Practice Final Exam PY 205 Monday 2004 May 3

Name____

- **There are THREE formula pages.**
- **Read all problems carefully before attempting to solve them.**
- **Your work must be legible, and the organization must be clear.**
- **Correct answers without adequate explanation will be counted wrong.**
- **Incorrect explanations mixed in with correct explanations will be counted wrong.**
- **Make explanations complete but brief. Do not write a lot of prose.**
- **Write on the backs of pages if you need more space, and write a note saying where we should look.**
- **Include diagrams!**
- **Show what goes into a calculation, not just the final number:** $\frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5\times10^{4}$
- **Give physical units with your results.**

If you cannot do some portion of a problem, invent a symbol for the quantity you can't calculate (explain that you're doing this), and do the rest of the problem.

Problem 1 (5 pts)

A mass of 0.7 kg connected to a spring of unknown stiffness oscillates with little friction, making one round trip in 1.2 seconds. What is the spring stiffness k_s?

Problem 2 (5 pts)

Making the approximation that the density of the Earth is roughly uniform, calculate the magnitude of the rotational angular momentum of the Earth around its own center.

Problem 3 (10 pts)

The Moon has a mass of 7×10^{22} kg and a radius of 1.75×10^{6} m. There is no air, so no air resistance. Suppose you launch a rocket from the surface of the Moon so that when it is very far from the Moon its speed is 1200 m/s. What is the required launch speed?

Problem 4 (10 pts)

A person jumps out of an airplane above the surface of the Earth, and falls a distance h.
(a) (2 pts) The work done on the person by the Earth is:
Positive, Negative, Zero, or Insufficient Information to Answer?

(b) (2 pts) The change in gravitational potential energy of the person+Earth system is:
Positive, Negative, Zero, or Insufficient Information to Answer?

(c) (2 pts) After falling a distance h, the person's parachute opens and he proceeds to fall an additional distance h, albeit at a much reduced speed. During this second phase of the fall, the work done on the person by the Earth is compared to that done in part (a). The work done is:
More Than, Less Than, The Same As, or Insufficient Information to Answer?

Pluto orbits the Sun in an elliptical orbit with a period of 249 years.

(d) (2 pts) During one full period of Pluto, the work done on Pluto by the Sun is:
Positive, Negative, Zero, or Insufficient Information to Answer?

(e) (2 pts) In its elliptical orbit, Pluto is currently moving away from the Sun. The work done on Pluto by the Sun during the next 10 days is:
Positive, Negative, Zero, or Insufficient Information to Answer?

Problem 5 (5 pts)

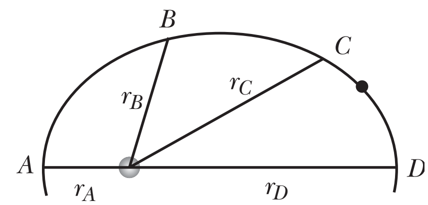
What is the total energy of an electron that is traveling at a speed of $0.97c$, where c is the speed of light? Give a numerical answer.

Problem 6 (10 pts)

An object had momentum $\langle 4, 6, -8 \rangle (\text{ kg} \cdot \text{m/s})$ when a net force $\langle 30, -20, 12 \rangle \text{ N}$ started to act on the object. After 0.02 seconds, what was the new momentum of the object?

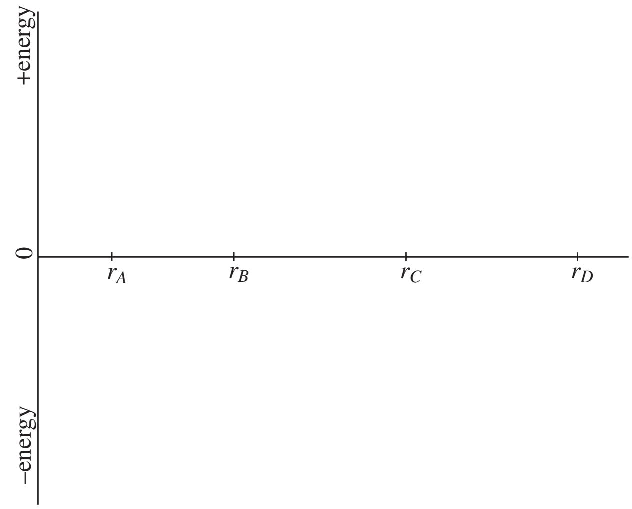
Problem 7 (10 pts)

Here is a portion of the orbit of an asteroid around the Sun in an elliptical orbit, moving from A to B to C to D.



