Mon.	9.1-.2, (.8) Energy and Momentum in Multi-particle Systems	RE 9.a
Tues.		HW8: Ch 8 Pr's 21, 23, 27(a-c)
Wed.	9.3 Rotational Energy Quiz 8	RE 9.b
Lab	L8 Energy Quantization Review Exam 2 (Ch 5-8)	Practice Exam 2 (bring to lab)
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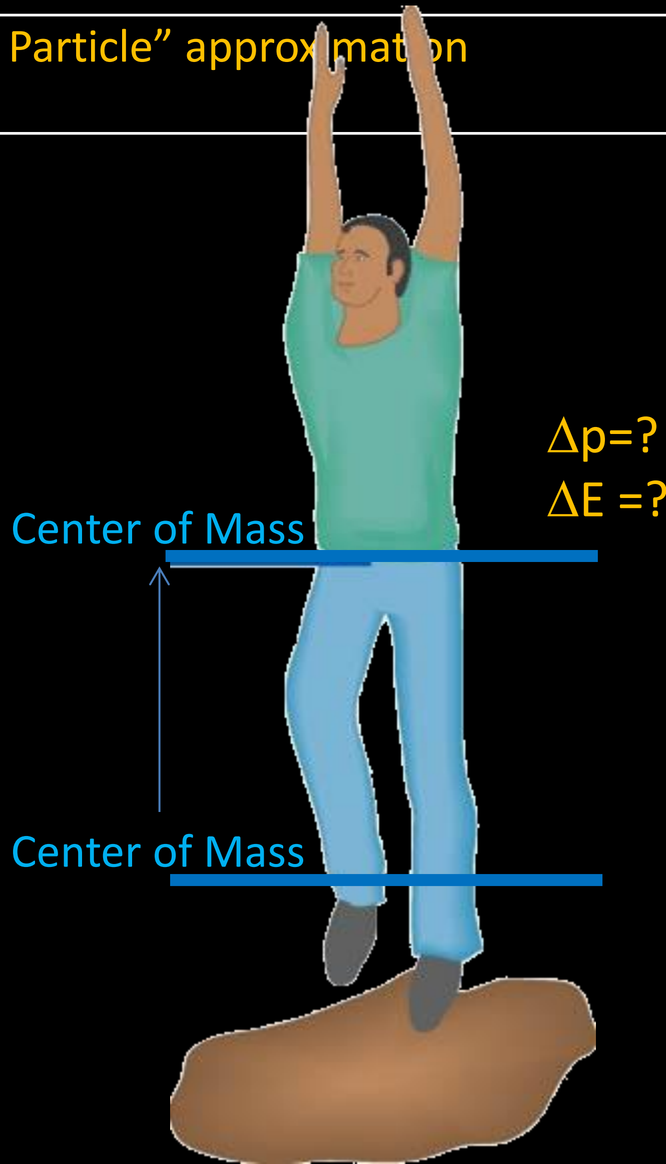
Mon.	9.4-.5 (.9) The "Point Particle" approximation	RE 9.c
Tues.		EP8, HW9: Ch 9 Pr's 34, 40, 43



Multi-Particle System's Momentum and Center of Mass

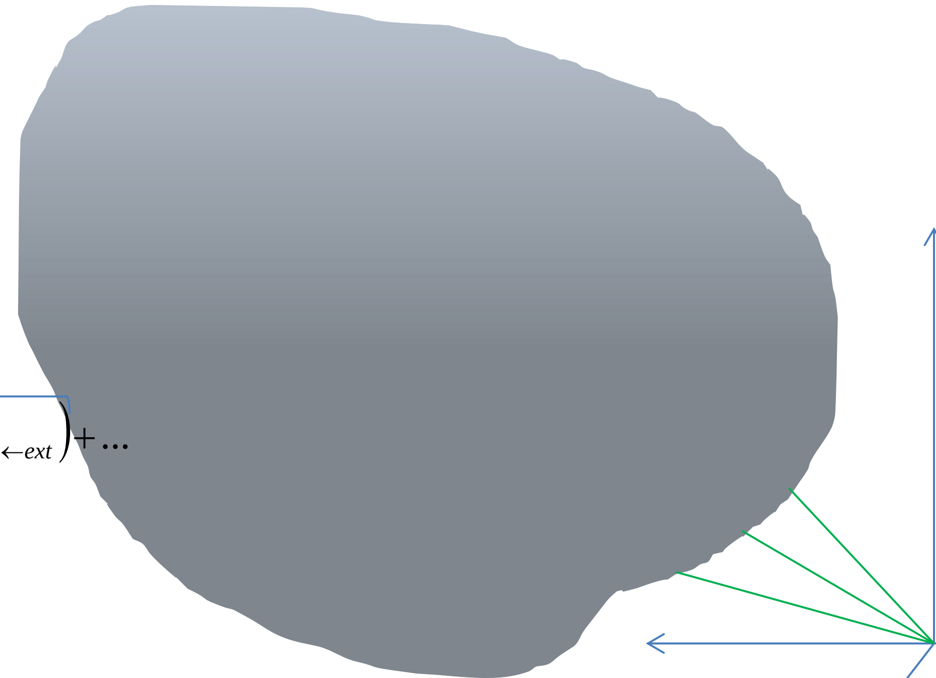
From chapter 3

$$\vec{p}_{system} \equiv \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots$$

