Calculus I: More Optimization

1) **Bunnies for profit.** Suppose that it costs 50 cents to manufacture one toy rabbit, and that each toy rabbit in a shipment of x toy rabbits sells for $100 - \frac{x}{200}$ cents.

(a) Write formulas for the cost $C(x)$, the revenue $R(x)$, and the profit $R(x)$ for a shipment of x toy rabbits. Cost $C(x)$ is the cost of manufacturing x toy rabbits. Revenue is the number of rabbits sold, times the price charged for each rabbit. Profit is the difference between revenue and cost ($P = R - C$).

(b) Graph the profit function, then use the graph to find the maximum profit and the number of toy rabbits that should be manufactured and shipped in order to attain it.

(c) Use the graph to find the maximum profit and the number of toy rabbits that should be manufactured and shipped if we may manufacture and ship no more than 1000 toy rabbits.

2) **Average daily temperatures in Alaska.** (Adapted from *Calculus,* Finney/Thomas) The sine function

$$T = 37\sin\left(\frac{2\pi}{365}(x-101)\right)+25$$

approximates the normal mean air temperature, in °F, in Fairbanks, Alaska, on day x of a typical 365-day year. Graph the function T over a one-year period. We shall hereafter refer to normal mean air temperature as "temperature." Remember that February has 28 days; April, June, September, and November have 30 days; and the remaining months have 31 days.

(a) What is the temperature on February 15?
(b) What is the highest temperature and on what day is the temperature the highest?
(c) What is the lowest temperature and on what day is the temperature the lowest?
(d) On what day is the temperature increasing the fastest and how fast is it increasing?
(e) On what day is the temperature decreasing the fastest and how fast is it decreasing?

3) **Air traffic control.** (Adapted from *The Calculus Reader,* Duke University) It is a calm winter day in southern California at the Redlands air traffic control (ATC) radar installation, except that there are some small, locally intense thunderstorms passing through the general area. Only two aircraft are in the vicinity of the station: American Flight 1003 from Seattle to Mexico City is approaching from the north-northwest and United Flight 366 from Honolulu to New York is approaching from the west-southwest. Both are on paths that will take them directly over the radar tower. There is plenty of time for the controllers to adjust the flight paths to insure a safe separation of the aircraft.

3) **Air traffic control.** (continued)

Suddenly lightning strikes a power substation five miles away, knocking out the power to the ATC installation. There is, of course, a gasoline powered auxiliary generator, but it fails to start. In desperation, a mechanic rushes outside and kicks the generator; it sputters to life. As the radar screen flickers on, the controllers find that both flights are at 33,000 feet. The American flight is 36 nautical miles from the tower and approaching it at a rate of 410 nautical miles per hour (knots), while the United flight is 41 nautical miles from the tower and approaching it at a rate of 455 knots.

(a) If the planes maintain their speed and direction, will they crash directly over the tower? If not, which plane will pass directly over the tower first? How far from the tower will the other plane be at this time? How many seconds later will the other plane pass over the tower? How far past the tower will the first plane be at this time?

(b) Even if you do not need it for part (a), it will be helpful in part (c) to have a formula for $x(t)$, the American flight's distance from the tower, in nautical miles, t hours after the controllers' screens flicker back on again, and also for $y(t)$, the United flight's distance from the tower. *Hints:* How far has the American flight traveled after 1 hour? After 2 hours? How far is it from the tower after 1 hour? After 2 hours? It may be helpful to allow negative quantities.

If you haven't already done so, draw a picture of the situation, perhaps a top view, that shows $x(t)$ and $y(t)$.

Big hint: What angle do the flight paths of the planes make with one another?

(c) The planes may not be closest to one another when they are directly over the tower. Assuming that the planes maintain their speed and direction, how close will they come to each other? Will they crash (perhaps not directly over the tower)? Will they violate the FAA's minimum separation requirement of 5 nautical miles? How many minutes do the controllers have before the time of closest approach? How far from the tower is each plane at the time when the planes are closest to one another?

Hints: The graph of your function $D(t)$ – or of $\left(D(t)\right)^2$, if you prefer – giving the distance between the two planes should look like one of the two graphs below. <u>Use</u> <u>calculus</u> to determine the shortest distance between the two planes and the time at which it occurs.