frac{2I}{r}(1+\pi) \text{ out of the page}$$

2. A conventional current I flows in the direction shown below. Determine the magnitude and direction of the magnetic field at point C.



By the RHR, all of the curved segments produce magnetic fields into the page at C. The straight segments produce no (zero) magnetic field at C since $d\vec{\ell} \parallel \hat{r}$ for those.

The outer arc (radius R) produces half the magnetic field of a loop (z=0):

$$\left|\vec{B}_{\text{outer}}\right| = \frac{1}{2}\left(\frac{m_0}{4p}\frac{2pI}{R}\right) = \frac{m_0}{4p}\frac{pI}{R}$$

The inner turns (radius r) produces 1.5 times the magnetic field of a loop (z=0):

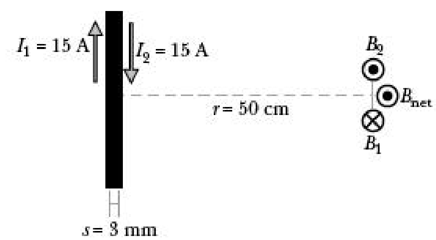
$$\left|\vec{B}_{\text{inner}}\right| = \frac{3}{2}\left(\frac{m_0}{4p}\frac{2pI}{r}\right) = \frac{m_0}{4p}\frac{3pI}{r}$$

Since the magnetic fields from the two parts are in the same direction (they are <u>vectors</u>!), the net magnetic field into the page is:

$$\left|\vec{B}_{\text{net}}\right| = \left|\vec{B}_{\text{outer}}\right| + \left|\vec{B}_{\text{inner}}\right| = \frac{m_0}{4p}\frac{pI}{R} + \frac{m_0}{4p}\frac{3pI}{r} = \frac{m_0}{4p}pI\left(\frac{1}{R}+\frac{3}{r}\right)$$

3. In a house, the maximum current carried by most wires is 15 amperes (circuit breakers flip off the power above that). Suppose two wires in a home power cord are about 3 mm apart as shown below and carry current in opposite directions at any instant (the directions alternate 60 times per second).
   (a) Calculate the maximum magnitude of the magnetic field 50 cm away from the center of the power cord.

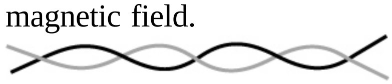
(a) At any instant the wires make magnetic fields in opposite directions. Since I_2 is closer, this results in a small net field. Assuming the wires are very long, the maximum magnitude of the time-varying magnetic field is this:



$$B_{\text{net}} = \frac{\mu_0}{4\pi}\frac{2I_2}{(r-s/2)} - \frac{\mu_0}{4\pi}\frac{2I_1}{(r+s/2)} = \frac{\mu_0}{4\pi}2I\left[\frac{1}{(r-s/2)} - \frac{1}{(r+s/2)}\right]$$

$$B_{\text{net}} = \frac{\mu_0}{4\pi}2I\left[\frac{s}{r^2-(s/2)^2}\right] \approx \frac{\mu_0}{4\pi}\frac{2Is}{r^2} = \left(10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)\frac{2(15\text{ A})(3\times10^{-3}\text{ m})}{(0.5\text{ m})^2} = 3.6\times10^{-8}\text{ T}$$

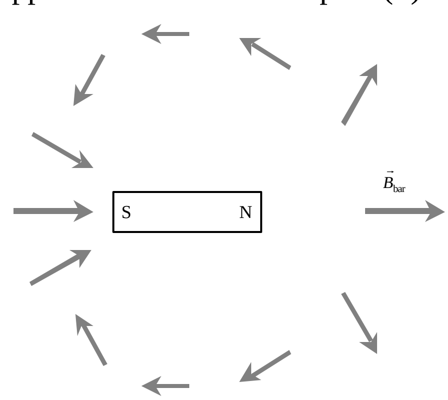
(b) Explain briefly why twisting the pair of wires into a braid as shown would minimize the magnetic field.



(b) If the wires are twisted then their fields should very nearly cancel, since the magnetic field contributions by regions where wire 1 is closer are nearly canceled by regions where wire 2 is closer.