so

$$\frac{d\vec{p}_{system}}{dt} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} + \frac{d\vec{p}_3}{dt} + \ldots$$

$$\left(\vec{F}_{1\leftarrow 2} + \vec{F}_{1\leftarrow 3} + \vec{F}_{1\leftarrow ext}\right) + \left(\vec{F}_{2\leftarrow 1} + \vec{F}_{2\leftarrow 3} + \vec{F}_{2\leftarrow ext}\right) + \left(\vec{F}_{3\leftarrow 1} + \vec{F}_{3\leftarrow 2} + \vec{F}_{3\leftarrow ext}\right) + \ldots$$



$$\frac{d\vec{p}_{system}}{dt} = \vec{F}_{1\leftarrow ext} + \vec{F}_{2\leftarrow ext} + \vec{F}_{3\leftarrow ext} \ldots$$

$$\frac{d\vec{p}_{system}}{dt} = \vec{F}_{net.ext}$$

So the sum of all momentum in the system varies in response to net external force just like the momentum of a point particle would.

Pushing the analogy further:

$$\vec{p}_{system} \approx m_{system} \text{"}\vec{v}_{system}\text{"}$$

What is this representative speed?

$$\frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} \approx \frac{\vec{p}_{system}}{m_{system}} \approx \text{"}\vec{v}_{system}\text{"} \approx \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

Mass-averaged velocity of the system

Multi-Particle System's Momentum and Center of Mass

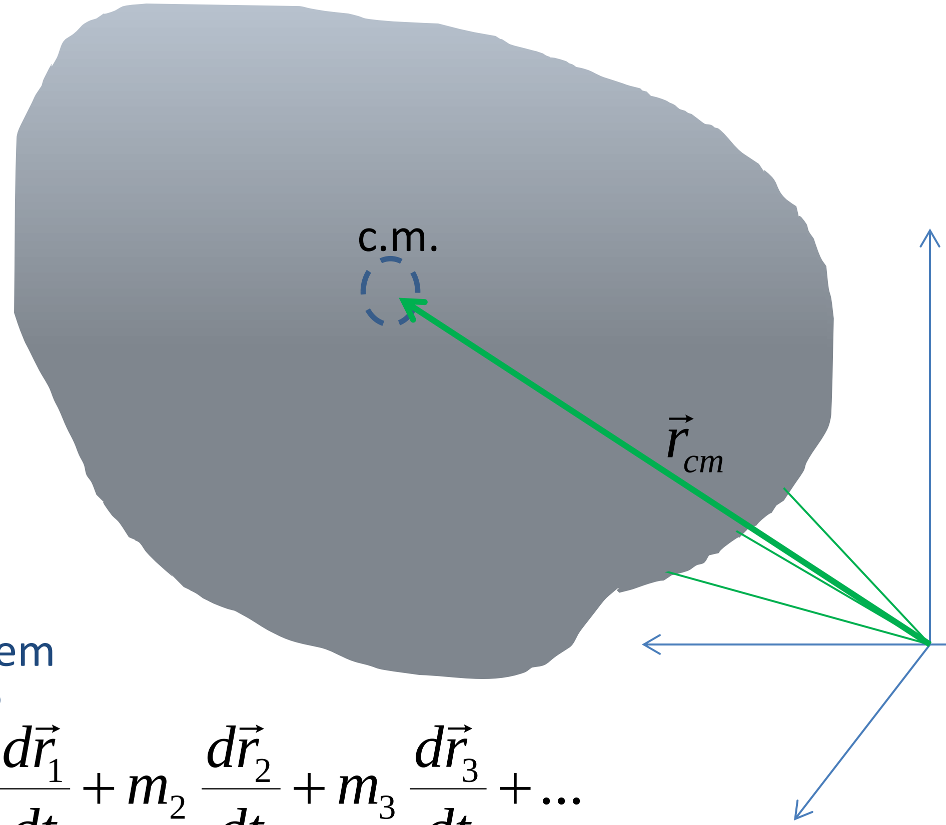
From chapter 3

$$\vec{p}_{system} \equiv \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots$$

$$\frac{d\vec{p}_{system}}{dt} = \vec{F}_{net.ext}$$

$$\vec{p}_{system} \approx m_{system} \text{"}\vec{v}_{system}\text{"}$$

$$\text{"}\vec{v}_{system}\text{"} \approx \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$



What representative location in the system moves with this representative velocity?

$$\vec{v} \equiv \frac{d\vec{r}}{dt} \quad \text{so} \quad \frac{d\text{"}\vec{r}_{system}\text{"}}{dt} \approx \frac{m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt} + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

$$\frac{d}{dt}\left(\text{"}\vec{r}_{system}\text{"}\right) \approx \frac{d}{dt}\left(\frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}\right)$$

apparently $\vec{r}_{system} \approx \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} \equiv \vec{r}_{cm}$

