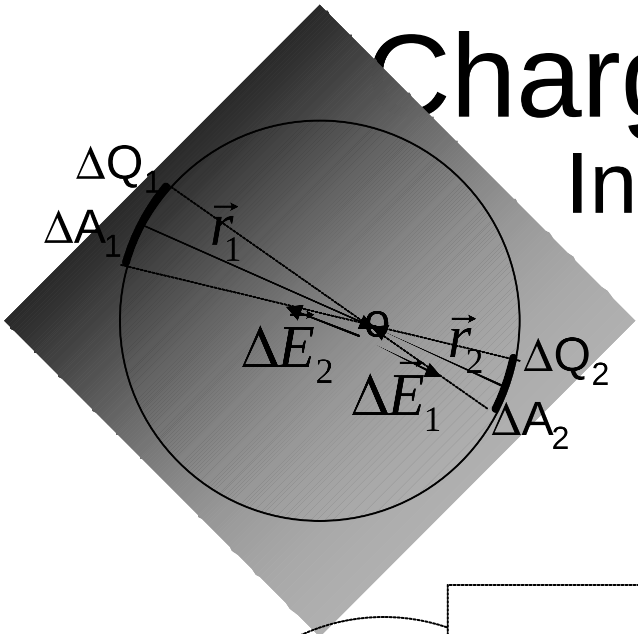
Electric Field of a Uniformly Charged Spherical Shell

Inside



$$|\Delta E_1| = \frac{1}{4pe_o}\left|\frac{\Delta Q_1}{r_1^{\,2}}\right|$$

where

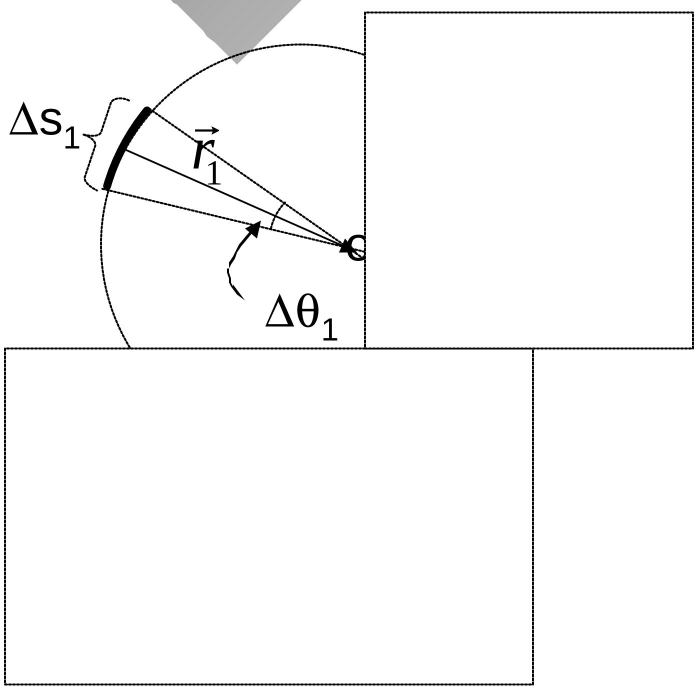
$$\frac{\Delta Q_1}{Q} = \frac{\Delta A_1}{A} \Rightarrow \Delta Q_1 = Q\,\frac{\Delta A_1}{A}$$