17.10 Magnetic field of a bar magnet

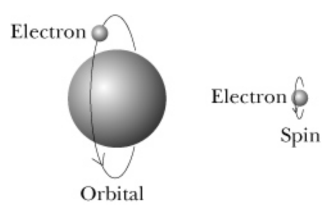
- The bar magnet produces a magnetic field very similar to the magnetic dipole (loop of current). The end of the magnet that the magnetic field points away from is called the north pole (N) and the opposite end is the south pole (S).



- **Demo: Solenoid – Magnet Equivalence.** Plug a solenoid into our power supply, lay compasses around it and see the direction of the field. Then replace the solenoid with a cow magnet.
- Note that the cow magnet is having an even stronger effect than was the solenoid with about 2 amps running through it. Somehow, there must be the equivalent of *over* 2 amps circulating around in this cow magnet!

Where is it?

- **Atomic structure of magnets**
  - **Magnitude.** The book estimates that one atom will contribute about $10^{-23}\ \mathrm{A\cdot m^2}$ (we won't repeat that calculation). The magnet has a mass of about 15 g, so if it is made of iron (56 g/mole) there are about $(15\ \mathrm{g})(6\times10^{23}\ \mathrm{atoms}/56\ \mathrm{g})\approx10^{23}\ \mathrm{atoms}$. That would yield a magnetic dipole moment of $(10^{23}\ \mathrm{atoms})(10^{-23}\ \mathrm{A\cdot m^2/atom})\approx 1\ \mathrm{A\cdot m^2}$, which is about what you'll find experimentally. However, orbital motion is not the entire explanation of the magnetic field from a permanent magnet!
  - The orbital motion of electrons is not sufficient to explain the magnetic dipole moment of a permanent magnet.
- **Electron Spin.** In addition to orbital motion, an object can also "spin" on its axis (as in what the Earth does about its axis), but it isn't *quite* the same. Unsatisfying as that is, I don't know anything much deeper (a little more, but not deeper) about electron spin.



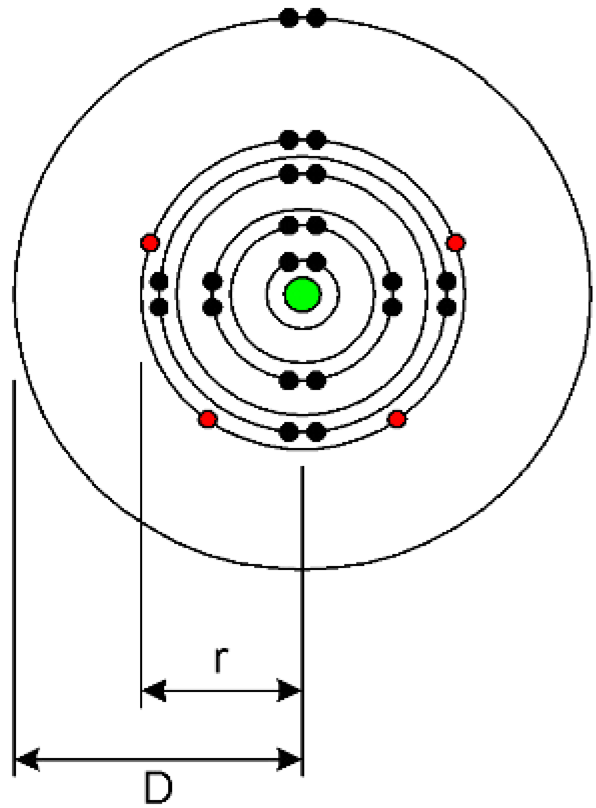
- This "spinning" charge produces a magnetic field because bits of charge are going in circles. The spin angular momentum of particles is quantized (i.e. it can only have certain values), :
  - $|S| = \frac{1}{2}\hbar = m_e v_{spin} r_{spin}$
    - Where $\hbar = h/2p = 1.05\times10^{-34}\ \mathrm{J\cdot s}$
  - Then again, the magnetic moment of a current loop is $m = IA = Ipr^2$, and the "current" of a single electron going around and around would be $I = e/T = e\dfrac{v}{2pr}$ where T is the period for one orbit.
  - So the magnetic moment is $m = evr$. Solving for vr above, $\frac{1}{2}\dfrac{\hbar}{m_e} = v_{spin} r_{spin}$, so
    - $m \approx \frac{1}{2}(e/m_e)\hbar = 1\times10^{-23}\ \mathrm{amp\ m^2}$,

- **Similarly for Protons and neutrons.** A similar argument holds for protons, and even for and neutrons (since they are made up of charged particles that circulate within it, but they have much larger masses so they have much smaller magnetic dipole moments.

