Physics 231 – Lab 5
Circular Motion and the Pendulum

Equipment: 2-meter stick, scale, stopwatch, plane (with batteries), pendulum

Objectives

This lab will cover applications of the momentum principle to:

- An object moving in a circle at a constant speed (called *uniform circular motion*)
- A pendulum

I. Circular Motion

A. Background

The momentum principle tells us that $\vec{F}_{net} = d\vec{p}/dt$. So, there are two ways of determining the net force applied to an object: directly measuring the forces or deducing them from the resulting change in motion (thanks to the momentum principle). You'll analyze the circular motion of toy plane from both angles – first you'll determine the net force by strictly considering the individual forces applied to the plane, and second you'll determine the rate of change of momentum by strictly considering the observed motion. Finally, you'll see how the two compare.

When an object executes uniform circular motion, only the direction (not the magnitude) of its momentum changes. It's most convenient then to resolve the momentum principle into components parallel to the motion and perpendicular to the motion since $\left.\frac{dp}{dt}\right|_{||} = 0$ and $\left.\frac{d\vec{p}}{dt}\right|_{\perp} = |p|\frac{d\hat{p}}{dt} = -|p|\frac{|v|}{r}\hat{r} \approx -\frac{m|v|^2}{r}\hat{r}$ where r is the radius of circle about which the object's orbiting.

When an object is uniformly orbiting in a circle, its speed is simply the circumference of that circle divided by the period of the orbit.

Note: This experiment is very similar to the circular pendulum discussed in section 5.7 of your text.

B. Experiment

- Measure the mass of the plane and the length of the string attached to it (note: the mass may already be written on the plane's underbelly.)
- Hang the plane and get it moving in a steady circular motion. Make the following two measurements close together so that the motion does not change in between.
  - Measure the distance *h* straight down from the ceiling to level at which the plane circles. (Be careful not to disturb the plane when you do this.)
  - Measure the time required for the plane to complete one revolution by measuring the time it takes to for ten revolutions and then dividing by 10. Be careful not to count "one" until the plane has gone around once (it can be tempting to say "one" the moment you start the watch.)
- On a whiteboard, make a diagram showing the string and the plane *viewed from the front at one instant* with the dimensions that you measured labeled so it's easy to see how *L* and *h* are related to *R* (the radius of the plane's orbit) and to θ, the angle at which the string hangs relative to the vertical.
- From your measurements of L and h, determine the radius of the orbit, *R,* and the tangent of the angle, tan(θ) (we aren't actually interested in the angle itself, just the tangent.)

C. Net Force

Returning to the white board, make a diagram showing all of the forces on the plane *as viewed from the front*. For consistency sake, $\vec{F}_s$ stands for the string's force on the plane and $\vec{F}_E$ stands for the Earth's force on the plane. If you can't think of an object exerting that a particular force, then it doesn't exist or belong on the diagram. Indicate the angel θ on this diagram too. While there is a force due to the propeller out of the page and a drag force into the page, those cancel out so you don't need to include them. If you're not quite sure what your picture should look like, there's a very similar one in section 5.7 of your text.

- Use the diagram, how components of the string's force are related to its magnitude and angle, and Newton's Second Law in the *vertical* direction to determine the magnitude of the string force, F_s, *symbolically* in terms of m, g, and θ.

- Use the diagram to find the net force in the horizontal or radial, a.k.a. centripetal, direction *symbolically* in terms of m, g, and θ. Show your work.

- Okay, calculate a value for the net radial force.

D. Rate of Change of the Momentum

- *Independently* of the calculations in the previous part, determine the magnitude of the rate of change of the momentum, $|d\vec{p}/dt|$ (magnitude *and* direction, because it is a vector) from the plane's *motion*, (not the known forces on it) symbolically in terms of m, R, and T.

- Okay, calculate a numeric value for the rate of change of momentum.

E. The Momentum Principle

According to the momentum principle, $\vec{F}_{net} = d\vec{p}/dt$. How well do the quantities that you calculated in the previous parts agree? (Calculate the percent difference.) If much greater than 10% difference, find and fix your mistake.