where

$$\Delta A_1 = p\left({}^{s_1}\!/_{2}\right)^2$$

where

$$s = r_1 \Delta q_1$$

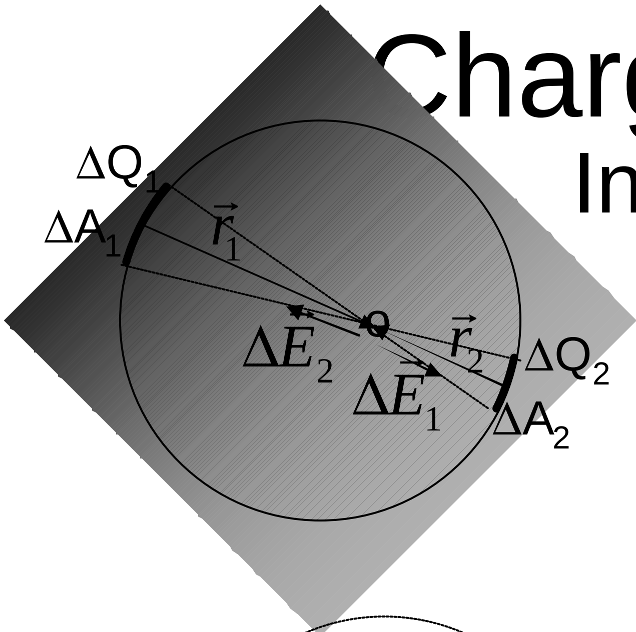


so

$$|\Delta E_1| = \frac{1}{4pe_o}\left|\frac{\left(Q\,\frac{p\left({}^{r_1\Delta q_1}\!/_{2}\right)^2}{A}\right)}{r_1^{\,2}}\right| = \frac{1}{4pe_o}\left|\frac{Qp\left(\Delta q_1\right)^2}{4A}\right|$$

Electric Field of a Uniformly Charged Spherical Shell

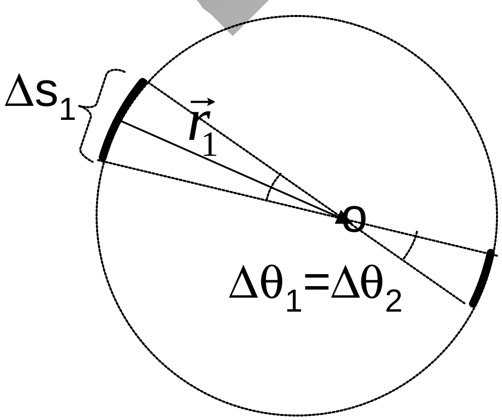
Inside $E_{shell} = 0$



$$\left|\Delta E_1\right| = \frac{1}{4pe_o}\left|\frac{Qp\left(\Delta q_1\right)^2}{4A}\right|$$

Ditto for ΔE_2

$$\left|\Delta E_2\right| = \frac{1}{4pe_o}\left|\frac{Qp\left(\Delta q_2\right)^2}{4A}\right|$$



but

$$\Delta q_1 = \Delta q_2$$

so

$$\left|\Delta E_2\right| = \frac{1}{4pe_o}\left|\frac{Qp\left(\Delta q_1\right)^2}{4A}\right| = \left|\Delta E_1\right|$$

Thus, the two are not just opposite direction, but also equal magnitude, so they cancel. This is true for ALL pairs of patches of the surface – they ALL CANCEL.

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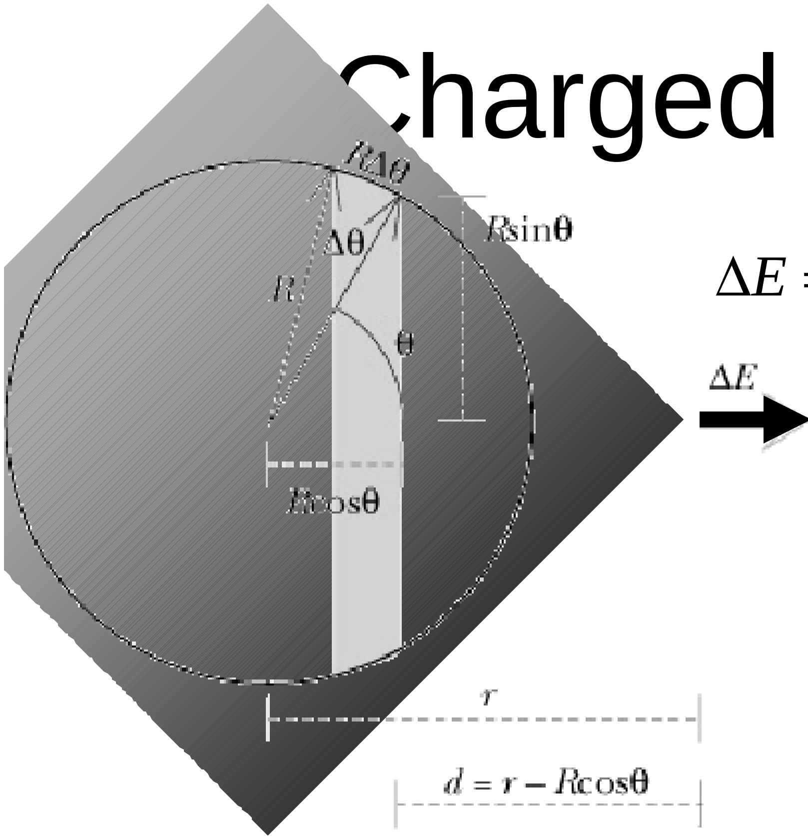