(a) (10 pts) For the system consisting of the Sun plus the asteroid, graph the gravitational potential energy U, the kinetic energy K, and the sum $K+U$, as a function of the separation distance between Sun and asteroid. ***Label each curve.*** Along the r axis are shown the various distances between Sun and asteroid.

(b) (0 pts) ***Did you remember to label the curves?***



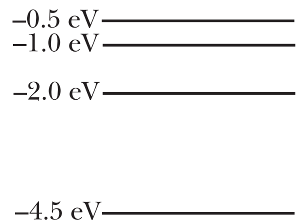
Problem 8 (10 pts)

Here is a portion of a program for a process similar to Rutherford scattering. An alpha particle interacts with a silver nucleus. Complete the missing pieces. (Not shown are the statements that set the initial conditions before the "while" loop.)

```
...
deltat = 1e-23
while t < 1e-20:
   r = alpha.pos - silver.pos
   rmag = sqrt(r.x**2 + r.y**2 + r.z**2)

   rhat =

   Fmag = 9e9*47*2*(1.6e-19)**2/rmag**2

   F =

   alpha.p =

   silver.p =

   alpha.pos = alpha.pos + (alpha.p/alpha.m)*deltat
   silver.pos = silver.pos + (silver.p/silver.m)*deltat
   t = t+deltat
```

Problem 9 (20 pts)

A particular quantum system has energy levels, $K+U$, shown in the diagram. A beam of high-energy electrons runs through a bottle that contains a large number of these systems, so that there are many systems in all of these energy states at all times, and photons are continually emitted from the bottle.



(a) (15 pts) What are the energies of the emitted photons? Indicate the transitions on the diagram.

(b) (5 pts) Next the electron beam is turned off, and the bottle is cold. A beam of light with a wide range of energies, from 0.1 to 10 eV, shines through the bottle. On the other side of the bottle, what photon energies in the beam are somewhat depleted ("dark spectral lines")? Explain with the help of a diagram.

Problem 10 (10 pts)

A spring whose stiffness is 3500 N/m is used to launch a 4 kg block straight up in the classroom. The spring is initially compressed 0.2 m, and the block is initially at rest when it is released. When the block is 1.3 m above its starting position, what is its speed?

Problem 11 (20 pts)

In outer space two rocks collide and stick together, without rotation. Here are the masses and initial velocities of the two rocks:

Rock 1: mass = 15 kg, initial velocity = $\langle 10, -30, 0 \rangle$ m/s

Rock 2: mass = 32 kg, initial velocity = $\langle 15, 12, 0 \rangle$ m/s

(a) (10 pts) Use the momentum principle to find the (vector) velocity of the stuck-together rocks after the collision.

(b) (10 pts) Use the energy principle, and your result from part (a), to calculate the increase in thermal energy of the rocks.

Problem 12 (30 pts)

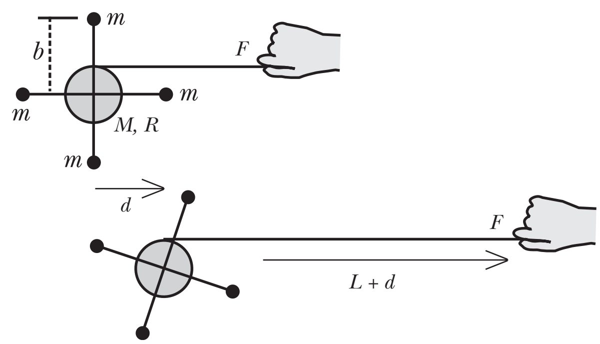
From Young's modulus for aluminum we found that the effective interatomic "spring" stiffness was 16 N/m. See formula sheet for additional data.

(a) (20 pts) For a nanoparticle consisting of 3 aluminum atoms (using the Einstein model of 9 independent oscillators), calculate the temperature when there are 5 quanta of energy in the nanoparticle. Show your work clearly.

(b) (10 pts) Calculate the heat capacity per aluminum atom at this temperature. Show your work clearly.

Problem 13 (20 pts)

A string is wrapped around a uniform-density disk of mass M and radius R. Attached to the disk are four low-mass rods, each with a small mass m at the end, a distance b from the center of the disk.



