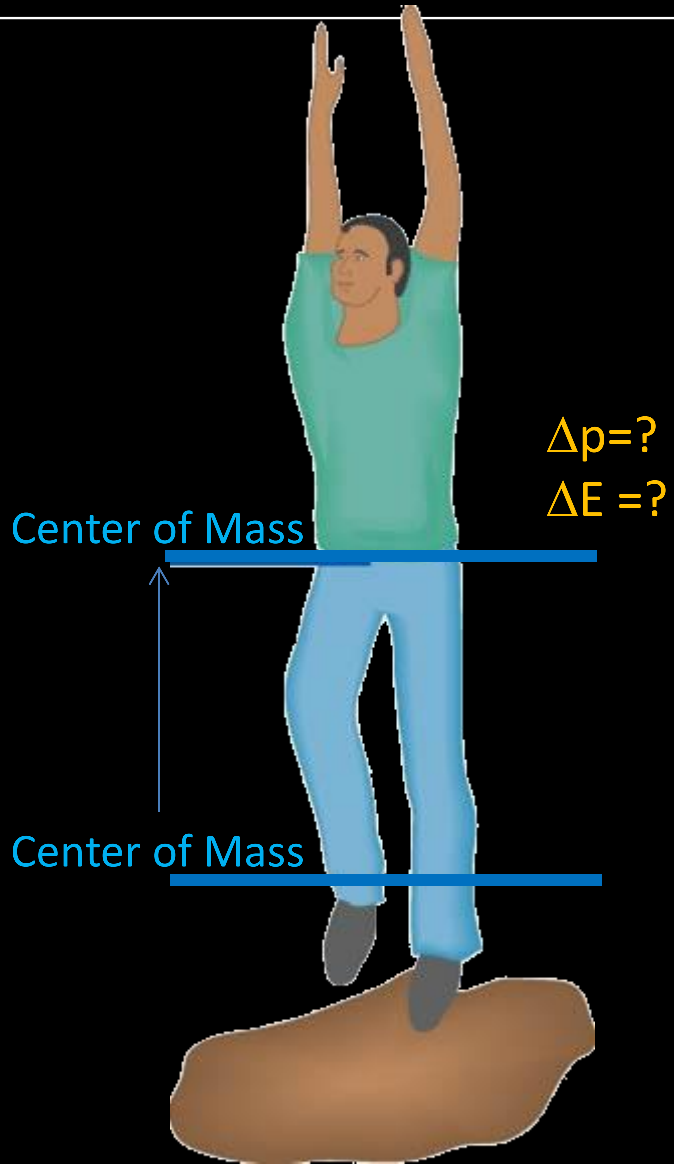
Wed.	9.3 Rotational Energy Quiz 8	RE 9.b
Lab	L8 Energy Quantization Review Exam 2 (Ch 5-8)	Practice Exam 2 (bring to lab)
Fri.,	Exam 2 (Ch 5-8)	
Mon.	9.4-.5 (.9) The "Point Particle" approximation	RE 9.c
Tues.		EP8, HW9: Ch 9 Pr's 34, 40, 43



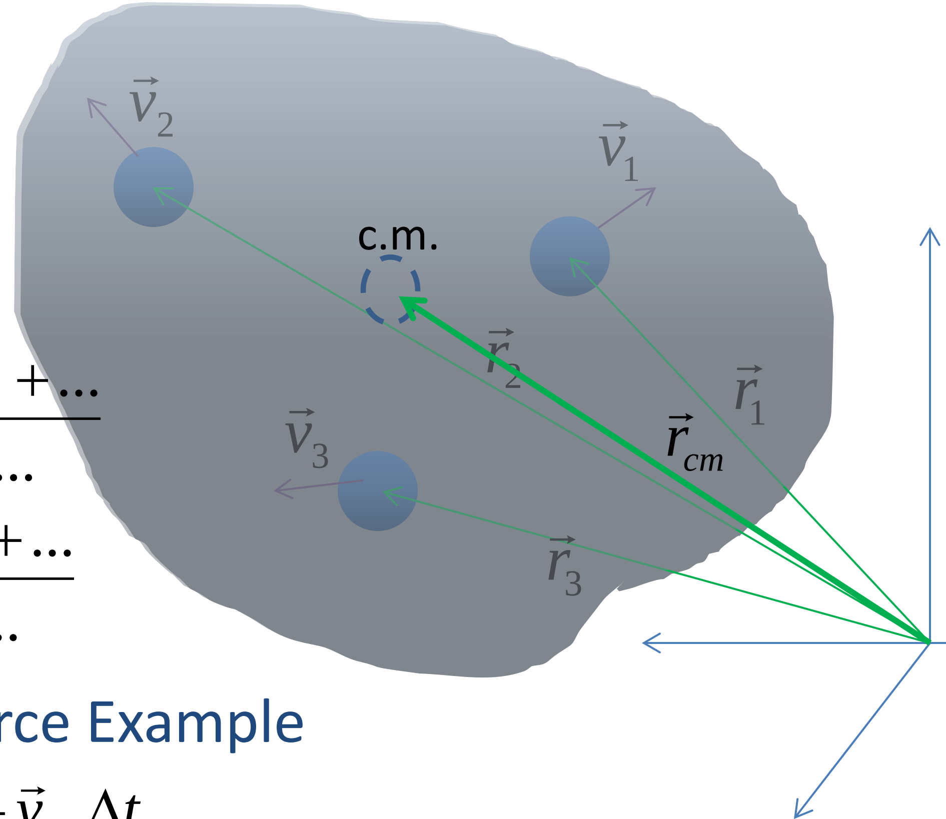
Multi-Particle System's Momentum and Center of Mass

$$\frac{d\vec{p}_{system}}{dt} = \vec{F}_{net.ext}$$

$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots \equiv \vec{p}_{system} \approx m_{system}\vec{v}_{cm}$$