$$\Delta E = \frac{1}{4\pi\varepsilon_0}\frac{\Delta Q d}{\left[(R\sin\theta)^2 + d^2\right]^{3/2}}$$ For a ring

where

$$\Delta Q = Q\frac{(\text{area of ring})}{(\text{area of sphere})} = Q\frac{2\pi R^2 \sin\theta\Delta\theta}{4\pi R^2}$$

$$\Delta Q = \frac{Q\sin\theta}{2}\Delta\theta$$

and

$$d = r - R\cos\theta$$

Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

so

$$\Delta E = \frac{1}{4\pi\varepsilon_0}\frac{(r - R\cos\theta)}{\left[(R\sin\theta)^2 + (r - R\cos\theta)^2\right]^{3/2}}\frac{Q\sin\theta}{2}\Delta\theta$$

Step 2: write an expression for ΔE

$$\Delta E = \frac{1}{4\pi\varepsilon_0}\frac{(r - R\cos\theta)}{\left[R^2 + r^2 - 2Rr\cos\theta\right]^{3/2}}\frac{Q\sin\theta}{2}\Delta\theta$$

Step 3: Add up all ΔE's to get the total E

Step 4: Check results

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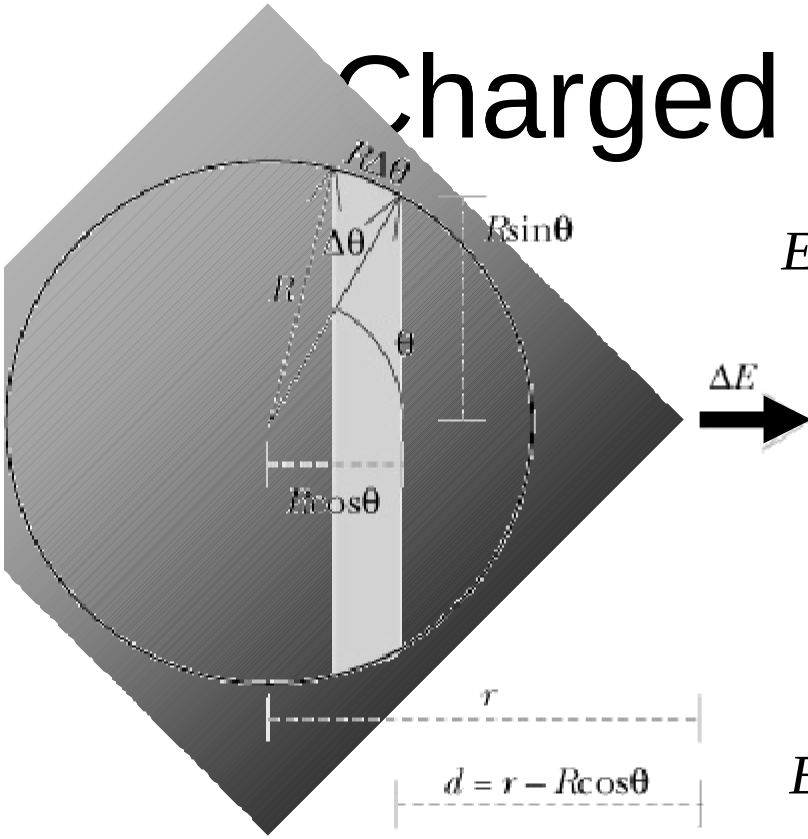

