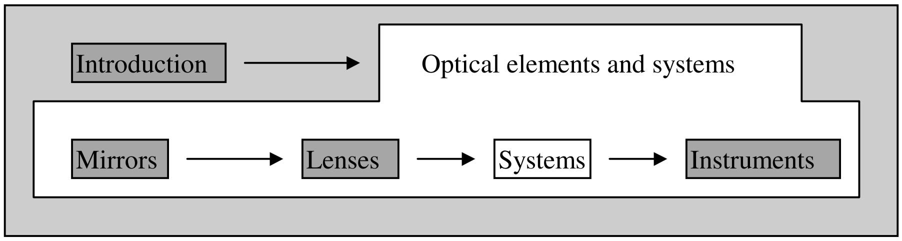
O4

Multiple Element Optical Systems



O4.1 Introduction

In this section we will be presented with a series of examples to illustrate and reinforce what we have learned. Specifically, we will take the skills learned in Chapters O2 and O3 and apply them to systems which have two optical elements (lenses and/or mirrors). There is less commentary as to why particular signs are chosen, but we will need to be honest and be sure that we understand why the values chosen in the numerical calculations have the values and signs that they do.

O4.2 A FINAL REAL IMAGE FROM TWO CONVERGING LENSES with a real image formed between the two lenses.

O4.3 DIVERGING AND CONVERGING LENSES with a virtual image formed by the first lens and a real image formed by the second lens.

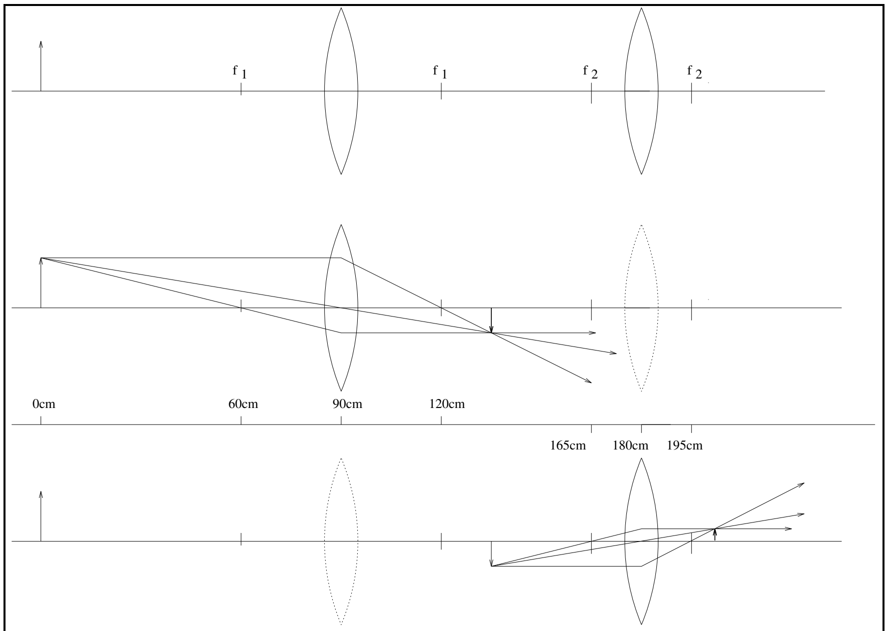
O4.4 A VIRTUAL OBJECT results from a real image in a system made up of two converging lenses.

O4.5 ONE CONVERGING LENS AND A CONCAVE MIRROR which form a three part problem because the rays travel through the lens, reflect, and pass through the lens a second time.

O4.2 A real image from two converging lenses.

To solve a multiple component problem, the "easiest" method is to break the problem into parts. Start with the object and the optical component closest to that object. Determine the location of the image produced by this lens (or mirror), and then use this image as the object for the next component that the light encounters. Repeat this process until the light rays from the final image encounter no additional optical components as they move off to infinity. We will step through an example of this process in the following example.

> Two converging lenses are placed 90 cm apart. The 30 cm focal length lens is to the left of the 15 cm focal length lens. A 15 cm tall arrow is placed 90 cm to the left of the 30 cm focal length lens. Where does the final image appear, and how tall is it?



O4.1 Two converging lenses with a real final image.

