Wed. Fri.	6.1 Magnetization 6.2 Field of a Magnetized Object	 HW10
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Linear Dielectrics

Point along field

Linearly proportional

Chunk of induced dipoles

for individual induced dipole

$$\vec{p} \approx \alpha \vec{E}$$

Everyone's field but its own

For chunk of induced dipoles

Polarization = Dipole density

$$\vec{P} \equiv \frac{d\vec{p}}{d\tau}$$

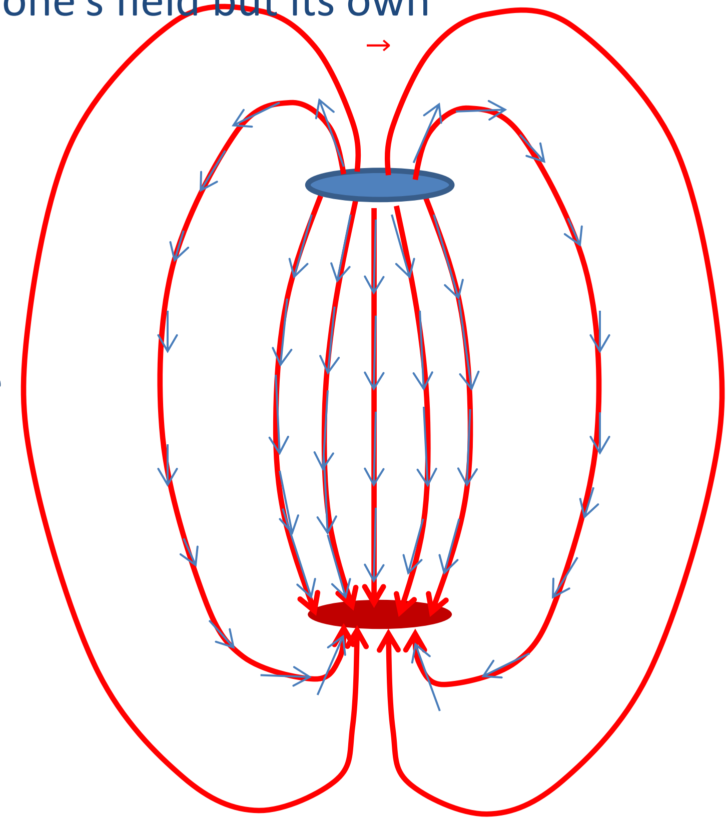
so

$$\vec{P} = \frac{d\left(\alpha \vec{E}\right)}{d\tau} = \left(\frac{d\alpha}{d\tau}\right)\vec{E}$$

Define "electric susceptibility" to be the proportionality constant (and provide convenient factor of ε_o.)

$$\chi_e \equiv \frac{1}{\varepsilon_o}\frac{d\alpha}{d\tau}$$

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$



Always / linear dielectric

$$\vec{D} = \underbrace{\varepsilon_o \vec{E} + \vec{P}}_{\text{Always}} = \underbrace{\varepsilon_o \vec{E} + \left(\varepsilon_o \chi_e \vec{E}\right)}_{\text{linear dielectric}} = \varepsilon_o \left(1 + \chi_e\right)\vec{E}$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{D} = \varepsilon_o \varepsilon_r \vec{E}$$

Permittivity (of not-so-free space)

$$\varepsilon \equiv \varepsilon_0 \left(1 + \chi_e\right)$$

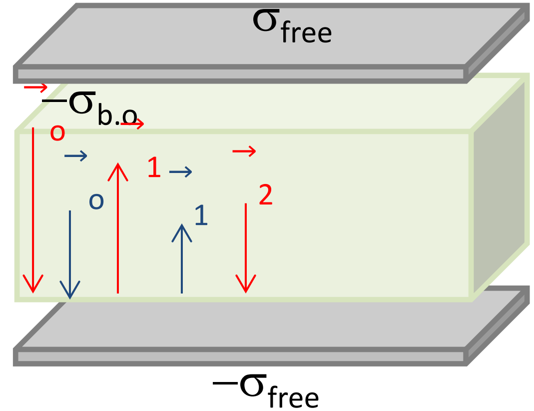
Or, in terms of polarization

$$\vec{D} = \left(\tfrac{1}{\chi_e} + 1\right)\vec{P} \quad \text{or} \quad \tfrac{\chi_e}{\chi_e + 1}\vec{D} = \vec{P}$$

Dielectric Constant

$$\varepsilon_r \equiv \tfrac{\varepsilon}{\varepsilon_o} = \left(1 + \chi_e\right)$$

Linear Dielectrics



Example: Alternate / iterative perspective on field in dielectric. Consider again a simple capacitor with dielectric. We'll find the electric field in terms of what it would have been without the dielectric. We'll do this iteratively and build a series solutions.