Ferro Magnets – Why some and not others.

- I've suggested that magnetism has to do with electron spin. We'll *everything's* got electrons, so why isn't everything magnetic?
- **Spin Ballencing & Hund's Rule.** As you might know from chemistry, an atom has equal number of protons and electrons. Thanks to the quantization of angular momentum and the Pauli Exclusion Principle, the electrons occupy different orbitals. If you imagine starting with a naked nucleus, then one-by-one adding electrons, the first two go into the 1s (one of them spin "up" and the other spin "down") the next two go into the 2s, the next 6 into the 2p,…
    - **Full shell – no magnetic field.** Each full shell have equal numbers of electrons with spin oriented in one direction as oriented in the opposite, so the magnetic fields of the electrons cancel.
    - **Partly filled-shell – Hund's rule & agnetic field.** So, we have to look to the *partly-filled* shells for an explanation. In these, there are "unpaired" electrons, so there's a chance of net spin and magnetic field. Better yet, as you might recall from chemistry, as you fill up a shell, *first* you add in all the electrons you can with *the same* spin, and then those with opposite spin. This is called Hund's Rule. It has to do with the fact that electrons repel each other, and so they try to spread-out as much as they can within a given shell: if one has an orbit stretched along the x-axis, the next two want to have orbits stretched along the y and z axes. Being same-spin orientation actually helps to ensure that they stay spread out this way. In terms of magnetism, that means that, say a ½-filled p shell will have 3 electrons *all with the same spin – all contributing to a net magnetic field.*
    - **Covalent bonding.** So, all isolated atoms except for the Nobel gases, which have only full shells, would be magnetic. Unfortunately, atoms don't tend to stay isolated – they bond up. And the first thing they like to do, is covalently bond – if I'm an oxygen with two vacancies in my outer, 2p orbital, I'm thrilled to team up with two hydrogens and let their electrons time-share my two vacancies, while some of my electrons time-share their vacancies. In this way, orbitals are filled, electrons are paired, and most molecules and solids are not strongly magnetic.
- **Unfilled *inner* orbitals.** In general, the orbitals with the smaller average radii are lower energy, and so they fill first. However, electron orbitals aren't really discrete rings, like the plants follow about the sun, they are more cloudy distributions, and those distributions can even be kind of lumpy. It so happens that, while the 4s has much larger *average* radius, than does the 3d, it actually

extends well within the 3d, and ends up having *lower* energy and filling first. Here's what Iron looks like.



- Iron (Fe)
  Atomic Number 26
  Electron configuration $1s^2 2s^2 2p^6 3s^2 3p^6 3d^6 4s^2$
  (www.coolmagnetman.com/maghow.htm)

All of the black electrons are paired – if one's spin up, the other's spin down, so their magnetic fields cancel each other, but look at the four red electrons – they're *un paired.* In a chunk of iron, the 4s orbit is large enough that it dominates bonding. The bonding mechanism is called "metallic" bonding, by which the 4s electrons of all the iron atoms join into something of an electron sea; there's no room for covalent bonding in the 4s, but energy is still lowered by flowing together. Still, the 3d orbital isn't so small as to be oblivious to the neighboring atoms – they overlap a little too. All the neighboring 3d orbitals meld into something of a supper 3d orbital that connects the atoms. Now, here's the key – just as it is preferable for the electrons in an individual orbital to be aligned (until it gets too full), it is preferable for the electrons in this super-orbital to be aligned.

There we have it! It is preferable for the unpaired electrons in a chunk of iron to align with each other, thus their magnetic fields add up.