The first step is to draw the lenses and object in a scale drawing (Figure O4.1, top part). This scale drawing should include the focal points for the optical elements as well as their location. Now, start with the object and the lens that light leaving the object will encounter first. (In this case, the first element encountered is the left lens.) Locate the image of the arrow after the light passes through the left lens. The middle section of Figure O4.1 shows the ray diagram for this image. Mathematically, the location and magnification are found using:

$$\frac{1}{90\,cm}+\frac{1}{S'}=\frac{1}{30\ cm} \Rightarrow S'=45\ cm \text{ and } M_1=-\left(45\ cm \big/ 90\ cm\right)=-0.5\,.$$

In summary, based on the sign conventions, the positive value for *S’* indicates that the image is real and on the opposite side of the lens from the object. The image is (15 cm)(-0.5)=-7.5 cm tall where the minus sign tells us that the image is inverted (upside down since the original object was erect).

Now, this image becomes the object for the second lens as the light rays from image 1 continue to the right and pass through the second lens. The lower part of Figure O4.1 shows the ray diagram for the second (and final) image. Mathematically, we need to know the location of the “object” relative to the right lens. Since the first image is 45 cm behind the left lens and the two lenses are positioned 90 cm apart, the “object” is 45 cm in front of the right lens. This allows us to find the location and height of the final image with:

$$\frac{1}{45\ cm}+\frac{1}{S'}=\frac{1}{15\ cm} \Rightarrow S'=22.5\ cm \text{ and}$$

$$M_2=-\left(22.5\ cm \big/ 45\ cm\right)=-0.5\,.$$

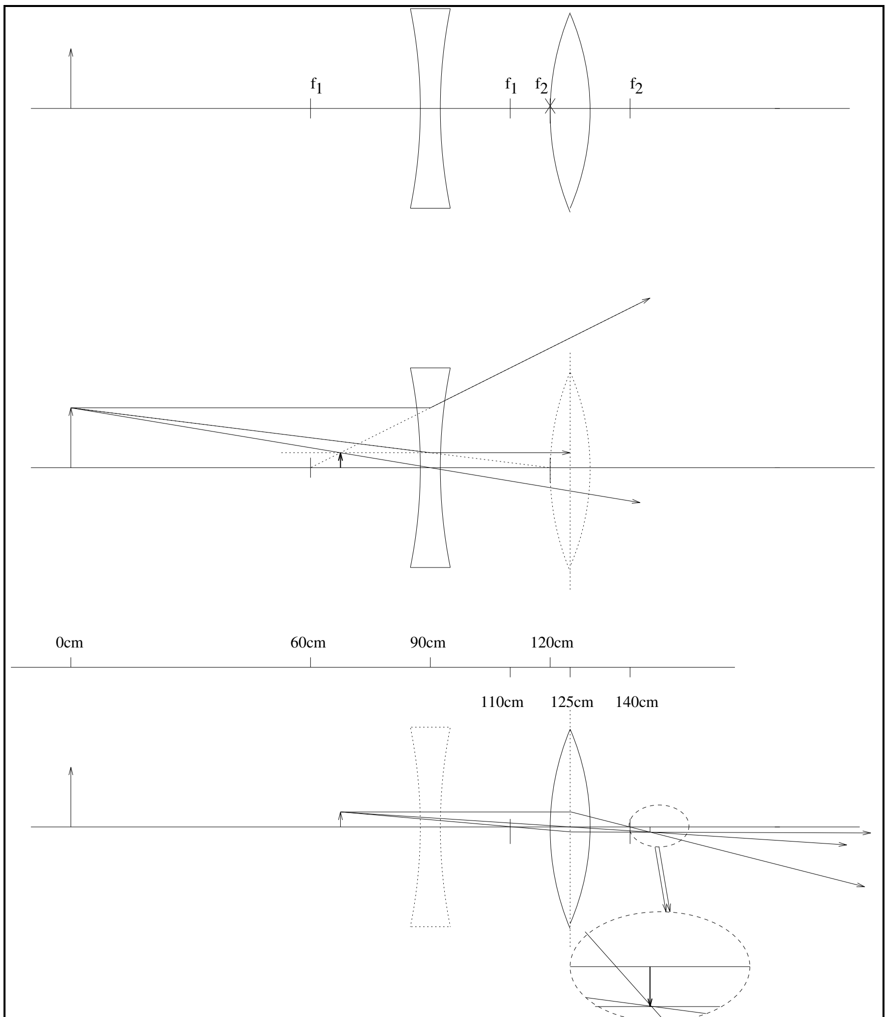
For this second step, we see that the sign conventions tell us that this image is real and on the opposite side of the lens. The height is (7.5 cm)(-.5)=-3.75 cm, and the minus sign indicates that the image is oriented in the opposite direction from the object. As a result, since the object pointed down, the image must point up.

