Calculus I: Population and Food

In "An Essay on the Principle of Population," written in 1798, the British economist Thomas Robert Malthus argued that food supplies grow at a constant rate, while human populations naturally grow at a constant *per capita* rate (that is, they grow exponentially). He therefore predicted that human populations inevitably would run out of food.

1) Let $t = 0$ correspond to the year 1798. Malthus estimated that in 1798 the population of Great Britain was about 7 million people, and the per capita growth rate was about 28 persons per year per thousand persons. This means that if P denotes the population of Great Britain (in millions) t years after 1798, then P is the solution to the initial value problem

$$P' = 0.028P, \qquad P(0) = 7$$

Write the formula for the solution $P(t)$ of this initial value problem.

$$P(t) =$$

2) Malthus assumed that in 1798 the population of Great Britain was adequately nourished, so he estimated that in that year the food supply was 7 million units of food, where one unit of food feeds one person for one year. Malthus' data suggested to him that the food supply in Great Britain was growing at a rate of about 0.28 million units per year. This means that if F denotes the number of units of food in Great Britain (in millions) t years after 1798, then F is the solution to the initial value problem

$$F' = 0.28, \qquad F(0) = 7$$

Write the formula for the solution $F(t)$ of this initial value problem.

$$F(t) =$$

3) Use your formulas from parts (a) and (b) to compute the population of Great Britain in 1848 and to compute the number of units of food in Great Britain in 1848. If the food is divided evenly among the citizens of Great Britain, will everyone have enough to eat? How much will each person have?

Source: *Calculus in Context,* by J. Callahan, K. Hoffman, *et al.,* W.H. Freeman, 1995