

## Calculus I: World Population Growth

**Goals:** To predict world population using the exponential and logistic models, and to compare the results.

United Nations demographers believe that the human population of the world reached 6 billion persons sometime during October of 1999. Assume world population was exactly 6 billion persons at the start of the day on January 1, 2000, and was increasing at a rate of 120 million people per year at that moment. Use this information to answer the following questions.

- 1) Use the exponential model for population growth to find a function  $P(t)$  giving the population of the world (in billions of persons)  $t$  years after 2000. Then use  $P(t)$  to predict world population in the years 2010, 2100, and 2500. In what year will the year 2000 population of 6 billion persons have doubled? What is the *doubling time* for the world's population?  
[Hints: Be sure to use the differential equation for exponential growth,  $P' = kP$  billion persons per year, to get all of the information you need for your formula  $P(t) = P_0 e^{kt}$  billion persons.  $P(0) = ?$   $P'(0) = ?$  ]
- 2) The total land surface area of the Earth is about  $1.8 \times 10^{15}$  square feet. According to the exponential model, how many square feet per person will there be in each of the years 2010, 2100, and 2500?
- 3) For a more realistic picture of long-term world population growth, consider the logistic model for world population growth. In this model, the rate of population increase is proportional both to the current population and to the number of people for which there is room left on the Earth. Although most demographers estimate the earth's carrying capacity to be much lower, some demographers believe that the very largest population the planet can support is 30 billion people. Use this very liberal estimate of Earth's carrying capacity to write a function  $Y(t)$  giving the population of the world (in billions)  $t$  years after 2000. Then use  $Y(t)$  to predict the world's population in the years 2010, 2100, and 2500, and compare your predictions with those given by the exponential model.

[Hints: Be sure to use the differential equation for logistic growth,  $Y' = kY \left(1 - \frac{Y}{C}\right)$  billion persons per year, where  $C$  is the earth's carrying capacity, to get all of the information you need for your formula  $Y(t) = \frac{C}{1 + Ae^{-kt}}$  billion persons, where  $A = \frac{C - Y_0}{Y_0}$ .  $Y(0) = ?$   $Y'(0) = ?$  ]

- 4) According to the logistic model, what will the world population be in the year after 2000 in which the world's population is increasing most rapidly?

Generalize this result to all logistic models.

[*Hints:* Use the differential equation for logistic growth, rather than its solution, to answer this question. Notice that you will solve for  $Y$ , not  $t$ .