Notice that the final height of the image is also given by the product of the magnifications and the original height. Thus, the final size is given by $h*M_1*M_2$=(15 cm)(-0.5)(-0.5)=3.75 cm. We conclude that the overall final image is oriented the same way as the original object, is 3.75 cm tall, and is a real image located 202.5 cm from the original object.

O4.3 Diverging and converging lenses.

Now, suppose that the first lens is a diverging lens rather than a converging lens. How does the solution change?

> We are given a diverging lens (*f*=-30 cm) placed 35 cm to the left of a converging lens (*f*=15 cm). An arrow is placed 90 cm to the left of the diverging lens. Where is the final image located and what is the total magnification of this lens combination?



O4.2 A real image from a diverging and converging lens.

As before, the first step is to draw the system of lenses and the object (Figure O4.2, top part). Starting with the object and the first lens encountered by the light rays, find the first image and its magnification both with ray diagrams (middle part of Figure O4.2) and mathematically with

$$\frac{1}{90\,cm}+\frac{1}{S'}=\frac{1}{-30\ cm}\Rightarrow S'=-22.5\ cm \text{ and}$$

$$M_1=-\left(\frac{-22.5\ cm}{90\ cm}\right)=0.25\,.$$

Notice that the image is virtual and its magnification is positive making it upright. Since this image is on the same side of the second lens as where the light rays originate, this virtual image becomes a **real** object for the second lens. Once again, ray diagrams (bottom part of Figure O4.2) will allow us to find the location of the final image. Mathematically we can verify this result using:

$$\frac{1}{57.5\ cm}+\frac{1}{S'}=\frac{1}{15\ cm}\Rightarrow S'=20.3\ cm \text{ and}$$

$$M_2=-\left(\frac{20.3\ cm}{57.5\ cm}\right)=-0.353\,.$$

Note that the final image has real light rays converging to form the image. Thus it is a **real** image located 145.3 cm from the original object. Its final magnification is negative and has a magnitude less than unity; M_1M_2=(0.25)(-0.353)=-0.0884. We conclude that the image is inverted from the original object and reduced in size.

Exercise O4.4.1: An arrow is placed 100 cm away from a converging lens (*f*=50 cm). On the other side of this lens is a second converging lens (*f*=40 cm) which is located 120 cm from the first lens. Where is the final image, what are its properties, and what is the overall magnification?

O4.4 A virtual object

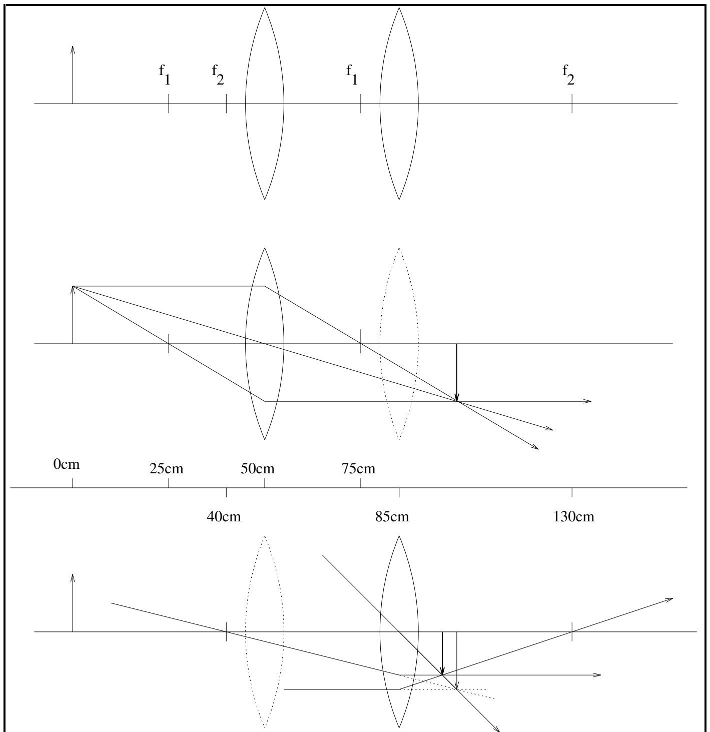
As in the past few chapters, we again encounter a virtual object. As before the virtual object occurs during one of the intermediate steps. In this problem we will see that the real image formed by the first lens becomes a virtual object for the second lens.