You might ask why Scandium, tin, Chromium, and Vanadium aren't magnetic too then – they're Iron's slightly lighter cousins. Well with fewer protons in the nucleus, the fewer electrons out in the 3d orbital are held less tightly, and so the orbital is larger and the electrons interact more with neighbors, leading to more covalent type bonding & thus pairing & cancelation.

You might ask why copper, zinc and gallium aren't magnetic. Well, with more protons in the nucleus, the d orbital is held *more* tightly and so the electrons in one atoms 3d don't interact strongly enough with those in the neighboring atoms to make a supper 3d and a strong preference for alignment.

Iron, cobalt, and manganese are just right.

In many materials, the net magnetic field due to orbital and spin magnetism is zero. In *ferromagnetic* materials (e.g. iron and nickel), the orbital and spin motions of some neighboring atoms line up to produce a magnetic field. Regions where this alignment occurs are called "magnetic domains."

Let's look at Iron, the poster child of ferromagnetism (and the origion of "ferro"). First, consider an individual iron atom:

This is just a schematic, don't take it too literally, but it does communicate some important information. The green dot at the center represents the nucleus containing all the neutrons and protons; its size is *greatly* exaggerated. The different rings represent the average radii of the different electronic levels. The dots (both black and red) represent the electrons occupying those levels.

Each level has a maximum allowed occupancy in accordance with the Pauli Exclusion Principle which says that no two electrons can be completely identical. What distinguishes electrons within the same atom are their spins, and the shapes/orientations & sizes of their orbitals – so, no two electrons can have the same orbits and spins. Naturally, the smaller radius orbits are generally more desirable, since they're closer to the nucleus and they have fewer repulsive electrons between them and the attractive nucleus. In the process of filling up a level, electrons take all compatible orientations and spins, to the effect that there is no *net* orbital or spinning motion – that is, however one electron may be orbiting, another has the opposite orbit. You'll notice that most of the dots are paired up, that represents there being two electrons with the same orbits but opposite spins. No net motion means no net magnetic field. Because it soon becomes important, I should note that electrons in the same level and the same orbit (but with opposite spin) are as close together for as much of the time as possible, and so they feel each other's electric repulsion more than do two electrons in the same level but different orbits. For this reason, it is preferable to be in *different* orbits, and this is ensured if electrons have the *same* spin – thus Hund's rule that a level is filled by first putting one electron in each possible orbital orientation, and with the same spin orientation, and only after that is done do opposite spin electrons get added into the different orientations.

Now look at the two outer most shells of the iron atom. The last shell is the 4s – 4 loosely refers its average radius and s refers to the orbit being spherically symmetric – with no net angular momentum. Only two electrons can occupy an s – spin up and spin down. The second to last shell is the 3d (d referrers to the magnitude of angular momentum, four different orientations are allowed with this particular magnitude; with two spins per orientation – the shell can hold 8 electrons). You'll notice two peculiar things, both of which are important for making iron Ferromagnetic.

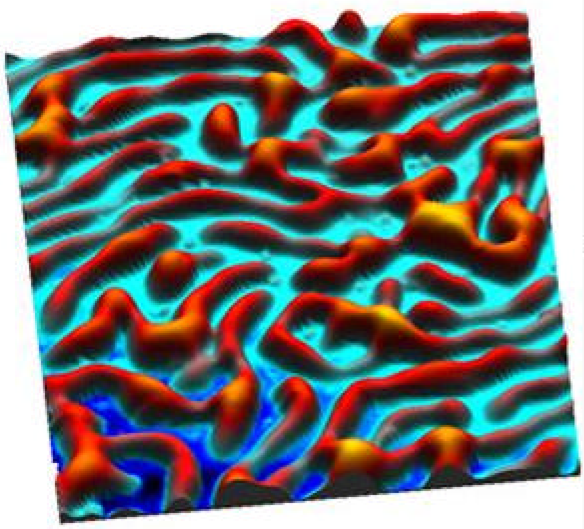
First, the 3d is only partially filled; four of its electrons are *unpaired.* By Hund's rule, their spins are all aligned, so their contributions to the magnetic field add up. Yippee, an individual Fe atom produces a magnetic field! Oh, but so would any other atom (except for noble gasses), so maybe that's not such a big deal in and of itself. What is important is that, somehow, when iron atoms bond up to form a solid they a) retain this unparedness and b) the unpaired electrons not only align with each other within the same atom, but also with each other in different atoms.