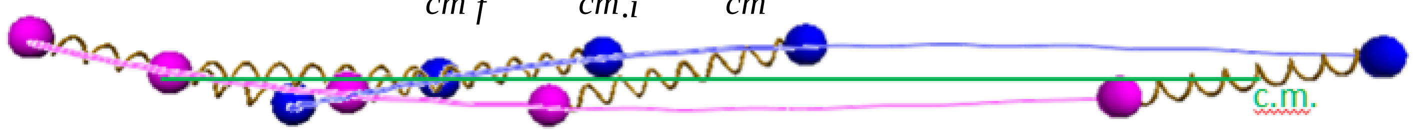
$$\vec{v}_{cm} \approx \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$



No External Force Example

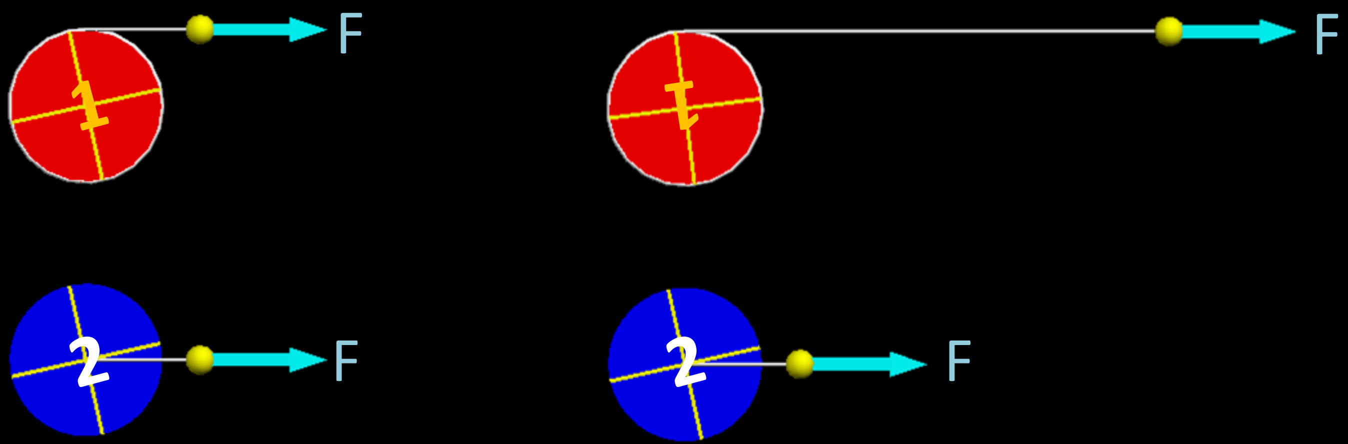
$$\vec{r}_{cm\,f} = \vec{r}_{cm.i} + \vec{v}_{cm}\Delta t$$

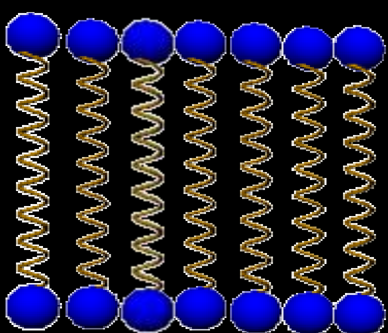


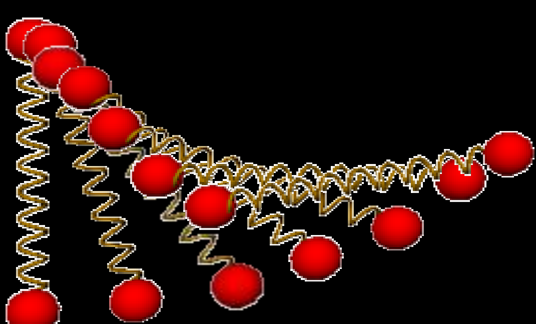
Constant External Force Example

$$\vec{r}_{cm\,f} = \vec{r}_{cm.i} + \vec{v}_{cm.i}\Delta t + \tfrac{1}{2}\left(\frac{\vec{F}_{net}}{m_{system}}\right)(\Delta t)^2$$

Keeping track of motion of and *within* a system







Momentum principle speaks only for *center of mass* motion

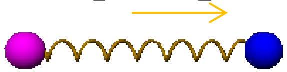
Sometimes you want to track motion *relative to* center of mass