> An arrow is placed 50 cm away from a converging lens (*f*= 25cm). On the other side of this first lens is a second converging lens (*f*= 45 cm) which is located 35 cm from the first lens. Where is the final object located, and what is its magnification?

Once again, sketch the system (top part of Figure O4.3) and complete a ray diagram for the first lens (middle part of Figure O4.3). Notice that the rays are drawn as if the second lens is not present and the image is formed to the right of the second lens. Numerically, the solution for the first lens is

$$\frac{1}{50\,cm}+\frac{1}{S'}=\frac{1}{25\ cm}\Rightarrow S'=50\ cm \text{ and } M_1=-\left(\frac{50\ cm}{50\ cm}\right)=-1.0\,.$$

Since the light rays will bend when they travel through the second lens, the light rays to the right of the second lens are really virtual rays. This image then becomes a virtual object for the second lens. Another clue that there is a virtual object in the problem is to note that the rays are converging as they approach the second lens. A ray diagram using this virtual object shows the location of the final image (bottom part of Figure O4.3). Numerically, we can verify the accuracy of the ray diagram with:



O4.3 The principle rays for two converging lenses.

$$\frac{1}{-15\ cm} + \frac{1}{S'} = \frac{1}{45\ cm} \Rightarrow S' = 11.25\ cm \text{ and}$$

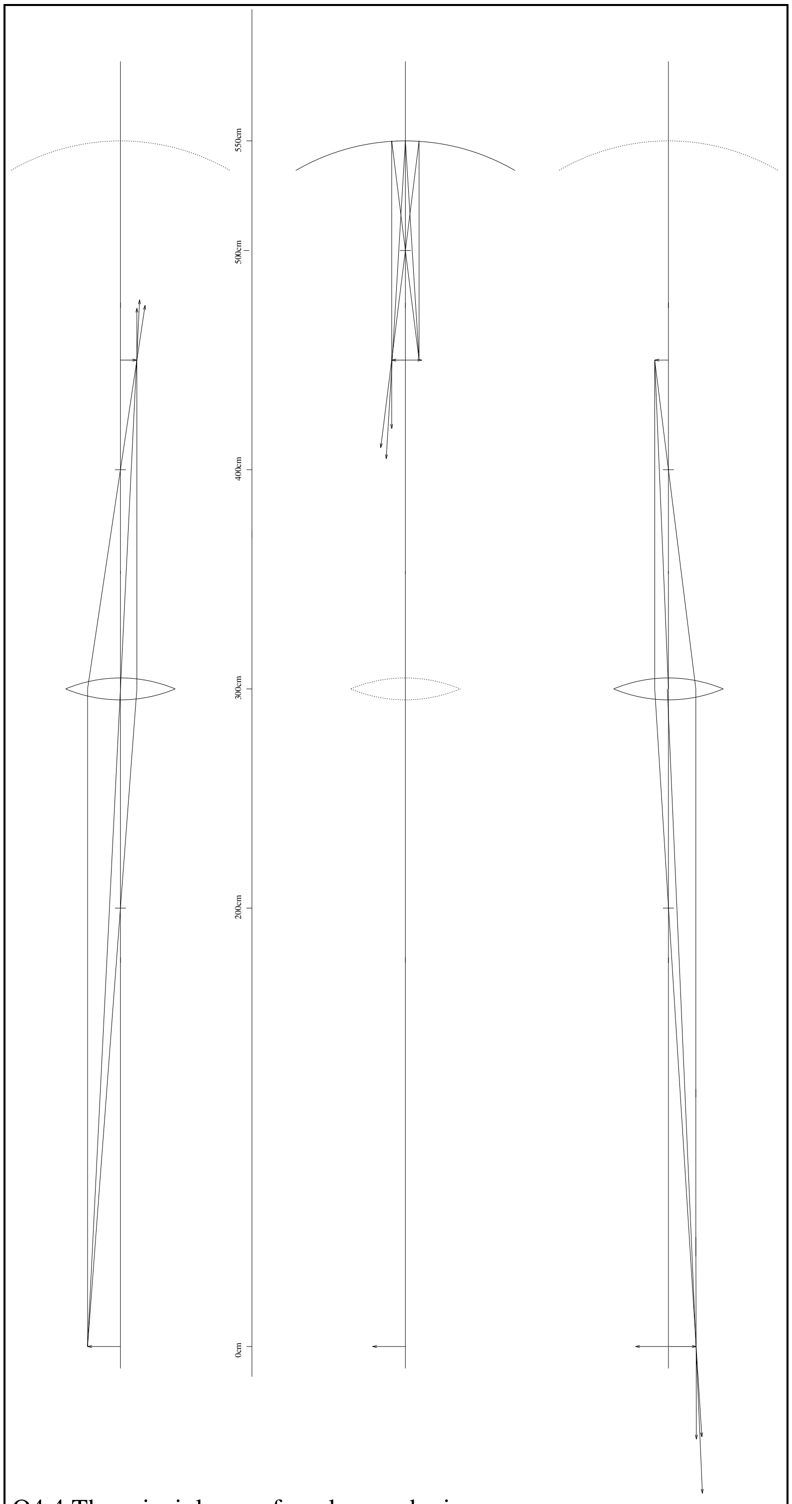
$$M_2 = -\left(11.25\ cm \big/ -15\ cm\right) = 0.75\ .$$

Overall the magnification is M_1M_2=(-1.0)(0.75)=-0.75 so we conclude that the overall final image is inverted from the original object and is a real image located 71.25 cm from the original object.

O4.5 One converging lens and a concave mirror.

How do mirrors complicate the system? Unlike the lens systems, mirrors may add a step to the problem. With mirrors, it becomes possible, depending on the specific arrangement of the optical elements, for the light to interact with the elements more than once. In the following example, we will see that light passes through a lens, reflects off of a mirror, and then passes back through the lens **AGAIN.**

A converging lens (*f*=100 cm) is placed 300 cm to the right of an arrow. A concave mirror (*f*=50 cm) is placed 550 cm to the right of the arrow. Find the location of the final image formed by this system and its overall magnification.



O4.4 The principle rays for a lens and mirror.

As always, starting with the component closest to the arrow and drawing a ray diagram determines the location of the first image (top part of Figure O4.4). Numerically, the solution for the first lens is

$$\frac{1}{300\ cm}+\frac{1}{S'}=\frac{1}{100\ cm}\Rightarrow S'=150\ cm \text{ and}$$

$$M_1=-\left(150\ cm \big/ 300\ cm\right)=-0.5\,.$$

The real image from the first lens becomes the real object for the mirror and is located 100 cm from the mirror. Once again a ray diagram will locate the second image which is produced by the mirror (middle of O4.4). Numerically, we have

$$\frac{1}{100\ cm}+\frac{1}{S'}=\frac{1}{50\ cm}\Rightarrow S'=100\ cm \text{ and}$$

$$M_2=-\left(100\ cm \big/ 100\ cm\right)=-1.0\,.$$

The real image from the mirror becomes the real object for the lens as the rays pass back through the lens. This image is 150 cm from the lens. As the ray diagram show (bottom of O4.4), a final real inverted image is produced. Numerically, the solution for the final step is

$$\frac{1}{150\ cm}+\frac{1}{S'}=\frac{1}{100\ cm}\Rightarrow S'=300\ cm \text{ and}$$

$$M_3=-\left(300\ cm \big/ 150\ cm\right)=-2.0\,.$$

Overall the magnification is $M_1M_2M_3$=(-0.5)(-1.0)(-2.0)=-1.0 so we conclude that the final image is inverted compared to the original object and is a real image located 200 cm from the original object.

Of course there are many, many more combinations of lenses and mirrors with real and virtual images and objects for the intermediate steps and the final answer. However, if we carefully work through element by element, paying strict attention to the sign conventions and verifying our answers by comparing the ray diagrams with the numerical answers, there should be little difficulty.

Exercise O4.4.2: The last exercise of this section deals with two mirrors. These systems can be difficult to study in the laboratory because any screen we use to look for the real images always seem to block the incoming light rays. However, systems of multiple mirrors are very common in telescopes, including the Hubble space telescope.

A concave mirror (*f*=20 cm) is placed with its reflective surface facing an arrow located 100 cm away. Between the arrow and the first mirror, a second concave mirror (*f*=5 cm) is placed so that its reflective surface faces the first mirror (and faces away from the arrow). The second mirror is located 20 cm from the first mirror. Find the location of the image formed by the second mirror and the overall magnification.

O4.6 HOMEWORK PROBLEMS

These homework problems attempt to cover a variety of situations that have real and virtual objects and real and virtual images. Be sure to observe the sign conventions for all quantities.

1. An object and two lenses are arranged so that the object is to the left of and 70 cm from a converging lens. The magnitude of the focal length of the first lens is 50 cm. A second converging lens is 100 cm to the right of the first lens and the magnitude of the focal length is 30 cm. Find the location of the final image relative to the original object, the overall magnification, and the character of the final image.

2. Draw two ray diagrams to verify the numerical result to the previous problem.
3. An object and two lenses are arranged so that the object is to the left of and 40 cm from a converging lens. The magnitude of the focal length of the first lens is 25 cm. A second converging lens is 100 cm to the right of the first lens and the magnitude of the focal length is 15 cm. Find the location of the final image relative to the original object, the overall magnification, and the character of the final image.
4. Draw two ray diagrams to verify the numerical result to the previous problem.
5. An object and two lenses are arranged so that the object is to the left of and 40 cm from a converging lens. The magnitude of the focal length of the first lens is 25 cm. A second converging lens is 40 cm to the right of the first lens and the magnitude of the focal length is 50 cm. Find the location of the final image relative to the original object, the overall magnification, and the character of the final image.
6. Draw two ray diagrams to verify the numerical result to the previous problem.
7. An object and two lenses are arranged so that the object is to the left of and 40 cm from a converging lens. The magnitude of the focal length of the first lens is 25 cm. A second converging lens is 100 cm to the right of the first lens and the magnitude of the focal length is 60 cm. Find the location of the final image relative to the original object, the overall magnification, and the character of the final image.
8. Draw two ray diagrams to verify the numerical result to the previous problem.
9. An object and two lenses are arranged so that the object is to the left of and 40 cm from a converging lens. The magnitude of the focal length of the first lens is 25 cm. A diverging lens is 100 cm to the right of the first lens and the magnitude of the focal length is 15 cm. Find the location of the final image relative to the original object, the overall magnification, and the character of the final image.
10. Draw two ray diagrams to verify the numerical result to the previous problem.
11. An object and two lenses are arranged so that the object is to the left of and 40 cm from a converging lens. The magnitude of the focal length of the first lens is 25 cm. A diverging lens is 40 cm to the right of the first lens and the magnitude of the focal length is 20 cm. Find the location of the final image relative to the original object, the overall magnification, and the character of the final image.
12. Draw two ray diagrams to verify the numerical result to the previous problem.
13. An object and two lenses are arranged so that the object is to the left of and 40 cm from a converging lens. The magnitude of the focal length of the first lens is 25 cm. A diverging lens is 40 cm to the right of the first lens and the magnitude of the focal length is 45 cm. Find the location of the final image relative to the original object, the overall magnification, and the character of the final image.
14. Draw two ray diagrams to verify the numerical result to the previous problem.
15. An object and two lenses are arranged so that the object is to the left of and 10 cm from a diverging lens. The magnitude of the focal length of the first lens is 25 cm. A converging lens is 20 cm to the right of the first lens and the magnitude of the focal length is 25 cm. Find the location of the final

image relative to the original object, the overall magnification, and the character of the final image.

16. Draw two ray diagrams to verify the numerical result to the previous problem.
17. An object and two lenses are arranged so that the object is to the left of and 10 cm from a diverging lens. The magnitude of the focal length of the first lens is 25 cm. A converging lens is 20 cm to the right of the first lens and the magnitude of the focal length is 50 cm. Find the location of the final image relative to the original object, the overall magnification, and the character of the final image.
18. Draw two ray diagrams to verify the numerical result to the previous problem.
19. An object and two lenses are arranged so that the object is to the left of and 10 cm from a diverging lens. The magnitude of the focal length of the first lens is 25 cm. A second diverging lens is 30 cm to the right of the first lens and the magnitude of the focal length is 20 cm. Find the location of the final image relative to the original object, the overall magnification, and the character of the final image.
20. Draw two ray diagrams to verify the numerical result to the previous problem.
21. An object, a lens, and a mirror are arranged so that the object is to the left of and 50 cm from a converging lens. The magnitude of the focal length of the first lens is 30 cm. A concave mirror is 25 cm to the right of the first lens and the magnitude of the radius is 20 cm. Find the location of the final image relative to the original object, the overall magnification, and the character of the final image.
22. Draw three ray diagrams to verify the numerical result to the previous problem.
23. An object and two mirrors are arranged so that the object is to the left of and 25 cm from the back of a special convex mirror that has a hole in the center. Just to be clear, the reflective side of this mirror is facing to the right. The magnitude of the radius of this mirror is 90 cm. A second convex mirror is 25 cm to the right of the first mirror and the magnitude of the focal length is 35 cm. The reflective side of this mirror is to the left and this is the first mirror from which rays from the object are reflected. Find the location of the final image relative to the original object, the overall magnification, and the character of the final image.
24. Draw two ray diagrams to verify the numerical result to the previous problem.

ANSWERS TO EXERCISES

OE4.4.1 The ray diagram is shown in Figure O4.5. Numerically, the solution for the first lens is

$$\frac{1}{100\ cm} + \frac{1}{S'} = \frac{1}{50\ cm} \Rightarrow S' = 100\ cm \text{ and}$$

$$M_1 = -\left(100\ cm \middle/ 100\ cm \right) = 1.0 .$$

The image from the first lens becomes the object for the second lens and is located 20 cm from the second lens. Numerically we have

$$\frac{1}{20\ cm}+\frac{1}{S'}=\frac{1}{40\ cm}\Rightarrow S'=-40\,cm \text{ and}$$

$$M_2=-\left(\frac{-40\ cm}{20\ cm}\right)=2.0\,.$$

Overall the magnification is M_1M_2=(-1.0)(2.0)=-2.0 so that we conclude that the overall final image is inverted compared to the original object and is a virtual image

O4.5 The ray diagrams for a virtual object formed from two converging lenses.

located 180 cm from the original object.

OE4.4.2 The ray diagram is shown in Figure O4.4.6. Numerically, the solution for the first mirror is

$$\frac{1}{100\ cm}+\frac{1}{S'}=\frac{1}{20\ cm}\Rightarrow S'=25\ cm \text{ and}$$

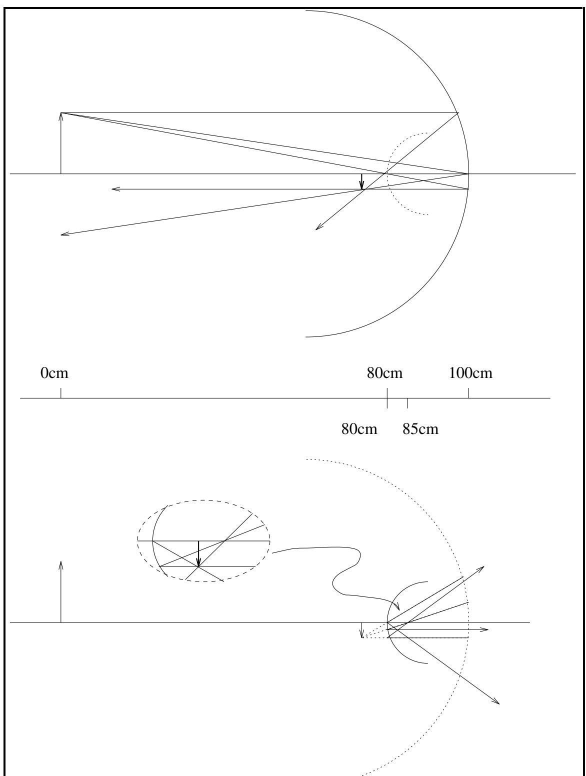
$$M_1=-\left(\frac{25\ cm}{100\ cm}\right)=-0.25\,.$$

The image from the first mirror becomes the object for the second mirror and is located 5 cm behind the second mirror. In this case we again encounter a virtual object. Numerically we have

$$\frac{1}{-5\ cm}+\frac{1}{S'}=\frac{1}{5\ cm}\Rightarrow S'=2.5\ cm \text{ and}$$

$$M_2=-\left(\frac{2.5\ cm}{-5\ cm}\right)=0.5\,.$$

Overall, the magnification is M_1M_2=(-0.25)(0.5)=-0.1255 so that we conclude that the overall final image is oriented inverted from the original object and is a real image located 2.5 cm from the second mirror. Finally, we check that the numerical solution is consistent with the ray diagram.



O4.6 The ray diagram for two concave mirrors.