The apparatus is initially at rest on a nearly frictionless surface. Then you pull the string with a constant force F. At the instant when the center of the disk has moved a distance d, a length L of string has unwound off the disk, so your hand has moved a distance $L+d$.

(a) (10 pts) Write out the energy principle for the point particle system. Use this to find the final speed v of the center of mass of the system.

(b) (10 pts) Write out the energy principle for the real system. Use this, and your result from part (a), to find the angular final speed ω of the apparatus.

FUNDAMENTAL PHYSICAL LAWS

Principle of relativity: Physical laws work in the same way for observers in uniform motion as for observers at rest. The superposition principle: the effective force on an object is the "net" force, the vector sum of all forces acting on the object, each force unaffected by the presence of other interactions.
The momentum principle, and the definition of momentum. (These must be memorized.)
The energy principle, and the definitions of work and energy. (These must be memorized.)
The angular momentum principle. (This must be memorized.)
The relationship among position, velocity, and time. (This must be memorized.)
The second law of thermodynamics or the entropy principle: If a closed system is not in equilibrium, the most probable consequence is that the entropy will increase.

MULTIPARTICLE SYSTEMS

$K_{\text{total}} = K_{\text{trans}} + K_{\text{rel to cm}} = K_{\text{trans}} + K_{\text{rot}} + K_{\text{vib}}$ $\quad K_{\text{trans}} \approx \frac{1}{2}M_{\text{total}}v_{\text{cm}}^2 = \frac{P_{\text{total}}^2}{2M}$ nonrelativistically

$\vec{P}_{\text{total}} = M_{\text{total}}\vec{v}_{\text{cm}}$ nonrelativistically $\quad K_{\text{rotation}} = \frac{L_{\text{rot}}^2}{2I} = \frac{1}{2}I\omega^2$

$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + ...$ $\quad \vec{L}_A = \vec{r}_A \times \vec{p}$ for a point particle $\quad \vec{\tau}_A = \vec{r}_A \times \vec{F}$

$\vec{L}_A = \vec{L}_{\text{trans},A} + \vec{L}_{\text{rot}} = (\vec{R}_{\text{cm,A}} \times \vec{P}_{\text{total}}) + \vec{L}_{\text{rot}}$ for a multiparticle system $\quad \vec{L}_{\text{rot}} = I\vec{\omega}$

EVALUATING GENERAL PHYSICAL QUANTITIES

$E^2 - (pc)^2 = (mc^2)^2$; $E = Pc$ for massless photon $\quad \omega = 2\pi/T \; v = \omega r$

$\Omega = \frac{(q+N-1)!}{q!(N-1)!}$ $\quad S = k\ln\Omega$ $\quad \frac{1}{T} = \frac{dS}{dE}$

EVALUATING SPECIFIC PHYSICAL QUANTITIES

$|\vec{F}_{\text{grav}}| = G\frac{m_1m_2}{|\vec{r}|^2}$; can be approximately mg $\quad U_{\text{grav}} = \frac{-GMm}{|\vec{r}|}$; change can be approximately $\Delta(mgy)$

$|\vec{F}_{\text{electric}}| = \frac{1}{4\pi\varepsilon_0}\frac{|Q_1Q_2|}{|\vec{r}|^2}$ $\quad U_{\text{electric}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{|\vec{r}|}$

$|\vec{F}_{\text{spring}}| = k_s|s|$, opposite the stretch $\quad U_{\text{spring}} = \frac{1}{2}k_ss^2 + U_0$

$\vec{F} = < -\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z} >$ $\quad$ Power = energy/time (watts = joules/second)

$\Delta E_{\text{thermal}} = mC\Delta T$ $\quad$ air resistance: $\vec{F} = -\frac{1}{2}C\rho Av^2\hat{v}$ $\quad$ sliding friction: $f \le \mu F_N$

oscillator: $E_N = N\left(\hbar\sqrt{\frac{k_s}{m}}\right)$; classically $\omega = \sqrt{\frac{k_s}{m}}$ $\quad$ hydrogen atom: $E_N = -\frac{13.6 \text{ eV}}{N^2}$ $(N = 1, 2, ...)$