Need a different tool: **Energy**

Multi-Particle System's Energy

Splitting up Kinetic

$$K_{total} = K_1 + K_2 + K_3 + \ldots$$

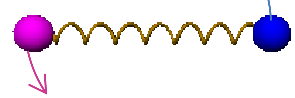


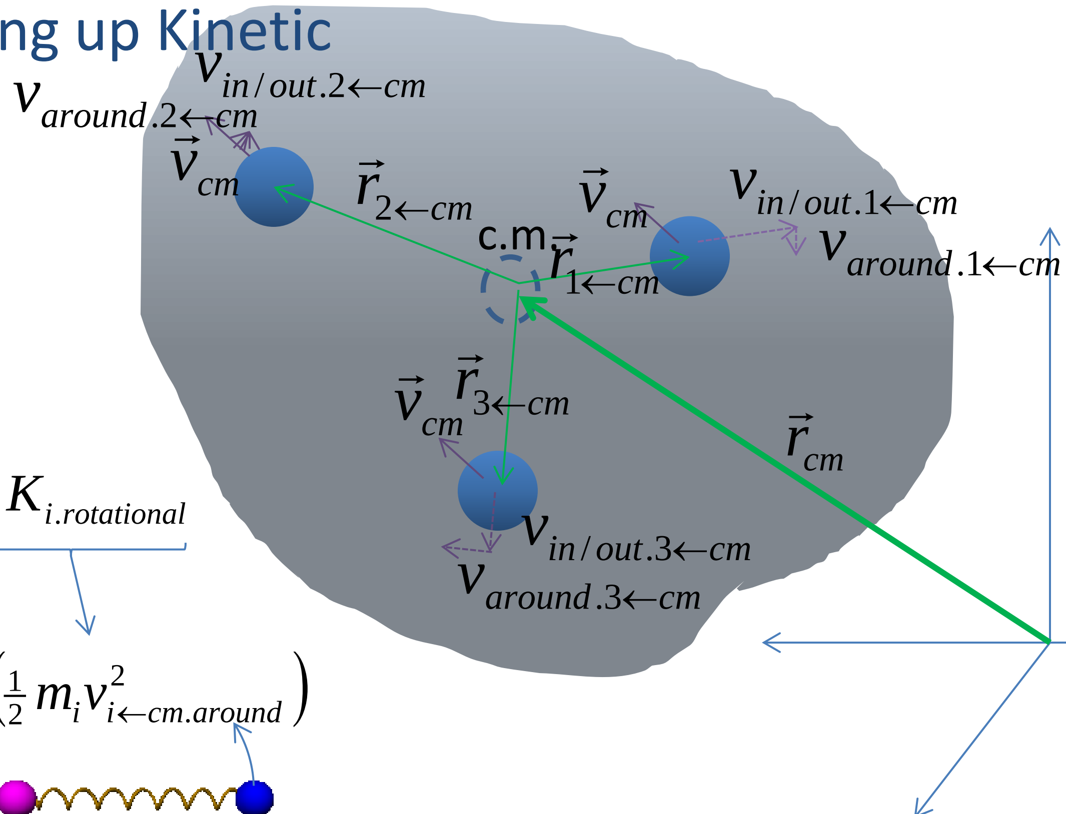
$$K_{total} \approx \tfrac{1}{2} m_{total} v_{cm}^2 + \sum_i \left(\tfrac{1}{2} m_i v_{i \leftarrow cm}^2\right)$$

$$K_{total} \approx K_{cm} + \sum_i K_{i.rel}$$

$$\sum_i K_{i.rel} = \sum_i K_{i.vibrational} + \sum_i K_{i.rotational}$$

$$\sum_i \left(\tfrac{1}{2} m_i v_{i \leftarrow cm.in/out}^2\right) + \sum_i \left(\tfrac{1}{2} m_i v_{i \leftarrow cm.around}^2\right)$$

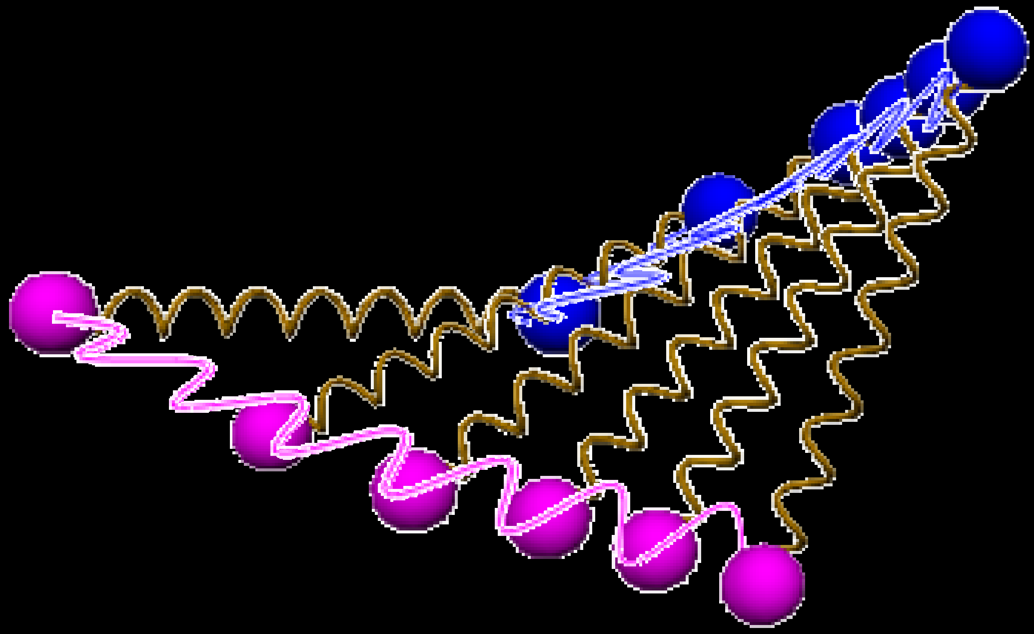




With interactions between parts of system, there's also potential energy

$$E_{system} = K_{cm} + \left(\sum_{all.parts} \left(K_{rel} + mc^2\right) + \sum_{all.pairs} U \right)$$