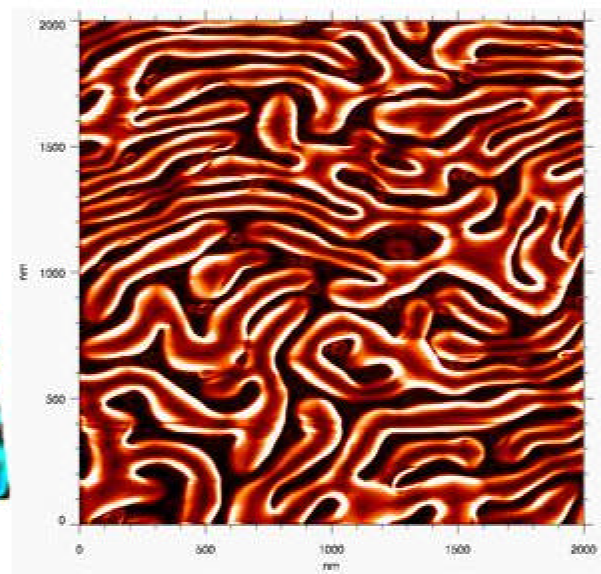
This relies on the other peculiar thing about iron atoms. Though the 4s orbit is on *average* larger than the 3d, the 4s is full and the 3d is not. This is because, while the 4s is *on average* larger than the 3d, that's just the average, and the radial distribution of the 3d orbital actually has some significant peaks well *inside* the 3d – giving it better visibility of and attraction to the nucleus than the 3d. Thus the 4s is actually the lower energy level, and so it gets occupied first. So, even though the iron atom has unpaired electrons, they *aren't* in the outer orbital, the electrons out there are paired, and it is the electrons in the *outer* orbital that dominate inter-atomic bonding. In truth, both 3d and 4s electrons do participate in the bonding, but since the 4s is *so* much larger than the 3d, it really dominates. Now, for *most* atoms, but not iron, it's the outer level that is only partially filled, which facilitates covalent bonding – that is, they essentially fill their vacancies by time-sharing outer electrons with their neighbors. Since these vacancies have the opposite spin orientations of the already occupied states, the bonding electrons are encouraged to anti-align, and thus there is no net spin, and no net field. The other extreme would be noble gases which have all their occupied shells full – their interactions are weak enough that they don't bond except at *very* low temperatures. However, iron has a full outer level *and* vacancies in its next level. The result is metallic bonding – that's where the electrons join up in an unlocalized electron sea (rather than pairing up in localized covalent bonds.) They *want* to do this because they get more room in the sea than when stuck to their own atoms, this spells a lowering of their energies (relative to being localized on their atoms). It's crucial to note that, the electron that occupies one of these extended 'sea' states, that stretches from atom to atom has got a particular spin and, locally, is still subject to Hund's rule – it's advantageous for it to have the same spin orientation as the other unpaired electrons in the sea!

Note: If the 3d level were less full than it is for iron, it would be larger (fewer protons in the nucleus attracting its electrons) and so 3d's unpaired electrons would play more prominent roles in the bonding, probably making the bonds more covalent like, and thus encouraging anti-alignment between bonding electrons. This is probably why K, Ca, Sc, Ti, and V aren't famous ferromagnets. Conversely, if it were more filled, there'd be less unpaired electrons, so less field, and the 3d would be smaller, so weaker interaction between the electrons in one atom's 3d and those in another's.

Hook and Hall's Solid State Physics has a useful discussion of Ferromagnetism in chapter 8. It discusses the origin of the "exchange" interaction, the existence of a conduction band, and suggests that the exchange interaction still holds for the electrons in that band. Fig. 8.4 gives the density of states for iron's 4s and 3d in the solid. Some books online show more detailed densities of states, but the same idea holds – more states aligned than unaligned below the Fermi energy.