$Y = \frac{F/A}{\Delta L/L} = \frac{k_{s,\text{ interatomic}}}{d_{\text{atomic}}}$ $\quad v_{\text{sound}} = d_{\text{atomic}}\sqrt{\frac{k_{s,\text{ interatomic}}}{m}}$

Circular motion at constant speed: $\frac{d\vec{p}}{dt} = -\frac{m\omega^2}{\sqrt{1-|\vec{v}|^2/c^2}}\vec{r}$, $\frac{d\vec{p}}{dt} \approx -m\omega^2\vec{r}$ for $|\vec{v}| << c$

where $\omega = \frac{d\theta}{dt} = \frac{2\pi}{T}$, $|\vec{v}| = \frac{2\pi|\vec{r}|}{T} = \omega|\vec{r}|$

$$x = A\cos\left(\sqrt{\frac{k_s}{m}}\,t\right) \qquad \omega = \sqrt{\frac{k_s}{m}} = 2\pi f = \frac{2\pi}{T}$$

sliding friction: $f \le \mu F_N$

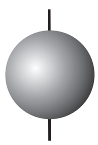
oscillator: $E_N = N\left(\hbar\sqrt{\frac{k_s}{m}}\right) + E_0$; classically $\omega = \sqrt{\frac{k_s}{m}}$ hydrogen atom: $E_N = -\frac{13.6 \text{ eV}}{N^2}$ $(N = 1, 2, \ldots.)$

$C = \frac{\Delta E}{\Delta T}$ (high-temperature limit: $\frac{1}{2}k$ per quadratic energy term; solid is $3k$/atom)

A component of angular momentum is an integer or half-integer multiple of $\hbar$.

The square magnitude of angular momentum has quantized values $L^2 = l(l+1)\hbar^2$,
where l has integer (0, 1, 2, ...) or half-integer values (1/2, 3/2, 5/2, ...).

Sphere

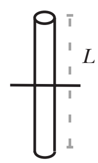


$I = \frac{2}{5}MR^2$

Cylinder or disk

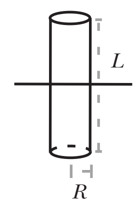
$I = \frac{1}{2}MR^2$

Thin rod (about axis shown)



$I = \frac{1}{12}ML^2$

Solid cylinder (about axis shown)



$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

CONSTANTS

$G = 6.7\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$ $g = 9.8$ N/kg $c = 3\times10^8$ m/s

$h = 6.6\times10^{-34}$ J·s $\hbar = \frac{h}{2\pi} = 1.05\times10^{-34}$ J·s $k = 1.4\times10^{-23}$ J/K

$\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$ proton or positron charge: $e = 1.6\times10^{-19}$ coulomb

1 eV $= 1.6\times10^{-19}$ J Avogadro's number $= 6\times10^{23}$ molecules/mole

$m_{\text{proton}} \approx m_{\text{neutron}} \approx m_{\text{hydrogen atom}} = \frac{1\times10^{-3}\text{ kg/mole}}{6\times10^{23}\text{ atoms/mole}} = 1.7\times10^{-27}$ kg $m_{\text{electron}} = 9\times10^{-31}$ kg

$M_{\text{Earth}} = 6\times10^{24}$ kg Radius of Earth $= 6.4\times10^6$ m

$M_{\text{Moon}} = 7\times10^{22}$ kg Distance from Earth to Moon $= 4\times10^8$ m

$M_{\text{Sun}} = 2\times10^{30}$ kg Distance from Earth to Sun $= 1.5\times10^{11}$ m

Typical atomic radius $r \approx 10^{-10}$ m Proton radius $r \approx 10^{-15}$ m, $R_{\text{nucleus}} \approx (1.3\times10^{-15}\text{ m})A^{1/3}$

Heat capacity of water = 4.2 (J/K)/gram. Energy of visible light is from about 1.8 eV to about 3.1 eV.

Effective stiffness of interatomic bond in aluminum is 16 N/m, lead is 5 N/m.

Mass of one mole of *atoms:*

carbon 12 grams (0.012 kg)

nitrogen 14 grams (0.014 kg)

oxygen 16 grams (0.016 kg)

aluminum 27 grams (0.027 kg)

iron 56 grams (0.056 kg)

lead 207 grams (0.207 kg)

CONVERSION FACTORS

1 kg near the Earth's surface weighs 2.2 pounds.

1 inch = 2.5 cm

1 foot = 30 cm