Mass-averaged *position* of the system : Center of Mass

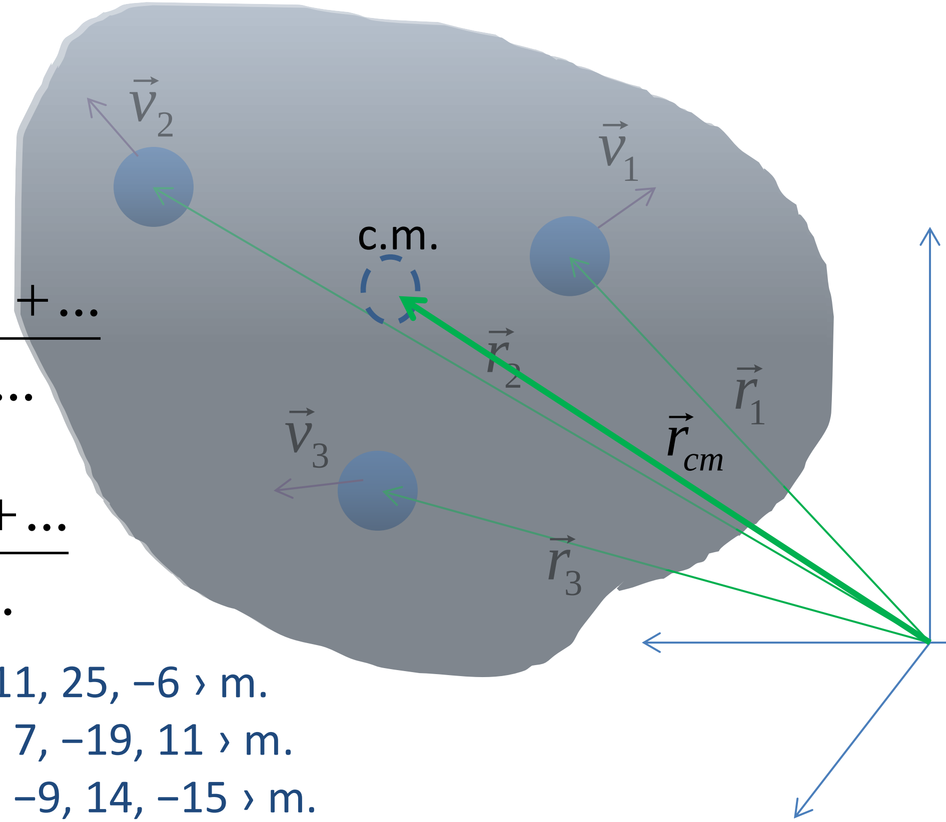
Multi-Particle System's Momentum and Center of Mass

$$\frac{d\vec{p}_{system}}{dt} = \vec{F}_{net.ext}$$

$$\vec{p}_{system} \approx m_{system}\vec{v}_{cm}$$

$$\vec{v}_{cm} \approx \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$



Example: say A 5 kg sphere is centered at ‹ 11, 25, −6 › m.
A 10 kg sphere is centered at ‹ 7, −19, 11 › m.
A 12 kg sphere is centered at ‹ −9, 14, −15 › m.

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} \qquad y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} \qquad z_{cm} = \frac{m_1z_1 + m_2z_2 + m_3z_3}{m_1 + m_2 + m_3}$$

$$x_{cm} = \frac{(5\text{kg})(11\text{m}) + (10\text{kg})(7\text{m}) + (12\text{kg})(-9\text{m})}{5\text{kg} + 10\text{kg} + 12\text{kg}} \qquad y_{cm} = \frac{(5\text{kg})(25\text{m}) + (10\text{kg})(-19\text{m}) + (12\text{kg})(14\text{m})}{5\text{kg} + 10\text{kg} + 12\text{kg}}$$

$$\vec{r}_{cm} = \langle 0.63, 3.81, -3.70 \rangle \text{m}$$

Multi-Particle System's Momentum and Center of Mass

No External Force Example

$$\vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt} \quad \Rightarrow \quad \vec{r}_{cm\,f} = \vec{r}_{cm.i} + \vec{v}_{cm}\Delta t$$

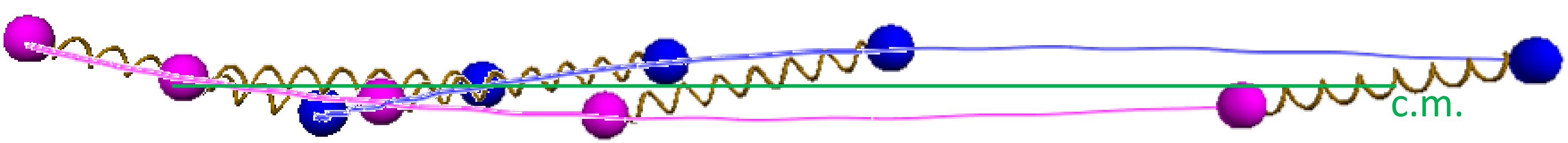
Demo: RotateVibrateTranslate.py (case 3 w & wo cm)