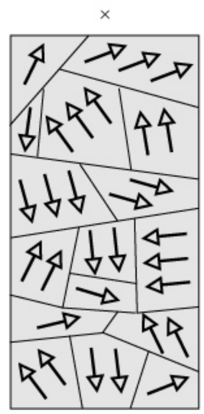
- **Domains.**
  - So, there's a preference to align, but there's not necessarily anything dictating *which way* everyone should align. So you end up with regions in which most of the atoms are aligned one way, and regions in which they're aligned another way. Here are two Magnetic Force microscopy images of domains on a Co/Pt.





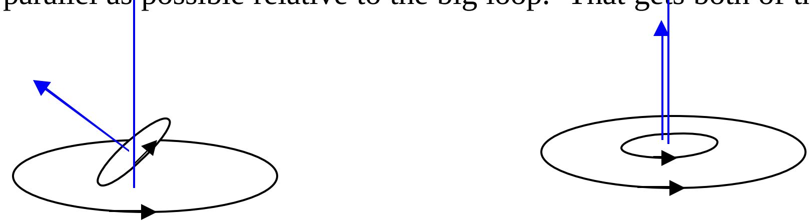
(http://www.omicron.de/index2.html?/rom/mfm_image_taken_at_a_co_pt_multilayer_surface/index.html~Omicron)

- The different apparent heights reflect how high the MFM's tip (cantilever with a microscopic magnet on the end) was pushed up or drawn down by the magnetic field of the surface – thus magnetic domains anti aligned with the tip push the tip away / are shown as red/yellow and high while domains aligned with the tip draw it down / are shown as blue/black and low.
- More schematically, we will represent this with several magnetic dipole moments (arrows) in the same direction. If there is no external magnetic field, the domains of iron are in random directions as shown below, so it produces no net magnetic field.



Multiplying Effect

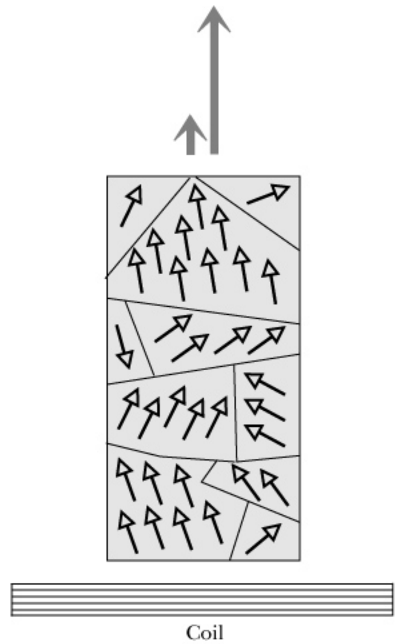
When an un-polarized chunk of ferromagnetic material is subjected to an external field, it induces polarization (ether whole-sale flipping unaligned domains, or more subtly expanding aligned domains.) we've yet to see how this works, since we've not really dealt with the magnetic force yet, but if you recall the demo I did the first day of this chapter, two parallel currents attract each other, so, say you've got a big, stationary current loop and a small current loop that's free to rotate, you can see that the one that can rotate would rotate to get its current as parallel as possible relative to the big loop. That gets both of their fields pointing the same way.



Demo: need a big current loop and a small rotating one (perhaps we have one in our magnet-wire pendulum kit.

Now, if we think of each magnetic domain as a current loop, we can see that it will rotate to align with the external current. In terms of field, that means that its aligns so that its field is parallel to the externally applied one. So, an externally applied field reorients the microscopic current loops (that already exist) in a Ferromagnet so as to increase the net field.

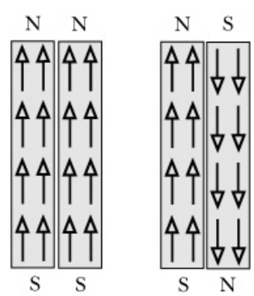
If an external magnetic field is applied (e.g. with a coil in the figure below), the domains will somewhat in the same direction. This will produce a magnetic field in the same direction as the external magnetic field (the "magnetic multiplier effect" of Exp. 17.11).



When the external field is removed, the domains mostly return to the disordered state for some materials (e.g. iron), but remain more aligned in other materials (e.g. Alnico).