II. The Pendulum

The pendulum has not been discussed, but its motion can be modeled using the Momentum Principle.

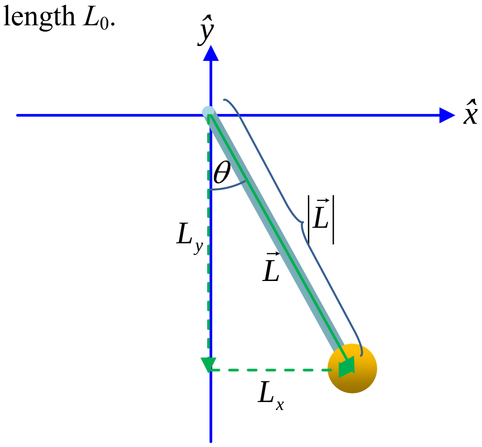
A. Computational Model

There are a number of approaches to predicting the motion of a pendulum. For example, in Problem 4.P.89 an approximation is made to the equation describing the motion so that it can be solved analytically. That gives an approximate expression that is valid for swings with small amplitudes. Instead, you will build a computational model of the motion. You already saw, in the most recent computational homework problem, that a spring that's free to move in 3-D swings like a pendulum while it bobs. So, to model a stiff pendulum, we'll treat it as a bob of mass *m* hanging from a spring with extremely high stiffness, *k*.

Of course the bob experiences the Earth's gravitational force down, $\vec{F}_E = m\vec{g}$, and the spring's force back along the spring. As you probably recall, the spring force is $\vec{F}_s = -k\left(\left|\vec{L}\right| - L_o\right)\hat{L}$; another way to put it is $\vec{F}_s = -k\left(\vec{L} - L_o\hat{L}\right)$, where $\vec{L}$ gives the position of the hanging mass relative to the other end of the spring, and L_o is the spring's equilibrium length.

When initially creating the bob and spring in your code, it's going to be handy to phrase the *x* and *y* components of bob's initial position in terms of the spring's length (which we'll start with being L_o) and it's *angle* off the vertical, θ. The figure below will help you to think how to do that.

In order to model the pendulum, you'll need to express the forces on the mass in vector notation. Suppose one end of a spring is attached to attached at the origin and the mass is at the location $\langle x, y, 0\rangle$ and the spring makes an angle θ with vertical. Also, assume that the length *L* of the spring is larger than its unstretched length L_0.



- Assuming that you'll start with the spring at its equilibrium length, determine an expression for the ball's initial position in terms of *Lo* and appropriate trig functions of θ.
- Draw and label the forces on the bob in the diagram above (assume that the spring / string has been stretched.) Make sure that their directions are accurate.

Fill in the missing parts of the program.

- Let the bob have a mass of 0.1 kg.
- Let the spring have a relaxed length (`L0`) of 0.5 m and a stiffness of 20 N/m.
- Set the initial a location of the bob so that the spring is relaxed and it makes an angle of 10° with vertical. It is useful to use the functions `sin()` and `cos()` in the program so that the angle can be changed easily.
    (a) **Note** that the angles used in those functions must be in *radians*. The constant `pi` is defined in VPython. If you want to specify an angle in degrees, you can include the conversion factor, $\frac{180°}{\pi \text{ radians}}$, within the trig functions.
- Have the bob start from rest.
- Make sure that the time interval (`dt`) is small enough to give accurate results (say, a millisecond.)
- Give it a go and make sure the behavior looks appropriate for a mass swinging from a spring.

Pause and consider: How does the motion of the pendulum change when you make the spring stiffness larger? Is this what you expect?

- Increase the spring stiffness to 10,000 N/m. Now, you've essentially modeled a mass swinging on a *string.*
- **Determining the Period.** The period of the pendulum is the amount of time that it takes the pendulum to repeat its motion (same position and velocity). To determine the period in the computational model, you'll add the following lines of code – they'll make the program determine and print out the period. each time the pendulum swings across the vertical (which it does twice a cycle). From these times, you can determine the period.