0. Say we start with no dielectric. Initially there's the field simply due to the free charge; E_o.

We insert the dielectric and that field induces a polarization,

$$\vec{P}_o = \varepsilon_o \chi_e \vec{E}_o$$

and the associated surface charges contribute a field of their own,

$$\vec{E}_1 = \frac{\sigma_{b.o}}{\varepsilon_o}\hat{z}$$ where $\sigma_{b.o} = \vec{P}_0 \cdot \hat{n}$ so $\vec{E}_1 = -\vec{P}_0 / \varepsilon_0 = -\chi_e \vec{E}_0$

in the opposite direction.

1. This field induces a little counter polarization,

$$\vec{P}_1 = \varepsilon_0 \chi_e \vec{E}_1 = -\varepsilon_0 \chi_e^2 \vec{E}_0$$

Which is means a surface charge and resulting field contribution of its own

$$\vec{E}_2 = -\vec{P}_1 / \varepsilon_0 = (-\chi_e)^2 \vec{E}_0$$

2. See a pattern?

$$\vec{E} = \vec{E}_o + (-\chi_e)\vec{E}_o + (-\chi_e)^2 \vec{E}_o + \ldots$$

$$\vec{E} = \sum_{n=0}^{\infty} (-\chi_e)^n \vec{E}_o$$

As long as $\chi_e < 1$, this converges to

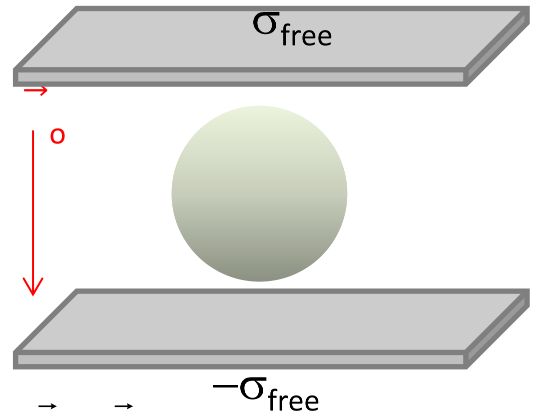
$$\vec{E}_{inside} = \left(\frac{1}{1+\chi_e}\right)\vec{E}_0 = \frac{1}{\varepsilon_r}\vec{E}_0$$

Same result as we got previously

Linear Dielectrics

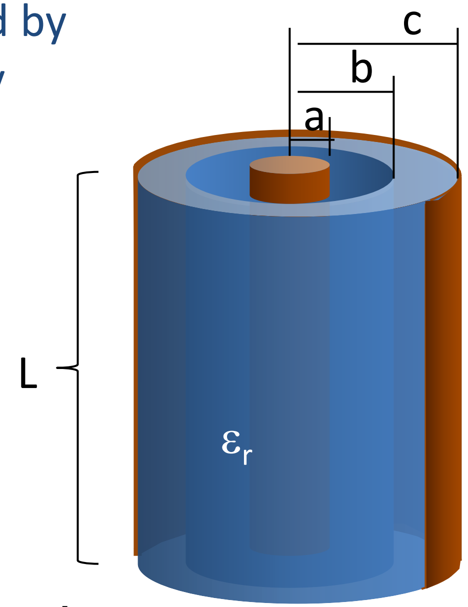
Exercise: Try it for your self. A sphere made of linear dielectric material is placed in an otherwise uniform electric field $\vec{E}_o$. Find the electric field inside the sphere in terms of the material's dielectric constant, ε_r.

You can take it as a given that a sphere of uniform polarization contributes field $\vec{E} = -\vec{P}/3\varepsilon_0$



$\vec{E} = \vec{E}_o +$

Example: A coaxial cable consists of a copper wire of radius *a* surrounded by a concentric copper tube of inner radius *c*. The space between is partially filled (from *b* to *c*) with material of dielectric constant ε_r as shown below. Find the capacitance per length of the cable.



