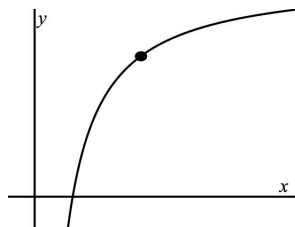
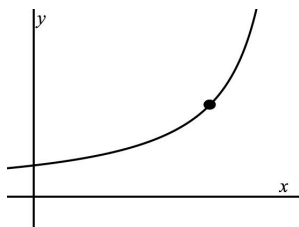


Calculus I: Concavity and the First and Second Derivatives

- 1) For each graph below, if the slope at the point shown is 1, what can you say about the slope at any point on the graph to the left of this point? At any point to the right of this point? As you read the graph from left to right, is the *slope* of the graph increasing or decreasing?



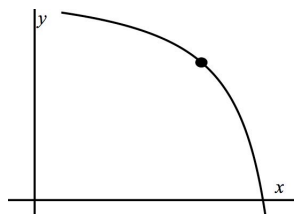
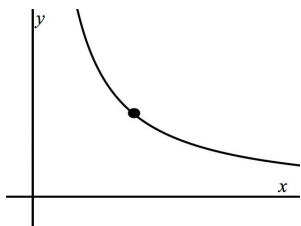
Assuming each graph is the graph of a function $f(x)$, as you read the graph from left to right, is $f'(x)$ increasing or decreasing? (Why is this a silly question?)

As a result, is $f''(x)$ positive or negative? (Read the hints!)

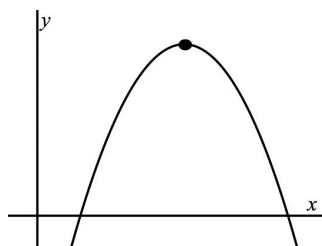
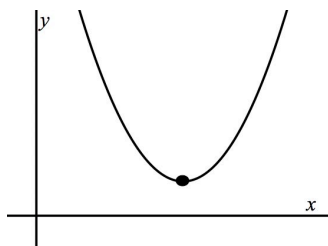
Hints: If the function f increases, then its derivative f' is positive.
If the function g increases, then its derivative g' is positive.
If the function f' increases, then its derivative f'' is positive.

Is the graph of f *concave up* (curved upward) or *concave down* (curved downward)?

- 2) Repeat Exercise 1 for the two graphs below. The slope at the point shown is -1 .



- 3) Repeat Exercise 1 for the two graphs below. The slope at the point shown is 0.



- 4) Based on your answers to Exercises 1–3, write two rules relating the concavity of the graph of f (concave up or concave down) to the increasing or decreasing behavior of the first derivative f' (increases or decreases).

Rule A. The graph of f is concave up if and only if f' _____.

Rule B. _____.

- 5) Based on your answers to Exercises 1–3 (or 4), write two rules relating the concavity of the graph of f (concave up or concave down) to the sign of the second derivative f'' (positive or negative).

Rule 1. The graph of f is concave up if and only if f'' is _____.

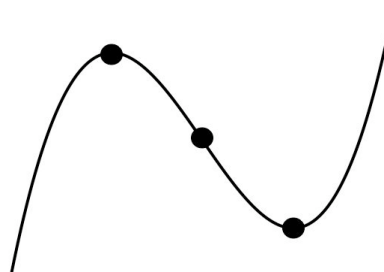
Rule 2. _____.

- 6) A point at which a graph changes concavity is called an *inflection point*. Your rules from Exercise 5 leave only two possibilities for such a point.

Rule 3. If the point $(x, f(x))$ is an inflection point, then $f''(x) =$ _____ or $f''(x)$ does not exist.

- 7) The function $f(x) = 2x^3 - 3x^2 - 12x + 8$ is a cubic function (polynomial of degree 3) with leading coefficient 2. Since 2 is positive, the general shape of the graph of f is as shown below. Do not graph this function on your calculator – at least not yet!

Notice that the graph has one local maximum point, one inflection point, and one local minimum point, and that all three of these points are marked on the graph. Use $f''(x)$ and Rule 3 to find the x -coordinate of the inflection point. What is the slope of the graph at the remaining two points? Use $f'(x)$ to find the x -coordinates of these two



points. Then find the y -coordinate of each point and the y -intercept of the graph, and sketch in x - and y -axes. Finally, check your work using your graphing calculator.

- 8) Repeat Exercise 7 for the function $f(x) = x^4 - 8x^2 + 5$. Since f is a quartic function (degree 4 polynomial), what general shape would you expect its graph to have? Use $f'(x)$ and $f''(x)$ to find the x - and y -coordinates of the three local maximum and minimum points and to find the two inflection points. Then sketch in x - and y -axes and, finally, check your work using your graphing calculator.