# Calculus I: Optimization without Calculus

## Quadratic functions and parabolas

The graph of the equation $y = ax^2 + bx + c$, $a \neq 0$, is a parabola.
If $a > 0$, the parabola opens upward (it's concave up).
If $a < 0$, the parabola opens downward (it's concave down).
Notice that $y'' = 2a$, so what we learned in pre-calculus classes is consistent with what we have learned in calculus class.

The vertex of the parabola has *x*-coordinate $x = \dfrac{-b}{2a}$.

Notice that $y' = 2ax + b = 0$ for $x = \dfrac{-b}{2a}$, so, again, what we learned in pre-calculus classes is consistent with what we have learned in calculus class.

## Sine and cosine functions

The largest value of the cosine and sine function is 1 and the smallest value is $-1$.
This means that $-1 \leq \cos(\theta) \leq 1$ and $-1 \leq \sin(\theta) \leq 1$ for every angle (or argument) $\theta$.
It also means, for instance, that the largest value of $f(x) = 17\cos(4x+7) - 12$ is $17 \cdot 1 - 12 = 5$ and the smallest value of $f(x) = 17\cos(4x+7) - 12$ is $17(-1) - 12 = -29$, and in fact that $-29 \leq 17\cos(4x+7) - 12 \leq 5$. Graph the function and see!

For what values of *x* do we have $f(x) = -29$? The answer is infinitely many values of *x*, but let's find just one! Since we get $f(x) = -29$ by making $f(x) = 17\cos(4x+7) - 12 = 17(-1) - 12 = -29$, we want to find *x* that makes

$$\cos(4x+7) = -1.$$

Since $\cos(\pi) = -1$, if we just make $4x + 7 = \pi$, we'll have it.

You can check that $x = \dfrac{\pi - 7}{4}$ results in $4x + 7 = \pi$, hence in

$$\cos(4x+7) = \cos(\pi) = -1, \text{ hence in}$$

$$f(x) = 17\cos(4x+7) - 12 = 17(-1) - 12 = -29.$$

Compute $f\left(\dfrac{\pi - 7}{4}\right)$ by hand. You should get $-29$.

Therefore, the point $\left(\dfrac{\pi - 7}{4}, -29\right)$ is a global minimum point on the graph of $f(x)$.

Other values of the cosine and sine functions that will come in handy are:

$$\cos\left(-\frac{\pi}{2}\right) = 0 = \cos\left(\frac{\pi}{2}\right) = \cos\left(\frac{3\pi}{2}\right), \quad \cos(0) = 1 = \cos(2\pi),$$

$$\sin\left(-\frac{\pi}{2}\right) = -1 = \sin\left(\frac{3\pi}{2}\right), \quad \sin(0) = 0 = \sin(\pi) = \sin(2\pi), \text{ and } \sin\left(\frac{\pi}{2}\right) = 1.$$