Individual parts may move erratically, but center of mass moves smoothly

Constant External Force Example

$$\frac{d\left(m_{system}\vec{v}_{cm}\right)}{dt} = \vec{F}_{net.ext} \quad \Rightarrow \vec{r}_{cm\,f} = \vec{r}_{cm.i} + \vec{v}_{cm.i}\Delta t + \tfrac{1}{2}\left(\frac{\vec{F}_{net}}{m_{system}}\right)(\Delta t)^2$$

Demo: baton

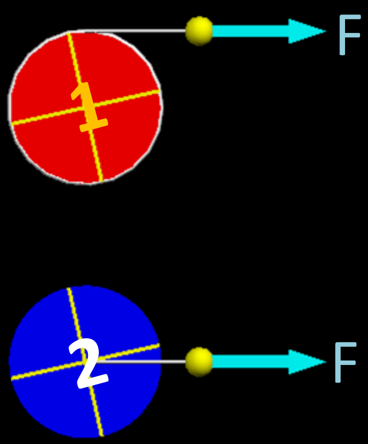


Demo: 09_RotateVibrateTranslate.py case 5

Note: momentum principle for multi-particle system doesn't care *at what point of system* the force is applied and doesn't say how *individual parts* move relative to center of mass

Two pucks lie on ice and can slide with little friction. A string is attached to each puck and each string is pulled with the same constant force F for the same time. The string is wound around the outer edge of puck 1 but attached to the center of puck 2. They both start from rest. Try to *imagine* what you would see as they move. What do you think will happen in the next 3 seconds?

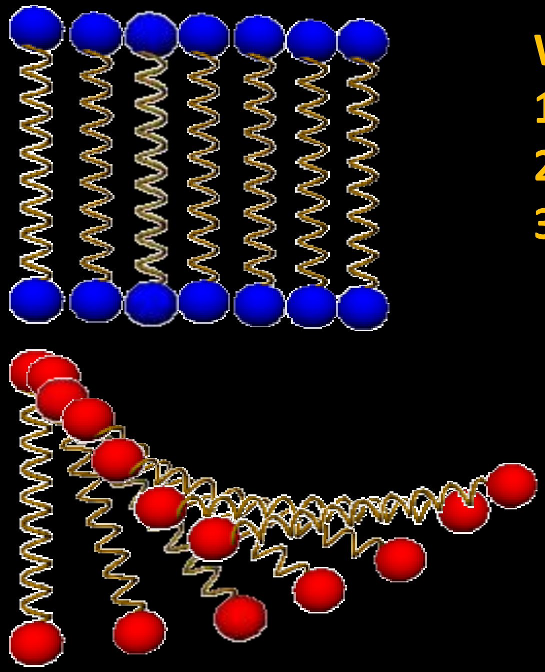
1) The center of 1 will move farther than 2
2) The center of 2 will move farther than 1
3) The centers of 1 and 2 will move the same distance



Now, *apply the momentum principle to each puck*. What does it predict should happen in the next 3 seconds?

1) The center of 1 will move farther than 2
2) The center of 2 will move farther than 1
3) The center of 1 and 2 will move the same distance

07_twopucks.py



Which object has the greater total momentum (magnitude)?

1) Top object (blue)

2) Bottom object (red)