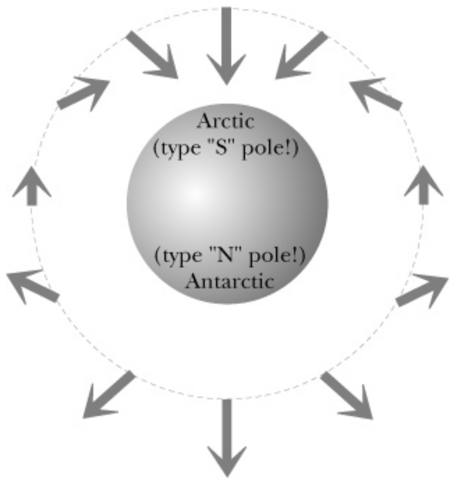
The atoms interact strongly at short range to cause neighboring atomic magnetic dipole moments line up with each other. This is the source of the magnetic domains.

Why don't all of the magnetic domains oriented in the same direction to form a single domain? That would be unstable. You can model the situation with two bar magnets. A magnet (or domain) will have tendency to line up with the field of a magnet next to it. If the magnets start side-by-side as shown below on the left, one will flip. Of course, the magnets will line up if they are end-to-end.

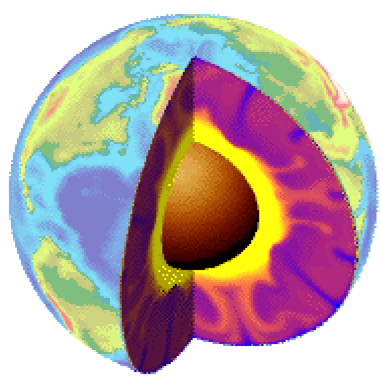


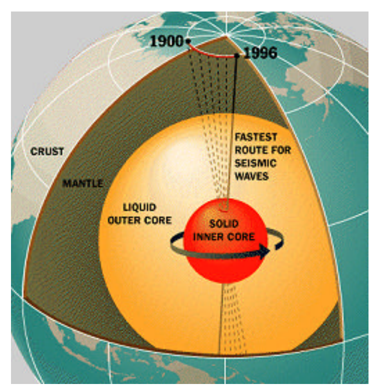
The interaction which causes the magnetic dipole moments to flip is a weaker, but has a longer range.

- **The Earth's Magnetic Field**
- **Currents Cause Magnetic fields.** Both with macroscopic currents in wires and microscopic ones in atoms of magnets, we can see that they are the sources of magnetic fields. What's the other big thing that has a field? The Earth.
    - o The Earth field is that of a dipole as shown below. Unfortunately, the magnetic S pole is in the Artic at the geographic north pole. The N pole of a compass needle is attracted to the S pole in the artic.



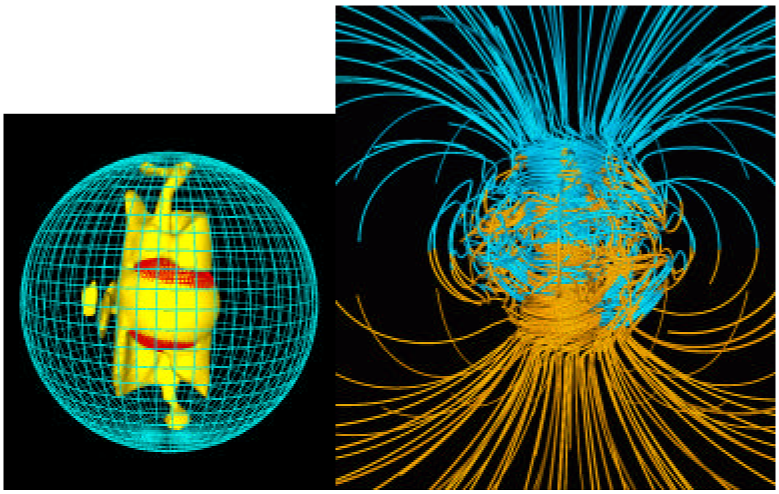
- **Q:** So, where's its current?
- Let's look inside



