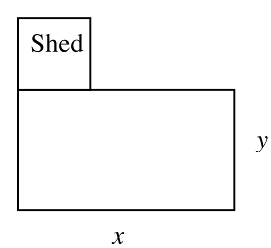
Calculus I: Optimization with Calculus

In the following problems, apply the Candidates Test if possible. If the Candidates Test does not apply, use the Second Derivative Test.

1. A farmer has 100 feet of fencing with which to build a rectangular enclosure. Use calculus to find the dimensions x and y of the enclosure resulting in maximum area. Begin by finding a formula and a domain for the area $A(x)$.

   $A(x) =$ ____________________, _____ $\leq x \leq$ _____.

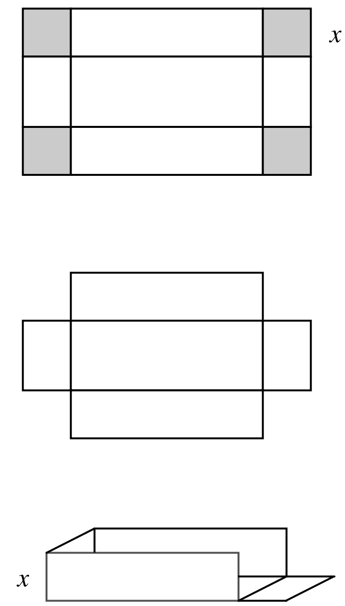
2. A farmer must use 100 feet of fencing and an existing 20 ft side of a shed to enclose a rectangular pen. Use calculus to find the dimensions x and y of the enclosure resulting in maximum area. Begin by finding a formula and a domain for the area $A(x)$. *Hints:* Why is $x = 0$ ft impossible? What about $x = 14$ ft? $x = 24$ ft? What value of x corresponds to $y = 0$ ft?

   $A(x) =$ ____________________, _____ $\leq x \leq$ _____.

3. A manufacturer wishes to build an open-top rectangular box by cutting out the corners and folding up the sides of a 5 inch by 7 inch piece of cardboard, as shown. Use calculus to find the dimensions (length, width, and height x) of the box with maximum volume. Begin by finding a formula and a domain for the volume $V(x)$. *Hint:* If $x = 1$ inch, what would the dimensions of the box be? (Build the box!) Why is $x = 3$ inches impossible?

$V(x) =$ ____________________, _____ $\le x \le$ _____.



4. A manufacturer wishes to build an open-top cylindrical container that holds exactly 40 in^3 of stuff. Use calculus to find the dimensions (radius r and height h) of the container constructed with the <u>least</u> amount of material. (Hint: Read and follow Section 4.5, Example 1.) Begin by finding a formula for the surface area $A(r)$. Notice that the Candidates Test does not apply, but the Second Derivative Test does!

Surface Area: $A(r) =$ ______________________, $r > 0$