In the case shown in the VPython program, what energy terms are nonzero for the system of the two balls and the spring?	$(K_{vib} + U_{spring})$ and K_{rot} and K_{trans}



09_RotateVibrateTranslate.py

Multi-Particle System's Energy

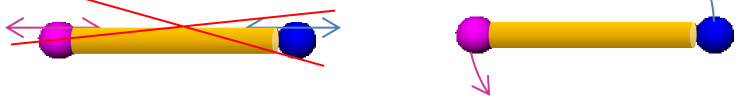
Splitting up Kinetic

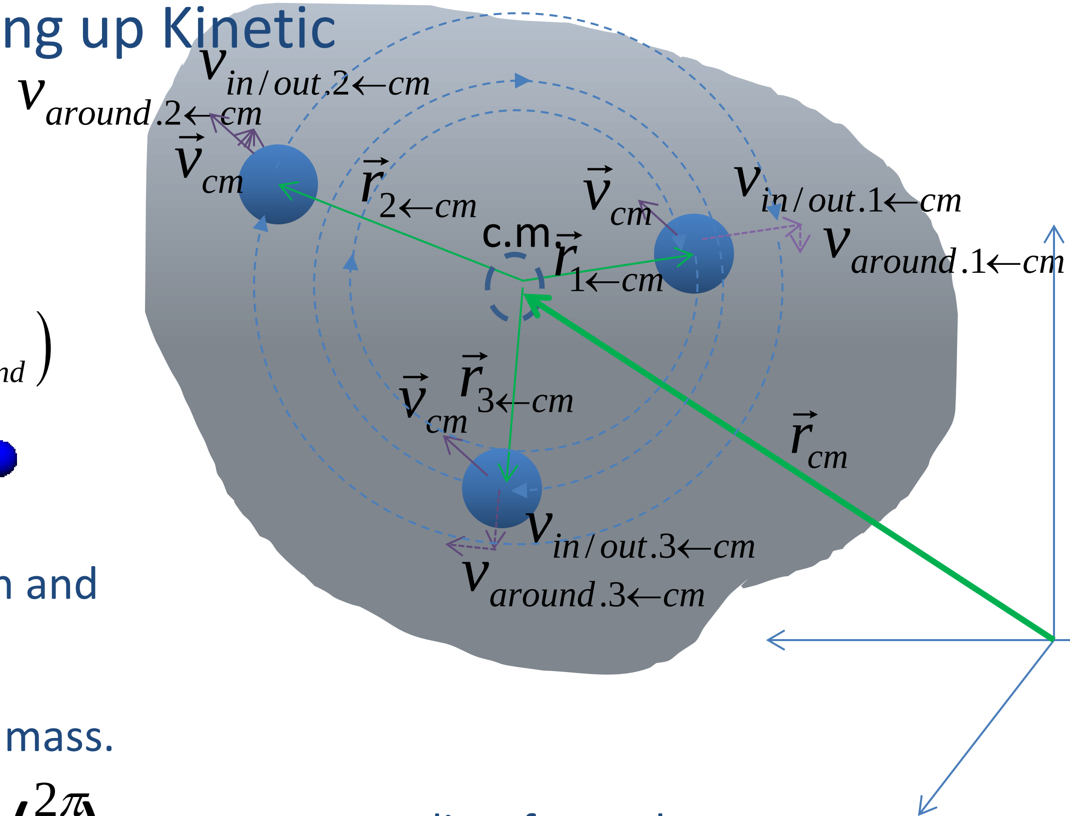
Focus on Rotational: *rigid object*

$$\sum_i K_{i.rel} = \sum_i K_{i.vibrational} + \sum_i K_{i.rotational}$$

0

$$\sum_i \left(\tfrac{1}{2} m_i v^2_{i\leftarrow cm.in/out}\right) + \sum_i \left(\tfrac{1}{2} m_i v^2_{i\leftarrow cm.around}\right)$$





For a **Rigid object**, particles don't move in and out from center of mass, just around

Each morsel of mass circles the center of mass.

$$v_{around.1\leftarrow cm} = \frac{\text{circumferance}_1}{\text{period}} = \frac{2\pi r_{1\leftarrow cm}}{T} = \left(\frac{2\pi}{T}\right) r_{1\leftarrow cm} = \omega r_{1\leftarrow cm}$$

ditto for each mass

$$v_{around.2\leftarrow cm} = \omega r_{2\leftarrow cm}$$

$$v_{around.3\leftarrow cm} = \omega r_{3\leftarrow cm}$$

etc.

Multi-Particle System's Energy

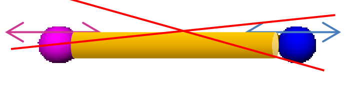
Splitting up Kinetic

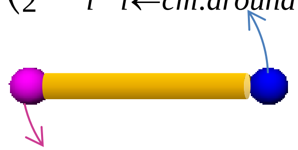
Focus on Rotational

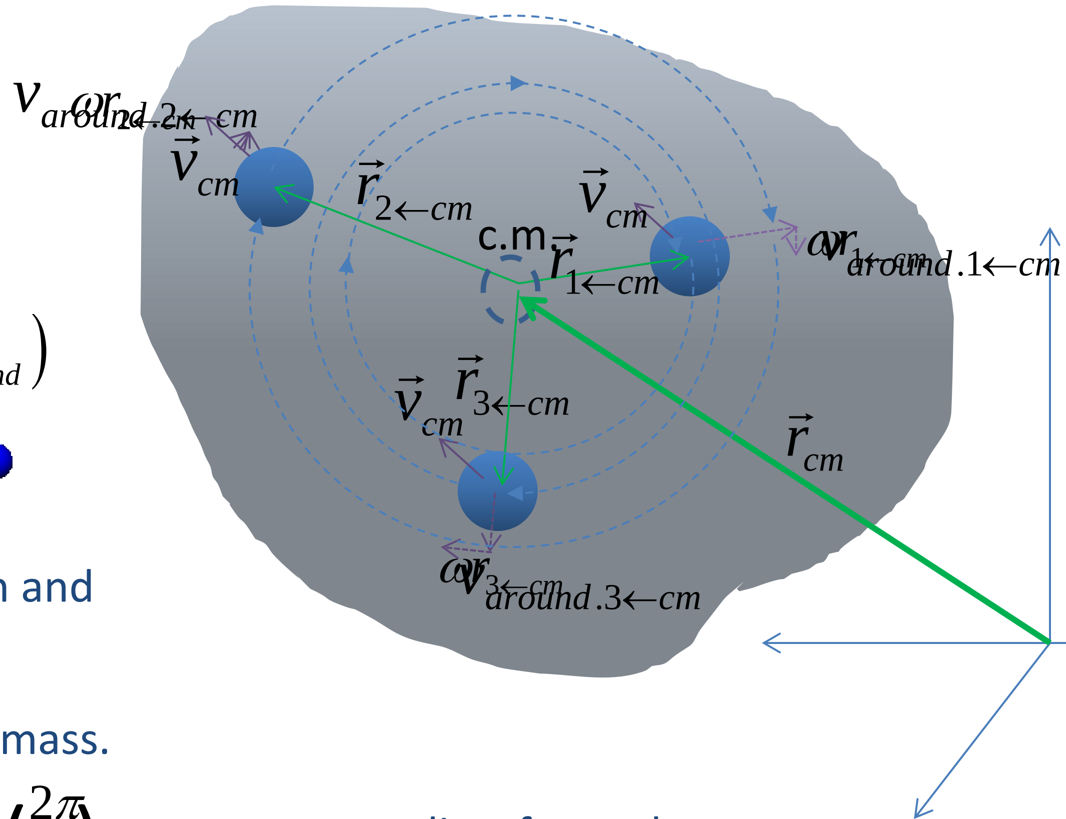
$$\sum_i K_{i.rel} = \sum_i K_{i.vibrational} + \sum_i K_{i.rotational}$$

0

$$\sum_i \left(\tfrac{1}{2} m_i v^2_{i\leftarrow cm.in/out}\right) + \sum_i \left(\tfrac{1}{2} m_i v^2_{i\leftarrow cm.around}\right)$$







For a **Rigid object**, particles don't move in and out from center of mass, just around

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ditto for each mass

$$v_{around.2\leftarrow cm} = \omega r_{2\leftarrow cm}$$

$$v_{around.3\leftarrow cm} = \omega r_{3\leftarrow cm}$$

etc.

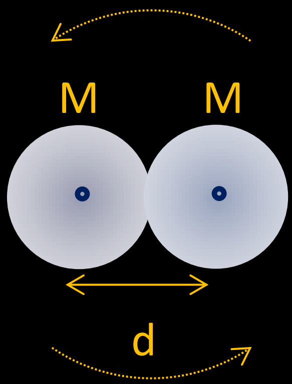
$$K_{rotational} = \sum_i K_{i.rotational} = \sum_i \left(\tfrac{1}{2} m_i v^2_{i\leftarrow cm.around}\right) = \sum_i \left(\tfrac{1}{2} m_i \left(\omega r_{i\leftarrow cm}\right)^2\right) = \tfrac{1}{2}\left(\sum_i m_i r^2_{i\leftarrow cm}\right)\omega^2 = \tfrac{1}{2} I \omega^2$$

$I_{about\ cm}$

Moment of Inertia

$$I_{about.cm} = \sum_i m_i r_{i \leftarrow cm}^2$$

A diatomic molecule such as molecular nitrogen (N_2) consists of two atoms each of mass M, whose nuclei are a distance d apart. What is the moment of inertia of the molecule about its center of mass?



