## Not a Group!

The following sets are not groups under the given operations. Why not?

1. $\mathbf{Z}_{4}=\{0,1,2,3\}$ under ordinary addition
2. Positive integers $\mathbf{Z}^{+}=\{1,2,3,4, \ldots\}$ under ordinary multiplication
3. $M=\left\{\left.\left[\begin{array}{ll}a & b \\ c & d\end{array}\right] \right\rvert\, a, b, c, d \in R\right\}$ under matrix multiplication
4. $\mathbf{Z}_{6} *=\{1,2,3,4,5\}$ under multiplication modulo 6
5. $S=\{1,2,3,4,5,6\}$ under the binary operation * given by the table

| $\boldsymbol{*}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | 1 | 2 | 3 | 4 | 5 | 6 |
| $\mathbf{2}$ | 2 | 3 | 4 | 5 | 6 | 1 |
| $\mathbf{3}$ | 3 | 4 | 1 | 6 | 2 | 5 |
| $\mathbf{4}$ | 4 | 5 | 6 | 1 | 3 | 2 |
| $\mathbf{5}$ | 5 | 6 | 2 | 3 | 1 | 4 |
| $\mathbf{6}$ | 6 | 1 | 5 | 2 | 4 | 3 |

