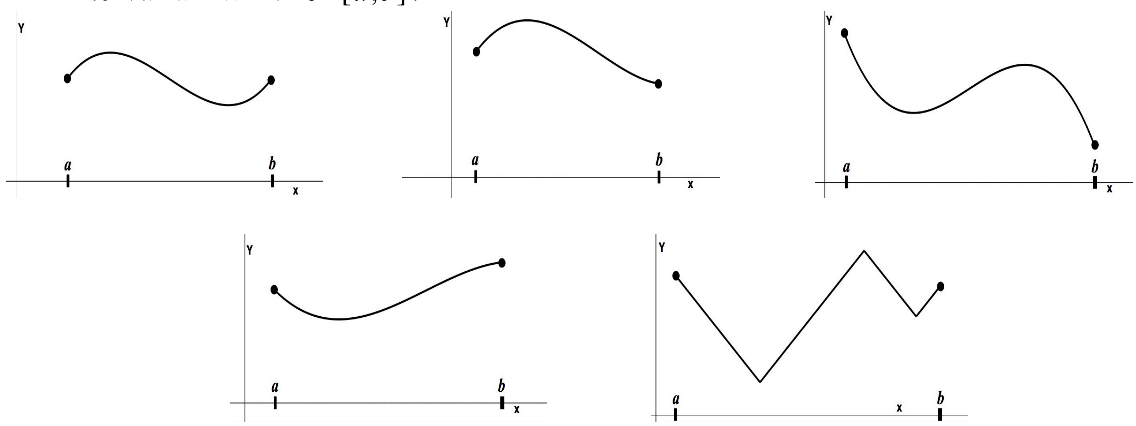
Calculus I: Candidates Test for Global Extrema

1) If a continuous function *f* is defined on a finite, closed interval, such as $-1 \le x \le 4$ or $[-1,4]$, or, more generally, $a \le x \le b$ or $[a,b]$, then *f* always has a global minimum value and a global maximum value on that interval. For instance, in the example at the top of the next page, $f(x) = x^2 - 3$ has global minimum value $f(0) = -3$ and global maximum value $f(4) = 13$ on the interval $-1 \le x \le 4$.
(*Note:* Mathematicians disagree over whether or not $f(-1) = -2$ should be considered a local maximum value for *f*. I would say that it is!)

Here are five possible graphs of a continuous function *f* defined on a closed interval $a \le x \le b$ or $[a,b]$.



(a) On each of the graphs above, mark the global minimum point and the global maximum point.

(b) In each of the graphs above, the global minimum point and the global maximum point occur at one of two types of "points," either a ________________ point (or number) or an ____________ point. (*Hints:* Three of the five global maximum points occur at ______________ points or numbers. The other two occur at __________ points of the interval $[a,b]$. Three of the five global minimum points occur at _____________ points, while the other two occur at __________ points of the interval $[a,b]$.)

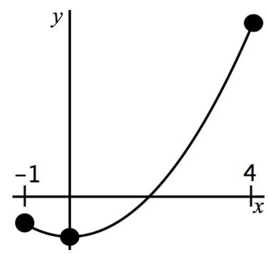
Therefore, to find the global minimum value and the global maximum value of a function *f* defined on a closed interval $a \le x \le b$:

Step 1. First find and list all of the "points" or numbers $x = c$ of these two types (the types listed in the blanks) that are in the interval $[a,b]$.

Step 2. Then compare the values $f(c)$ of the function at all of these numbers. The smallest value is the global minimum value of the function *f* and the largest value is the global maximum value of *f*.

This two-step procedure is called the **Candidates Test for Global Extrema** (global minimum and maximum values) of a function *f* on a closed interval $a \le x \le b$ or $[a,b]$.

Example. For $f(x) = x^2 - 3$ on the interval $-1 \le x \le 4$, the result of Step 1 of the **Candidates Test for Global Extrema** would be the list $x = -1, x = 0, x = 4$. In Step 2, we would compare $f(-1) = -2$, $f(0) = -3$, and $f(4) = 13$, and select $f(0) = -3$ as the global minimum value and $f(4) = 13$ as the global maximum value.



2) Apply the **Candidates Test for Global Extrema** in order to find the global minimum value and global maximum value of the function $f(x) = x^4 - 4x^3 + 20$ on each interval $a \le x \le b$.

$-1 \le x \le 4$ Step 1.

Step 2.

Global minimum value: Global maximum value:

Global minimum point: Global maximum point:

$-1 \le x \le 2$ Step 1.

Step 2.

Global minimum value: Global maximum value:

Global minimum point: Global maximum point:

$0 \le x \le 4$ Step 1.

Step 2.

Global minimum value: Global maximum value:

Global minimum point: Global maximum points:

Note that there is just one global maximum value, but two global maximum points.

In Exercises 3-8, use the **Candidates Test for Global Extrema** to identify the global minimum and maximum values and points of the function *f* on the closed interval $a \le x \le b$ or $[a,b]$.

3) $f(x) = x^3 - 6x^2 + 9x + 5$ on the interval $0 \le x \le 4$

Step 1.

Step 2.

Global minimum value: Global maximum value:

Global minimum points: Global maximum points:

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4) $f(x) = x + \frac{7}{x}$ on the interval $1 \le x \le 3$

Step 1.

Step 2.

Global minimum value: Global maximum value:

Global minimum point: Global maximum point:

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5) $f(x) = x + \frac{7}{x}$ on the interval $1 \le x \le 2$

Step 1.

Step 2.

Global minimum value: Global maximum value:
Global minimum point: Global maximum point:

6) $f(x) = x + 2\sin(x)$ on the interval $0 \le x \le 2\pi$

Step 1.

Step 2.

Global minimum value: Global maximum value:

Global minimum point: Global maximum point:

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7) $s(r) = a(R - r)r^2$ on the interval $0 \le x \le R$, where s is the speed of the air leaving your windpipe as you cough, r is the current radius of your windpipe, and R is the resting radius of your windpipe. In this formula, a and R are positive constant numbers, while s and r are variables (s is like y and r is like x). *Hint:* Multiply out the right-hand side before you take the derivative $s'(r)$ of the function.

Step 1.

Step 2.

Global minimum value: Global maximum value:

Global minimum points: Global maximum point:

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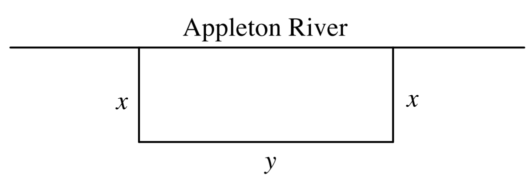
8) $A(x) =$ ____________________ on the interval $0 \le x \le 50$, where A is the area of a rectangular field enclosed by 100 feet of fencing on three sides and a straight river on the remaining side, as shown.



Step 1.

Step 2.

Global minimum value: Global maximum value:

Global minimum points: Global maximum point: