More Examples of Groups

Write a group table (Cayley table) for each of the first five groups listed below.

1. $Z_4 = \{0, 1, 2, 3\}$ under addition modulo 4

2. $U = \{1, -1, i, -i\}$ under ordinary multiplication. Here, i is the square root of -1.

3. $M = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \right\}$ under matrix multiplication

4. $U(5) = Z_5{}^* = \{1, 2, 3, 4\}$ under multiplication modulo 5

5. $Z_6 = \{0, 1, 2, 3, 4, 5\}$ under addition modulo 6

6. $Z_n = \{0, 1, 2, 3, \ldots, n-1\}$ under addition modulo n

7. Integers $\mathbf{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ under ordinary addition

8. $\mathbf{7Z} = \{\ldots, -21, -14, -7, 0, 7, 14, 21, \ldots\}$ under ordinary addition

9. Nonzero rational numbers $\mathbf{Q}{*}$ under ordinary multiplication

10. Positive real numbers $\mathbf{R}^+$ under ordinary multiplication