1) Md^2

2) $2Md^2$

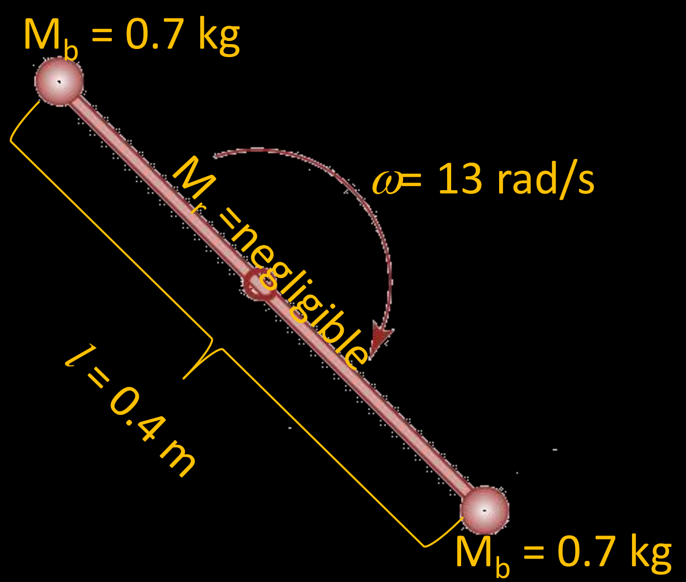
3) $\frac{1}{2}Md^2$

4) $\frac{1}{4}Md^2$

5) $4Md^2$

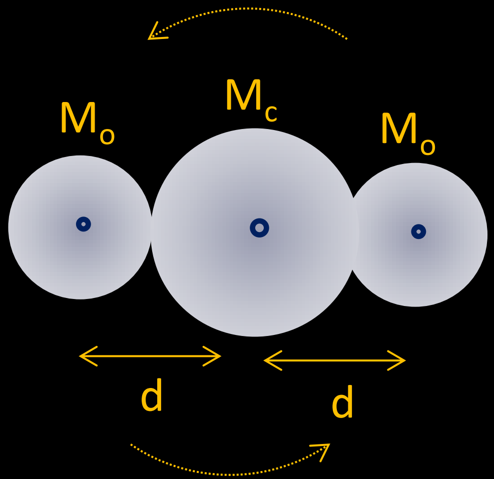
$$I_{aboutcm} = \sum_i m_i r_{i\leftarrow cm}^2$$

Two balls of mass 0.7 kg are connected by a low mass rigid rod of length 0.4 m. The object rotates around a pivot at its center, with angular speed 13 radians/s. What is the rotational kinetic energy of this object?	a) 484 J b) 4.73 J c) 2.37 J d) 0.056 J e) 0 J



$$I_{about\,cm} = \sum_i m_i r_{i\leftarrow cm}^2$$

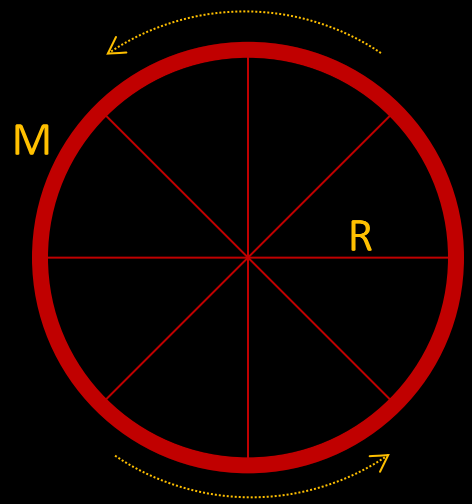
Example: A linear tri-atomic molecule such as carbon dioxide (CO_2) What is the moment of inertia of the molecule about its center of mass?



$$I_{about\,cm} = \sum_i m_i r_{i\leftarrow cm}^2$$

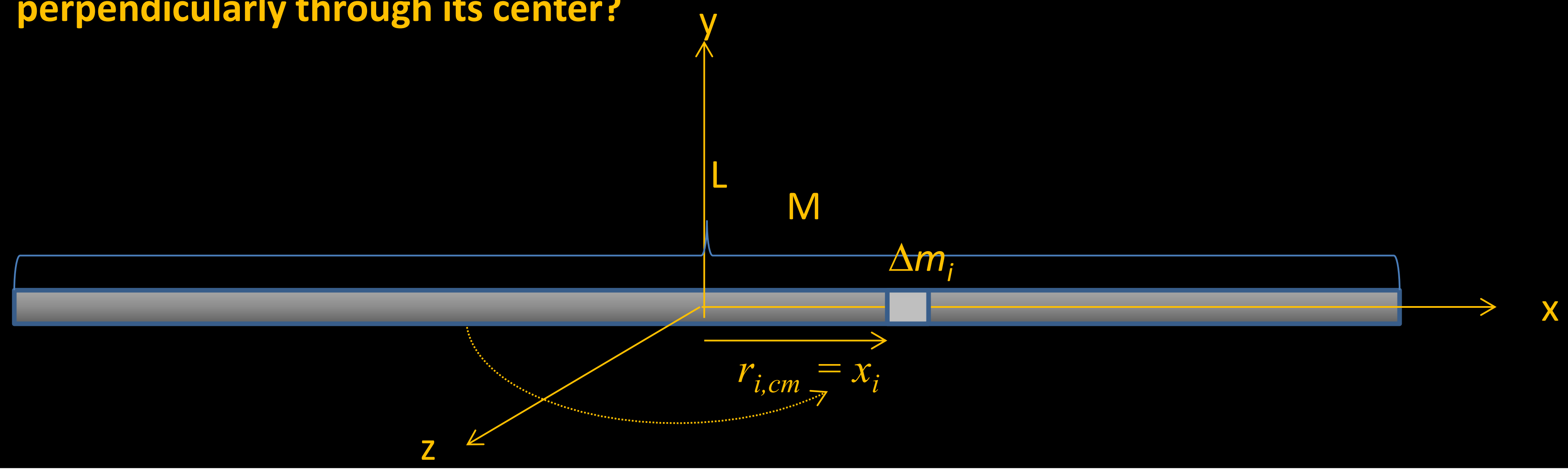
The spokes of a bicycle wheel have low mass, so almost all of the mass of the wheel is concentrated in the rim. What is the moment of inertia of a bicycle wheel of radius R and mass M?