3) Their total momentum is the same

Momentum principle speaks only for *center of mass* motion

Sometimes you want to track motion *relative to* center of mass

Need a different tool: Energy

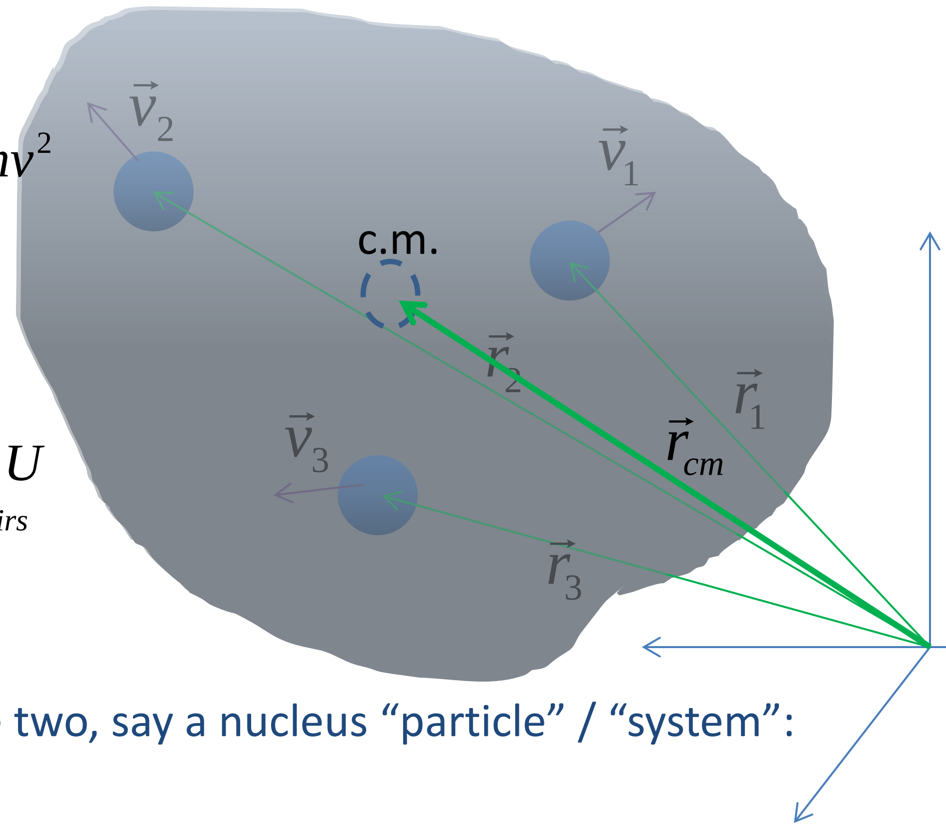
09_Krel.py

Multi-Particle System's Energy

Already familiar: $E_{particle} = K + E_{rest}$

$$K = (\gamma - 1)mc^2 \approx \tfrac{1}{2}mv^2$$

$$E_{rest} = mc^2$$



For system of particles:

$$E_{system} = \sum_{all.parts}\left(K + mc^2\right) + \sum_{all.pairs} U$$

Without thinking too deeply, we've blurred these two, say a nucleus "particle" / "system":

$$E_{particle} = K + E_{rest}$$

$$E_{system} = \sum_{all.parts}\left(K + mc^2\right) + \sum_{all.pairs} U$$

It seems reasonable to expect we can break up the system's energy as:

$$E_{rest} = E_{\text{int}}$$

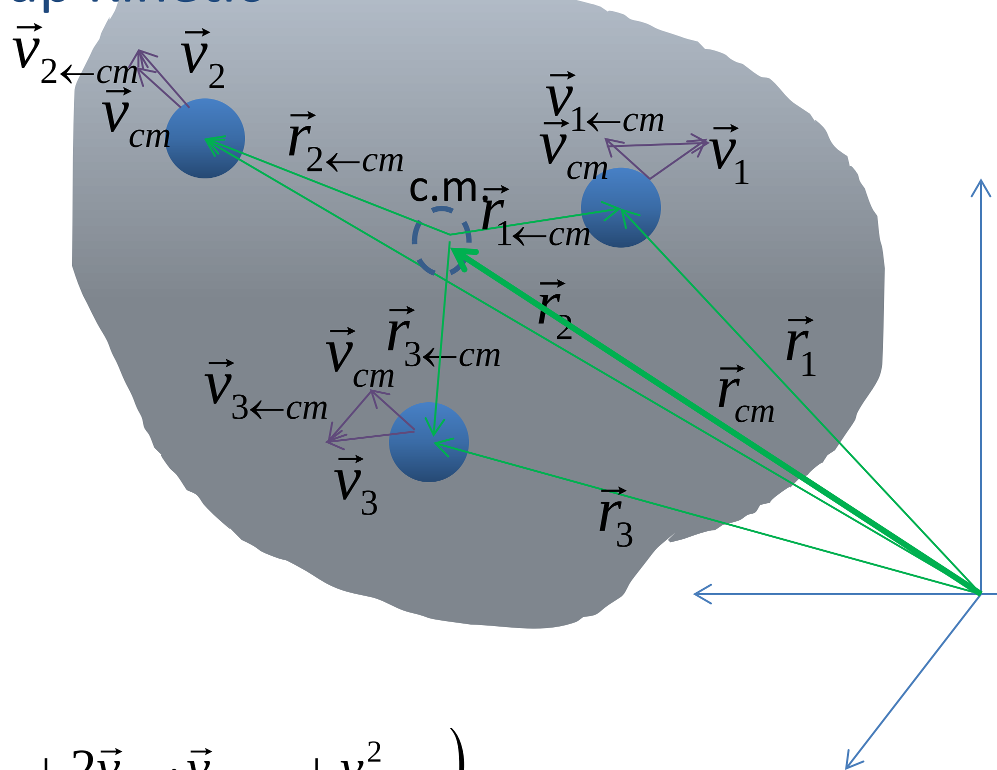
$$E_{system} = K_{cm} + \left(\sum_{all.parts}\left(K_{rel} + mc^2\right) + \sum_{all.pairs} U\right)$$

Multi-Particle System's Energy

Splitting up Kinetic

$$K_{total} = K_1 + K_2 + K_3 + \ldots$$

$$K_{total} \approx \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 + \tfrac{1}{2} m_3 v_3^2 + \ldots$$



Where $\vec{v}_1 = \dfrac{d}{dt}\vec{r}_1$, ect.