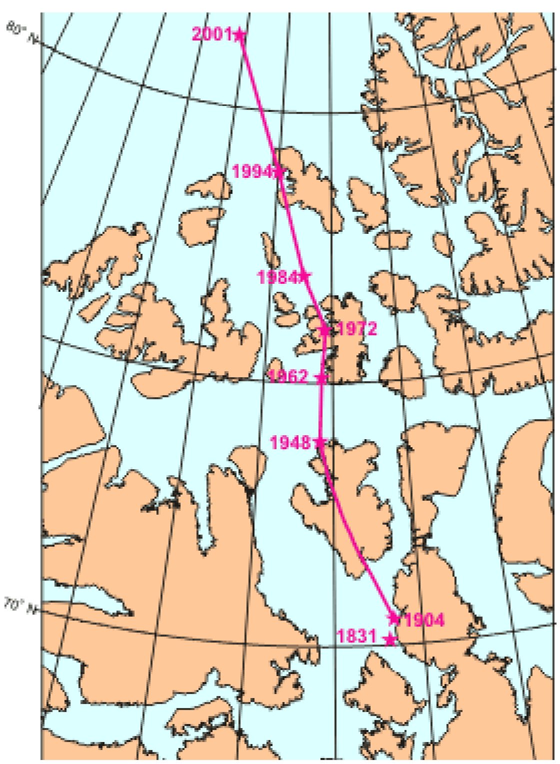


- (http://science.nasa.gov/headlines/y2003/29dec_magneticfield.htm)
- **Iron-rich liquid outer core.** The planet's liquid outer core is rich in iron.
- **Need electrons and protons to move separately.** Smooth rotation of neutral metal liquid would not constitute a current any more than would spinning a metal ring. Instead, we need differently charged particles to be moving in different directions.
- **How electrons move separately.** Here's how that arises.
    - o **Ionizing temperatures.** First, it's quite hot in the liquid outer core – the inner core is about as hot as the surface of the sun – that's hot enough to ionize atoms, and thus allow positive ions and negative electrons to circulate seperately.

- **Turbulance.** Second there's a lot of turbulence, which locally gives some circulation as well as additional heating.
  - **Differential rotation.** This arises because the solid inner core and the planet surface rotate at different rates
  - **Convection.** Because heat generated with in the core (inner and outer, due to friction as well as radioactive decay) convects out toward the surface - drawing hot materials up (and cooling in the process). So we end up with quite a seething mess, not too unlike our atmospheric weather. It is these 'hurricanes' and such that generate the strongest magnetic fields.

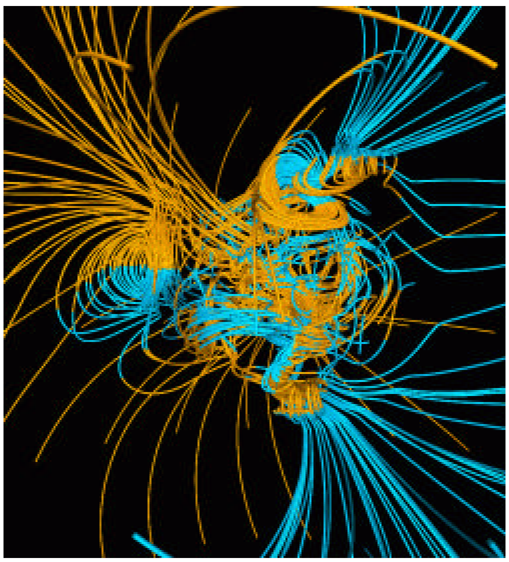
- Here's a simulation highlighting the regions where the circulation would be expected to be greatest and the resulting magnetic field
- 
- http://www.es.ucsc.edu/%7Eglatz/geodynamo.html

- But since this depends on something chaotic like storms, maybe it's not completely surprising that its unstable. Not only can it migrate



- (http://science.nasa.gov/headlines/y2003/29dec_magneticfield.htm)

- But it can migrate all the way upside down. In the process, the Earth's magnetic field can get pretty gnarly.

- 

- The book points our that the earth's field isn't just parallel to the Earth's surface, it's that of a dipole, so it has vertical as well as horizontal components.

- **Demo:** our 3-D compass