$$E = \sum_{sphere} \Delta E$$

where

$$\Delta E = \frac{1}{4\pi\varepsilon_0} \frac{(r - R\cos\theta)}{\left[R^2 + r^2 - 2Rr\cos\theta\right]^{3/2}} \frac{Q\sin\theta}{2} \Delta\theta$$

so

$$E = \sum_{\theta=0}^{\theta=\pi} \frac{1}{4\pi\varepsilon_0} \frac{(r - R\cos\theta)}{\left[R^2 + r^2 - 2Rr\cos\theta\right]^{3/2}} \frac{Q\sin\theta}{2} \Delta\theta$$

so

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{2} \int_0^{\pi} \frac{(r - R\cos\theta)}{\left[R^2 + r^2 - 2Rr\cos\theta\right]^{3/2}} \sin\theta \, d\theta$$

Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

Step 2: write an expression for ΔE

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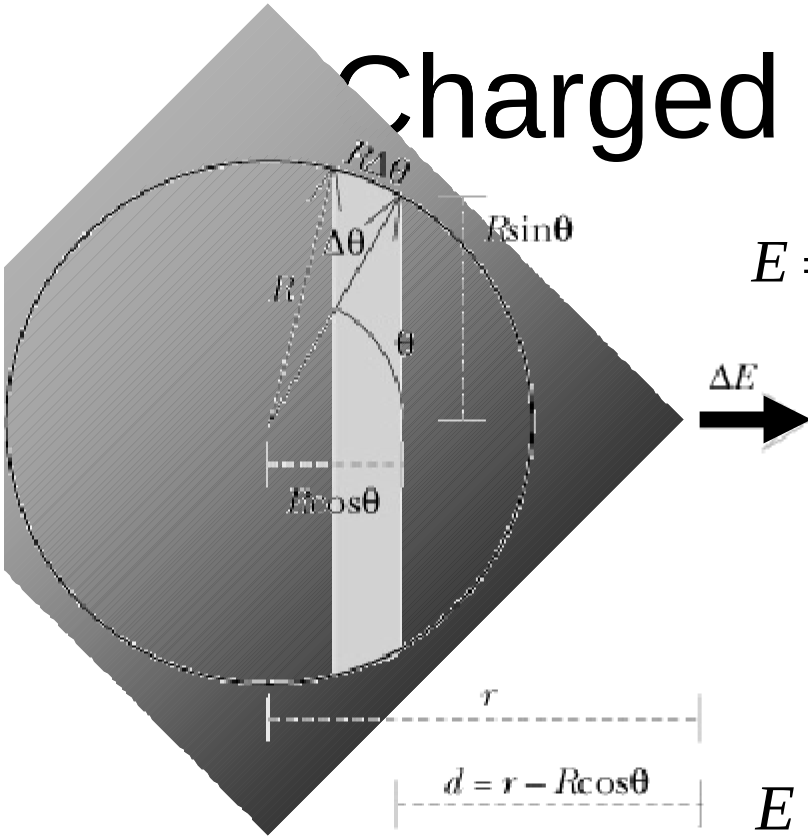

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2}\int_0^{\pi}\frac{(r - R\cos\theta)}{\left[R^2 + r^2 - 2Rr\cos\theta\right]^{3/2}}\sin\theta\, d\theta$$

Change of variables

$$u \equiv \cos\theta \qquad du/d\theta = -\sin\theta$$

$$\theta = 0 \rightarrow u = 1 \qquad \theta = \pi \rightarrow u = -1$$

so

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2}\int_1^{-1}\frac{(r - Ru)}{\left[R^2 + r^2 - 2Rru\right]^{3/2}}\, du$$

Step 1: cut up charge distribution and *draw* it's contribution to the field: ΔE

so

Step 2: write an expression for ΔE

Step 3: Add up all ΔE's to get the total E

Step 4: Check results

Note:

$$(r - Ru) = \frac{(R^2 + r^2 - 2Rru) - (R^2 - r^2)}{2r}$$

Can thus simplify integrand