But $\vec{r}_1 = \vec{r}_{cm} + \vec{r}_{1\leftarrow cm}$, etc.

so $\vec{v}_1 = \dfrac{d}{dt}\vec{r}_{cm} + \dfrac{d}{dt}\vec{r}_{1\leftarrow cm}$

Or $\vec{v}_1 = \vec{v}_{cm} + \vec{v}_{1\leftarrow cm}$, etc.

so

$$\tfrac{1}{2} m_1 v_1^2 = \tfrac{1}{2} m_1 \left(\vec{v}_{cm} + \vec{v}_{1\leftarrow cm}\right)^2 = \tfrac{1}{2} m_1 \left(v_{cm}^2 + 2\vec{v}_{cm}\cdot\vec{v}_{1\leftarrow cm} + v_{1\leftarrow cm}^2\right)$$

$$= \tfrac{1}{2} m_1 v_{cm}^2 + m_1 \vec{v}_{cm}\cdot\vec{v}_{1\leftarrow cm} + \tfrac{1}{2} m_1 v_{1\leftarrow cm}^2$$

then $K_{total} \approx \tfrac{1}{2}\left(\sum_i m_i\right) v_{cm}^2 + \vec{v}_{cm}\cdot\left(\sum_i m_i \vec{v}_{1\leftarrow cm}\right) + \sum_i \left(\tfrac{1}{2} m_i v_{i\leftarrow cm}^2\right)$

$$K_{total} \approx \tfrac{1}{2} m_{total} v_{cm}^2 + \sum_i \left(\tfrac{1}{2} m_i v_{i\leftarrow cm}^2\right)$$

recall $\vec{v}_{cm} = \dfrac{\sum_i m_i \vec{v}_i}{m_{total}}$

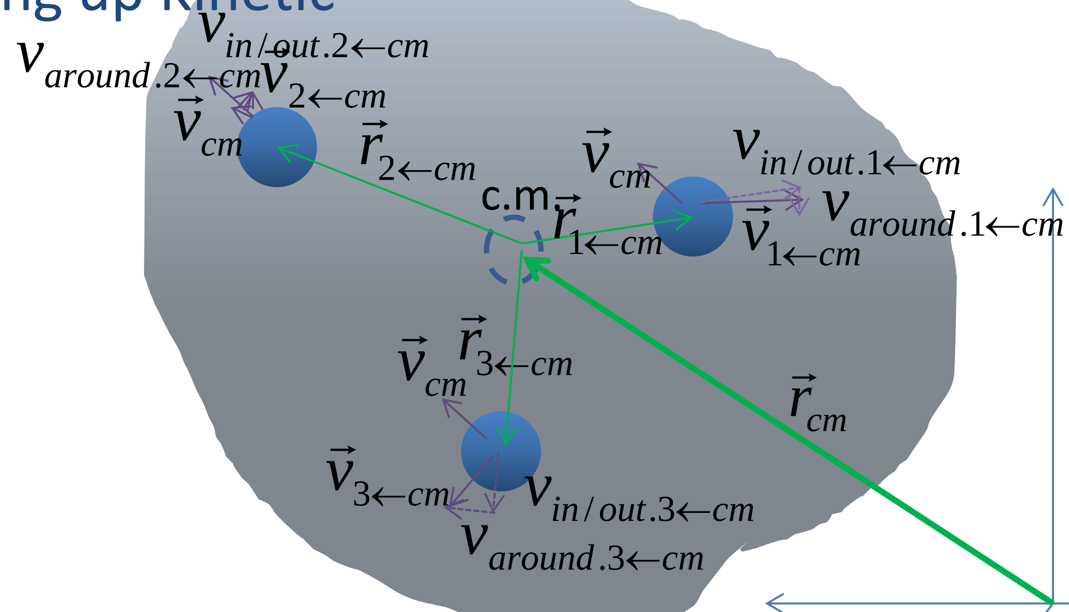
recall $\vec{v}_1 = \vec{v}_{cm} + \vec{v}_{1\leftarrow cm}$

so $\vec{v}_{1\leftarrow cm} = \vec{v}_1 - \vec{v}_{cm}$

$$\sum_i m_i \left(\vec{v}_i - \vec{v}_{cm}\right) = \sum_i m_i \vec{v}_i - \left(\sum_i m_i\right)\vec{v}_{cm} = m_{total}\vec{v}_{cm} - m_{total}\vec{v}_{cm} = 0$$

Multi-Particle System's Energy

Splitting up Kinetic



$$K_{total} = K_1 + K_2 + K_3 + \ldots$$

$$K_{total} \approx \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 + \tfrac{1}{2} m_3 v_3^2 + \ldots$$

$$K_{total} \approx \underbrace{\tfrac{1}{2} m_{total} v_{cm}^2} + \sum_i \underbrace{\left(\tfrac{1}{2} m_i v_{i\leftarrow cm}^2\right)}$$

$$K_{total} \approx K_{cm} + \sum_i K_{i.rel}$$

Often convenient to resolve $\vec{v}_{i\leftarrow cm}$ into perpendicular components

Towards/away from center of mass

Around center of mass $\vec{v}_{i\leftarrow cm} = v_{i\leftarrow cm.in/out}\hat{r}_{i\leftarrow cm} + v_{i\leftarrow cm.around}\hat{\phi}_{i\leftarrow cm}$

$$\left(\vec{v}_{i\leftarrow cm}\right)^2 = \left(v_{i\leftarrow cm.in/out}\hat{r}_{i\leftarrow cm} + v_{i\leftarrow cm.around}\hat{\phi}_{i\leftarrow cm}\right)^2 = v_{i\leftarrow cm.in/out}^2 + v_{i\leftarrow cm.around}^2$$