1) MR^2
2) $2\pi MR^2$
3) $2\pi RM$
4) $(1/2)MR^2$
5) πMR^2



$$I_{aboutcm} = \sum_i m_i r_{i\leftarrow cm}^2$$

Example: Thin, uniform rod, length L, mass M; what is its moment of inertia about an axis perpendicularly through its center?



$$I_{cm} = \sum_i m_i r_{i\leftarrow cm}^2$$

Think of in limit of very small morsels of mass Δm and visualize a representative one

$$I_{cm} = \sum_i (\Delta m_i) r_{i\leftarrow cm}^2$$

Rephrase mass and position in terms of coordinates

$$\frac{\Delta m}{\Delta x} = \frac{M}{L} \Rightarrow \Delta m = \left(\tfrac{M}{L}\right)\Delta x$$

$$I_{cm} = \sum_i \left(\Delta x\left(\tfrac{M}{L}\right)\right) x_{i\leftarrow cm}^2$$

Send to differential limit

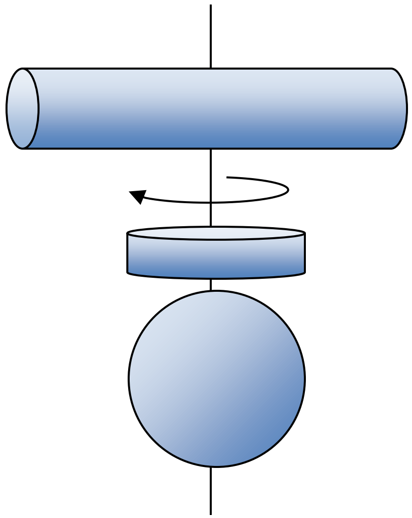
$$I_{cm} = \left(\tfrac{M}{L}\right) \int_{x_{\min}=-L/2}^{x_{\max}=L/2} x^2 dx$$

Integrate

$$I_{cm} = \left(\tfrac{M}{L}\right)\tfrac{1}{3} x^3 \Big|_{-L/2}^{L/2} = \left(\tfrac{M}{L}\right)\tfrac{1}{3}\left(\left(\tfrac{L}{2}\right)^3 - \left(\tfrac{-L}{2}\right)^3\right) = \tfrac{1}{12} ML^2$$

$$I_{about\,cm} = \sum_i m_i r_{i\leftarrow cm}^2$$

Other simple shapes and their Moments of Inertia



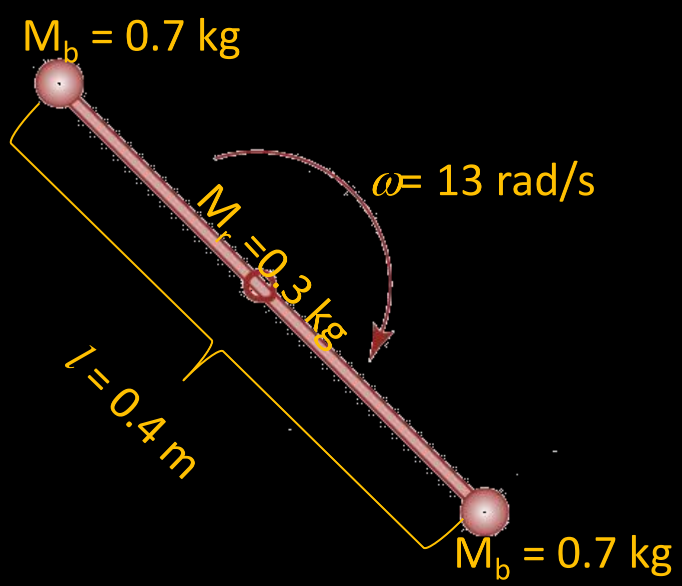
$$I_{cylinder.c} = \tfrac{1}{12} ML^2 + \tfrac{1}{4} MR^2$$

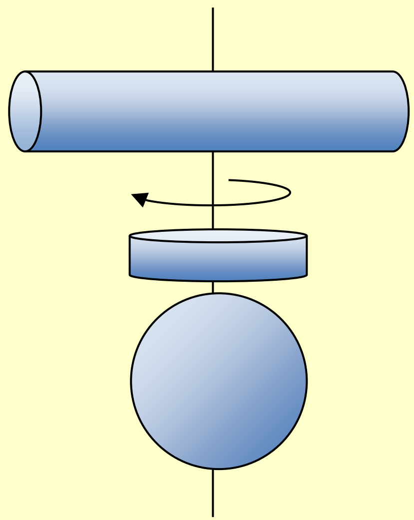
$$I_{disc.c} = \tfrac{1}{2} MR^2$$

$$I_{sphere.c} = \tfrac{2}{5} MR^2$$

$$I_{about\,cm} = \sum_i m_i r_{i \leftarrow cm}^2$$

Two balls of mass 0.7 kg are connected by a rigid rod of length 0.4 m and mass 0.3 kg. The object rotates around a pivot at its center, with angular speed 13 radians/s. What is the rotational kinetic energy of this object?





$$I_{cylinder.c} = \tfrac{1}{12} ML^2 + \tfrac{1}{4} MR^2$$

$$I_{disc.c} = \tfrac{1}{2} MR^2$$

$$I_{sphere\,c} = \tfrac{2}{5} MR^2$$

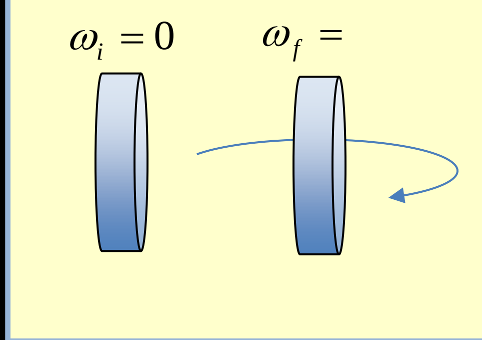
Example: Work to spin a quarter

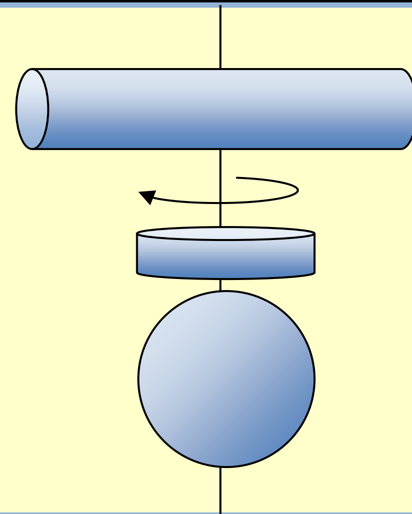
How much work does it take to spin a quarter from rest to spinning 10 times per second?

R ~0.01m

L~0.001m

M~ 0.01kg





$$I_{cylinder.c} = \tfrac{1}{12} ML^2 + \tfrac{1}{4} MR^2$$

$$I_{disc.c} = \tfrac{1}{2} MR^2$$

$$I_{sphere.c} = \tfrac{2}{5} MR^2$$

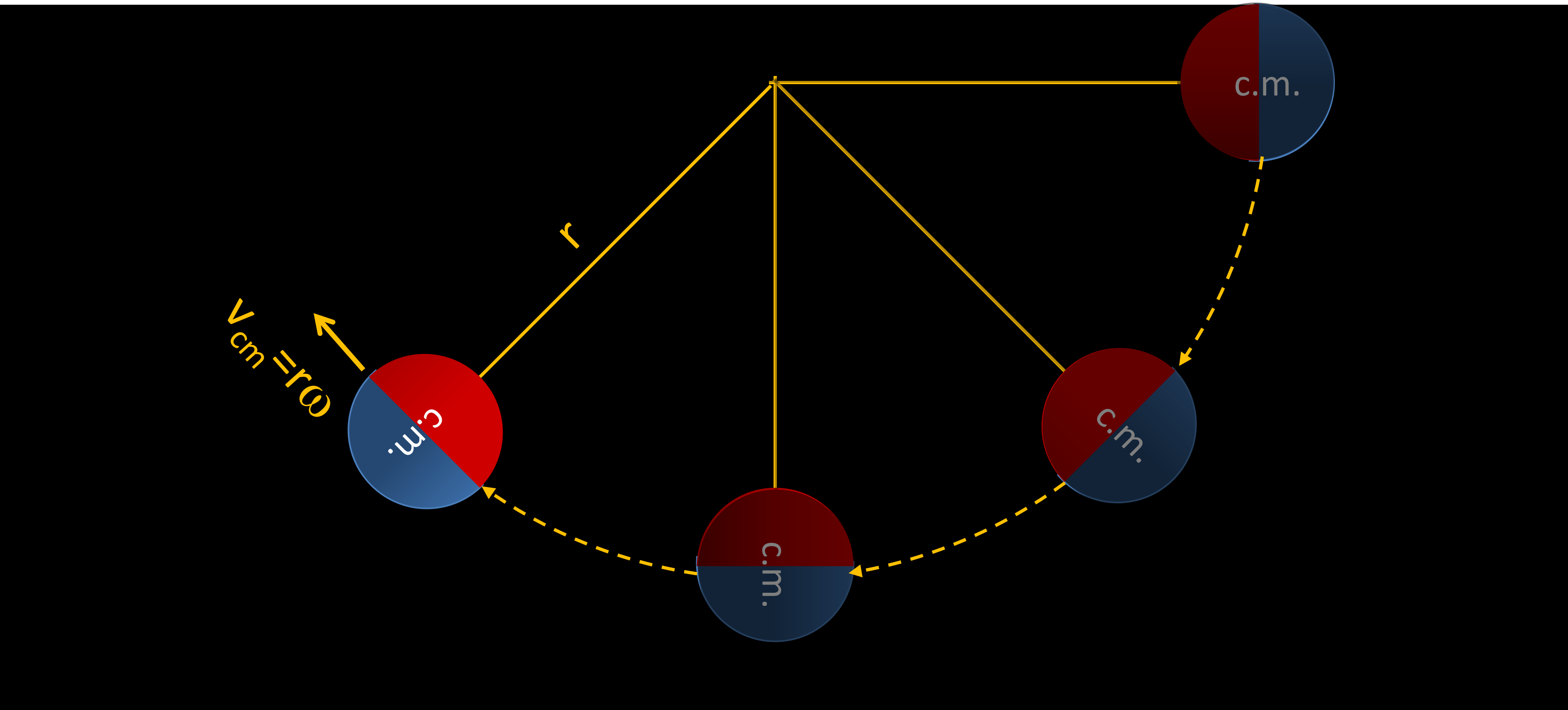
Rotation *not* about the center

$$K_{total} \approx K_{cm} + \sum_i K_{i.rel}$$

$$K_{total} \approx \tfrac{1}{2} M v_{of.cm}^2 + \tfrac{1}{2} I_{about.cm} \omega^2$$

$$K_{total} \approx \tfrac{1}{2} M (r\omega)^2 + \tfrac{1}{2} I_{about.cm} \omega^2$$

$$K_{total} \approx \tfrac{1}{2} \left(Mr^2 + I_{about.cm}\right)\omega^2 = \tfrac{1}{2}\left(I_{\text{about.other.point}}\right)\omega^2$$



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