Calculus I:
Review of Tests for Local Extrema and Concavity

For each function f:

(a) Find all critical points (or critical numbers) of f.

(b) Use the First Derivative Test for Local Extrema to classify each critical point as corresponding to a local minimum point or a local maximum point or neither. Use your sign chart for the first derivative $f'(x)$ to determine the intervals on which the graph of f increases and decreases.

(c) Use the Second Derivative Test for Local Extrema to classify each critical point $x = c$ for which $f'(c) = 0$ as corresponding to a local minimum point or a local maximum point. (*Note:* Part (c) should confirm some of your results from part (b).)

(d) Use the Second Derivative Test for Concavity to determine whether each value $x = c$ for which $f''(c) = 0$ or $f''(c)$ is not defined is an inflection point or not. Use a sign chart for $f''(x)$ to determine the intervals on which the graph of f is concave up and concave down.

(e) Use the results of parts (a) through (d) to sketch the graph of the function f from approximately $x = -3$ to $x = 3$. It will be helpful to find the y-coordinate of each point you considered in parts (a) through (d).

1) $f(x) = \dfrac{x}{e^x}$ (*Hint:* Look for a horizontal asymptote.)

2) $f(x) = x + \dfrac{1}{x}$, $x \neq 0$ (*Hint:* Look for vertical and oblique asymptotes.)