0 cross-term since they're perpendicular components

$$\sum_i \left(\tfrac{1}{2} m_i v_{i\leftarrow cm}^2\right) = \sum_i \underbrace{\left(\tfrac{1}{2} m_i v_{i\leftarrow cm.in/out}^2\right)} + \sum_i \underbrace{\left(\tfrac{1}{2} m_i v_{i\leftarrow cm.around}^2\right)}$$

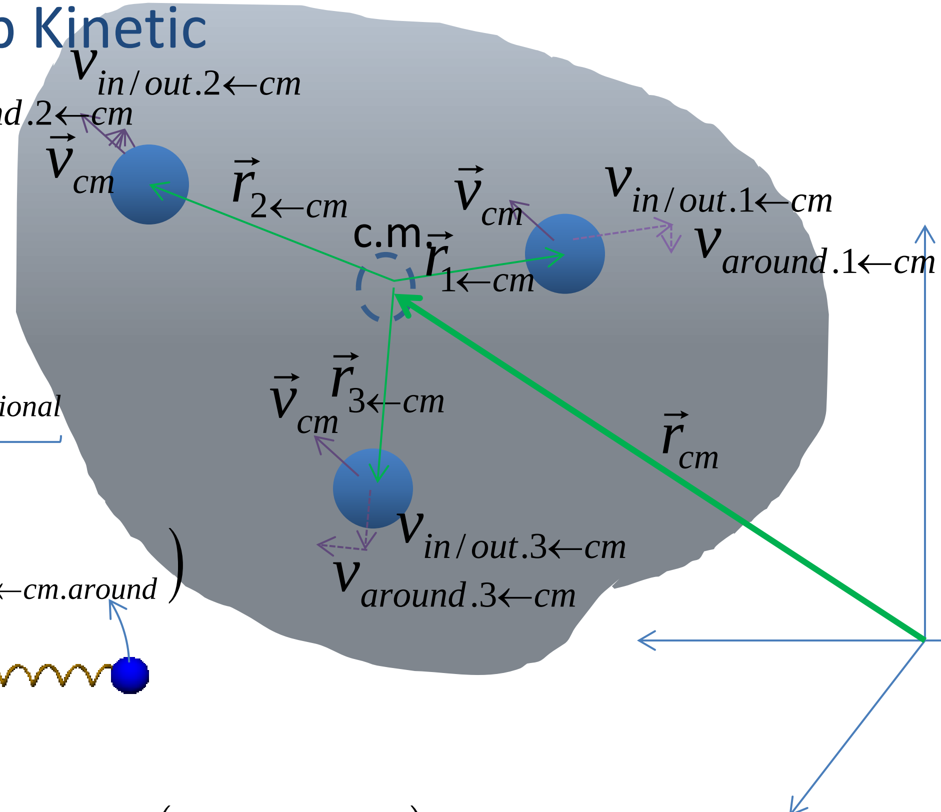
$$\sum_i K_{i.rel} = \sum_i K_{i.vibrational} + \sum_i K_{i.rotational}$$

Multi-Particle System's Energy

Splitting up Kinetic

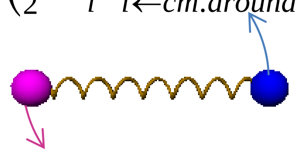
$$K_{total} \approx \tfrac{1}{2} m_{total} v_{cm}^2 + \sum_i \left(\tfrac{1}{2} m_i v_{i\leftarrow cm}^2 \right)$$

$$K_{total} \approx K_{cm} + \sum_i K_{i.rel}$$



$$\sum_i K_{i.rel} = \sum_i K_{i.vibrational} + \sum_i K_{i.rotational}$$

$$\sum_i \left(\tfrac{1}{2} m_i v_{i\leftarrow cm.in/out}^2 \right) + \sum_i \left(\tfrac{1}{2} m_i v_{i\leftarrow cm.around}^2 \right)$$



With vibrating a spring (or bond approximated as a spring), there's also potential

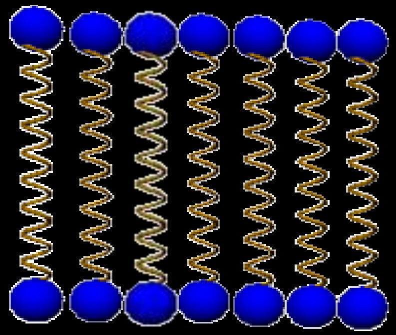
$$U = \sum_{all.springs} \left(\tfrac{1}{2} k_s s^2 + U_{eq} \right)$$

$$K_{rel} = K_{vib} + K_{rot}$$

$$E_{system} = K_{cm} + \left(\sum_{all.parts} \left(K_{rel} + mc^2 \right) + \sum_{all.pairs} U \right) = K_{cm} + E_{rest}$$

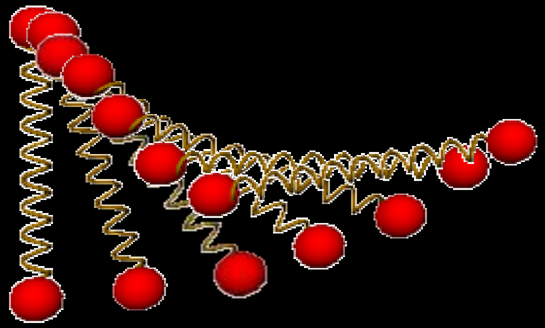
Just like for a particle

$$E_{\text{int}} = E_{rest}$$



Which object has the greater translational kinetic energy (K_{TRANS})?

