Calculus I: Tests for Global Extrema

We review one test and introduce two new tests for global extreme values.

Candidates Test for Global Extrema on a Closed Interval: To find the global minimum value and the global maximum value of a continuous function *f* defined on a closed interval $a \le x \le b$:

Step 1. First find and list all of the critical numbers $x = c$, along with the endpoints $x = a$ and $x = b$.

Step 2. Then compare the values of the function at all of these numbers. The smallest value is the global minimum value of the function *f* and the largest value is the global maximum value of *f*.

1) Fill in the missing test.

First Derivative Test for Global Minimum Value: If $x = c$ is a critical number for a continuous function *f* and $f'(x) \le 0$ for every x in the domain of *f* with $x < c$ and $f'(x) \ge 0$ for every x in the domain of *f* with $x > c$, then the function *f* has the global minimum value $f(c)$ at $x = c$ and $(c, f(c))$ is a global minimum point on the graph of *f*.

First Derivative Test for Global Maximum Value: If $x = c$ is a critical number for a continuous function *f* and …

2) Fill in the missing test.

Second Derivative Test for Global Minimum Value: If $f'(c) = 0$ and $f''(x) > 0$ for <u>every</u> x in the domain of the continuous function *f*, then *f* has the global minimum value $f(c)$ at $x = c$ and $(c, f(c))$ is a global minimum point on the graph of *f*.

Second Derivative Test for Global Maximum Value: If $f'(c) = 0$ and …

In Exercises 3 – 11, use the Candidates Test, if possible, to find the global minimum and maximum values and the global minimum and maximum points. If this is impossible, say why and use the Second Derivative Test, if possible, to find either a global minimum or maximum value and point. If this is impossible, say why and use the First Derivative Test to find either a global minimum or maximum value and point.

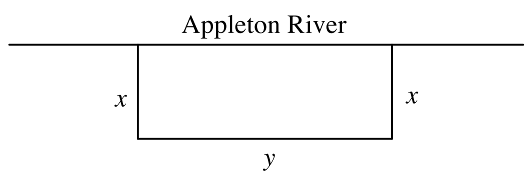
3) $f(x) = x + \dfrac{1}{x},\ x > 0$ The Candidates Test does not apply because …

4) $f(x) = \dfrac{x}{e^x}$ The Candidates Test does not apply because …

The Second Derivative Test does not apply because …

5) $f(x) = x^4 - 4x^3 + 20$

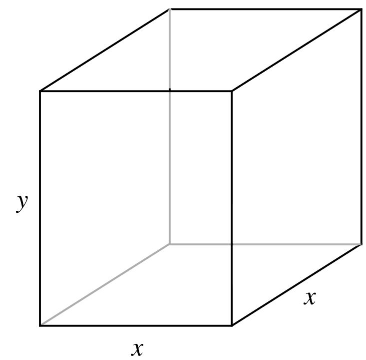
6) $A(x) =$ ____________________, $0 \le x \le L/2$ (explain!), where A is the area of a rectangular field enclosed by L feet ($L > 0$) of fencing on three sides and a straight river on the remaining side, as shown.



7) $L(x) =$ ____________________, $x > 0$, where L is the length of fence, in feet, enclosing on three sides a rectangular field of area 1250 ft^2, as shown in Exercise 6.

8) $L(x) =$ ____________________, $x > 0$, where L is the length of fence, in feet, enclosing on three sides a rectangular field of area A ft^2 ($A > 0$), as shown in Exercise 6.

9) $V(x) =$ ______________________, $0 \le x \le 5\sqrt{3}$ (explain!), where V is the volume of a closed-top rectangular box with a square base constructed from 150 ft^2 of plywood, as shown.



10) $V(x) =$ ______________________, $0 \le x \le$ _________, where V is the volume of a closed-top rectangular box with a square base constructed from A ft^2 $(A > 0)$ of plywood, as shown in Exercise 9.

11) $M(r) = 2\pi r^2 + \dfrac{80}{r}$, $r > 0$, from Example 1, pages 198-199. You may use the critical number found there, $r = \sqrt[3]{\dfrac{20}{\pi}} \approx 1.85$ inches, corresponding to $h \approx 3.7$ inches (note that $h = 2r$) and $M \approx 64.7$ in^2. All that's left to do is to apply the Second Derivative Test for the Global Minimum Value.