Before entering the while loop, add the following

```
t1 = 0.0 # used in the loop to record when the pendulum crosses
vertical
```

Just *before* your position-update line, add the following

```
x = 1*bob.x #used to determine when vertical is crossed
```

Just *after* your position-update line, add the following (make sure to properly indent so it's considered inside the while loop)

```
# keep track of each time bob passes vertical from right to left
if x/bob.x <0 and bob.p.x<0:
    #the difference between current time and last time vertical was
    #crossed form the right is one period
      T = t - t1
      #only ¼ of a period passes before the first time the vertical
      #is crossed, so avoid printing a "T" that time.
      if t1 > 0
            print("period =", T,"s.")
      #updates t1 to keep track of this vertical-crossing time
      t1 = 1*t
```

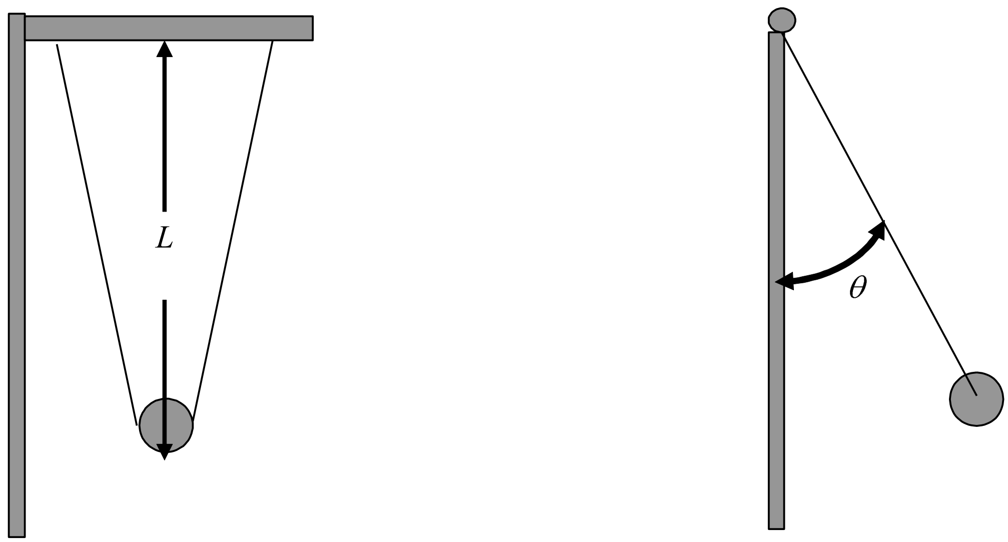
Run the program with these new lines of code and, from the times printed, determine the period.

Save your code as pendulum.py and upload it.

- To see how they affect the pendulum's period, vary some of the parameters in your program (the mass, the length, the initial angle, and the gravitational field strength).
- **Pause and consider:** for each variation, discuss with your partners *why* the pendulum's period should depend (or not depend) on the parameters as you observe.

B. Experiment

- Set up a pendulum as shown below using the metal bob. The string going through the ball should be adjusted so that it is the same length on each side.



- Measure the length (L) of the pendulum, which is the shortest distance between the center of the ball and the axis around which it swings. This is *not* the length of the strings.
- Release the pendulum from an angle of about 10° (exact angle doesn't matter, as long as it's fairly small.) To get a good measurement of the period, repeat the following 5 times and average: Measure the time required for 10 complete swings (back and forth), then divide the measured times by 10 to get the period. (warning: it's tempting to start your count at "one" the moment you release the bob – that should be "zero").

C. Comparison of the Computational Model and the Experiment

If the simulation accurately models nature, then it should reproduce the experimental results if you set it up with the same parameters (mass, length, etc.). Test that.

- Change the program so that the values of the mass, length, and initial angle (and g) are the same as in the experiment (warning: don't forget that *cos* and *sin* in the program expect angle values to be in *radians* unless you included the conversion factor.)
- Run the program to determine the period predicted by the computational model. Be sure that the spring stiffness is set to a very large value.