1) Top object (blue)
2) Bottom object (red)
3) Their translational kinetic energy is the same



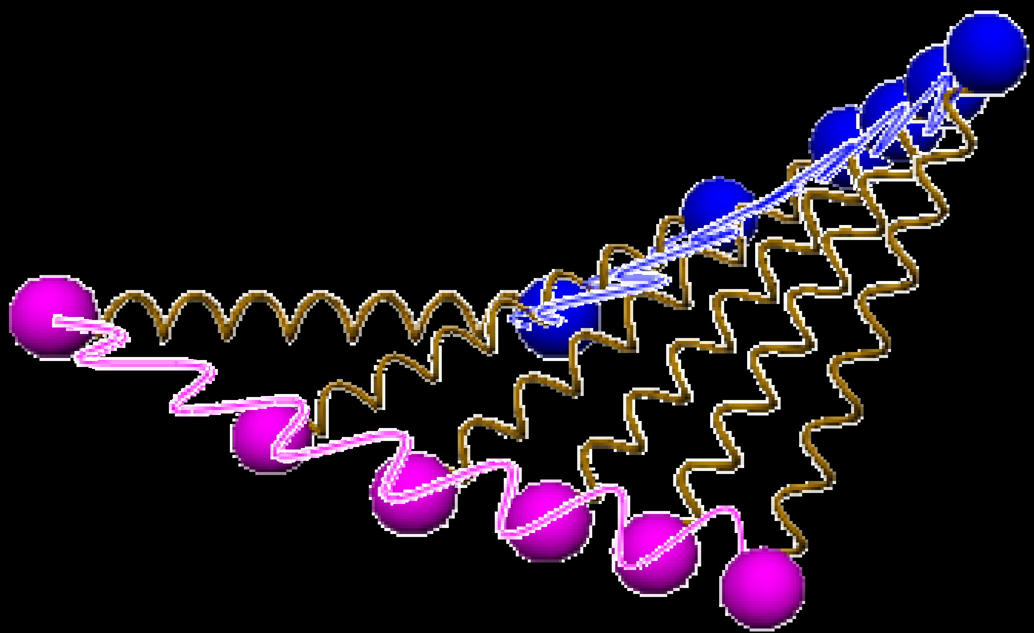
Which object has the greater total kinetic energy?

1) Top object (blue)
2) Bottom object (red)
3) Their total kinetic energy is the same

09_Krel.py

In the case shown in the VPython program, what energy terms are nonzero for the system of the two balls and the spring?

1) K_{trans}
2) $(K_{vib} + U_{spring})$
3) K_{rot}
4) $(K_{vib} + U_{spring})$ and K_{rot}
5) K_{trans} and K_{rot}
6) K_{trans} and $(K_{vib} + U_{spring})$
7) $(K_{vib} + U_{spring})$ and K_{rot} and K_{trans}



09_RotateVibrateTranslate.py

Consider a system consisting of three particles:

$m_a = 3$ kg, $\vec{v}_a = <11, -8, 15>$ m/s

$m_b = 3$ kg, $\vec{v}_b = <-12, 11, -5>$ m/s

$m_c = 5$ kg, $\vec{v}_c = <-23, 36, 18>$ m/s

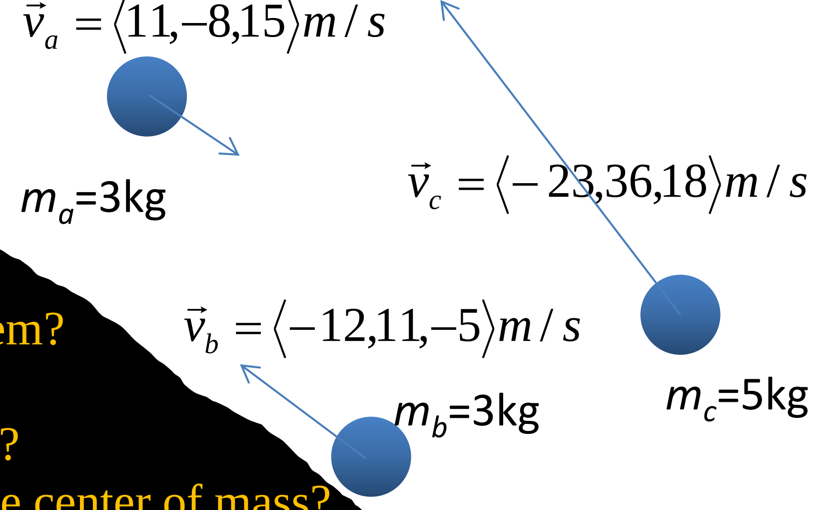
(a) What is the total momentum of this system?

(b) What is the velocity of the center of mass of this system?

(c) What is the total kinetic energy of this system?

(d) What is the translational kinetic energy of this system?

(e) What is the kinetic energy of this system relative to the center of mass?



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