For the sake of reasoning this out, say there's charge Q uniformly distributed along the surface of the central wire.

$$C \equiv \left|\frac{Q}{\Delta V}\right|$$

$$\Delta V = -\int_a^c \vec{E}\cdot d\vec{l}$$

$$\vec{E} = \begin{cases} \frac{1}{\varepsilon_o}\vec{D} & a<s<b \\ \frac{1}{\varepsilon_o\varepsilon_r}\vec{D} & b<s<c \end{cases}$$

Gaussian cylinder of some radius a<s<c.

$$\oint \vec{D}\cdot d\vec{a} = Q_{f.encl}$$

$$D2\pi sL = Q$$

$$D = \frac{Q}{2\pi sL}$$

$$\vec{E} = \begin{cases} \frac{Q}{\varepsilon_o 2\pi sL}\hat{s} & a<s<b \\ \frac{Q}{\varepsilon_o\varepsilon_r 2\pi sL}\hat{s} & b<s<c \end{cases}$$

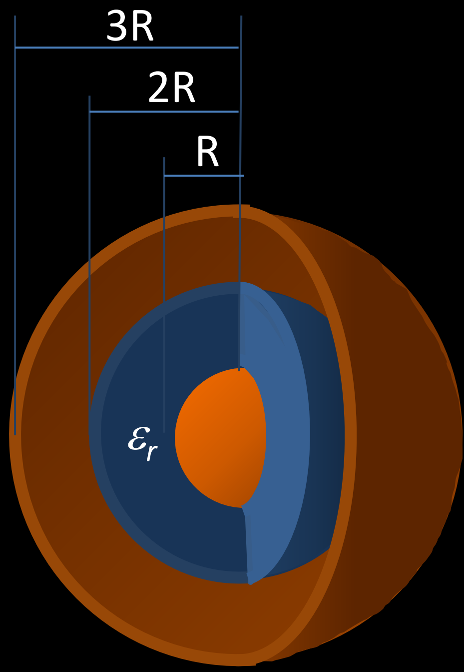
$$\Delta V = -\int_a^c \vec{E}\cdot d\vec{l} = -\int_a^b \vec{E}\cdot d\vec{l} - \int_b^c \vec{E}\cdot d\vec{l}$$

$$\Delta V = -\int_a^b \frac{Q}{\varepsilon_o 2\pi sL}ds - \int_b^c \frac{Q}{\varepsilon_o\varepsilon_r 2\pi sL}ds$$

$$\Delta V = -\frac{Q}{\varepsilon_o 2\pi L}\left(\ln\left(\tfrac{b}{a}\right) + \tfrac{1}{\varepsilon_r}\ln\left(\tfrac{c}{b}\right)\right)$$

$$\frac{C}{L} \equiv \frac{\varepsilon_o 2\pi}{\left(\ln\left(\tfrac{b}{a}\right) + \tfrac{1}{\varepsilon_r}\ln\left(\tfrac{c}{b}\right)\right)}$$

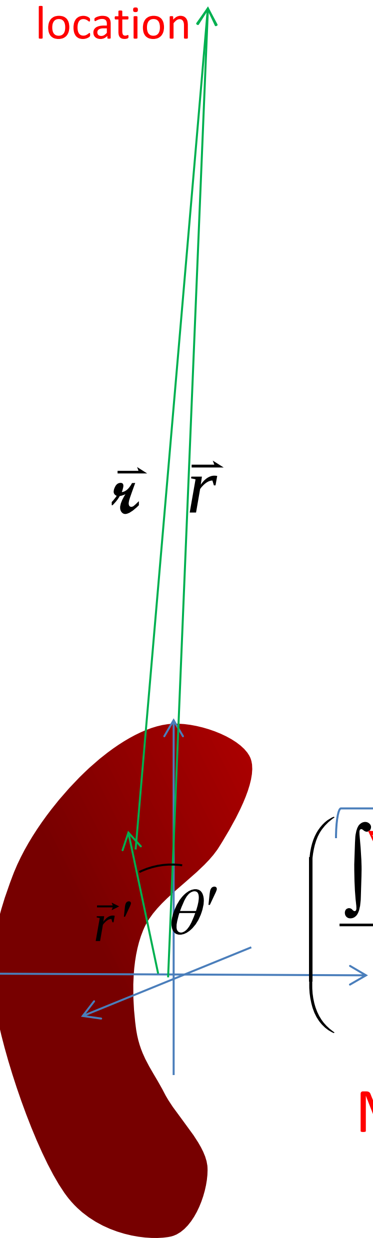
Exercise: There are two metal spherical shells with radii R and 3R. There is material with a dielectric constant $\varepsilon_r = 3/2$ between radii R and 2R. What is the capacitance?



Recall Multi-pole Expansion of *Vector* Potential

Continuous *current* distribution

Observation location



$$\vec{A}(r) = \frac{\mu_o}{4\pi}\int \frac{dq\vec{v}}{\mathscr{r}}$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{r'}{r}\right)^n P_n(\cos\theta')$$

nth Legendre polynomial

$$P_n(\cos\theta')$$

$$P_0 = 1$$

$$P_1(u) = u$$

$$P_2(u) = \frac{(3u^2-1)}{2}$$

$$\vec{A}(r) = \frac{\mu_o}{4\pi}\int\left(\left(\frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{r'}{r}\right)^n P_n(\cos\theta')\right)\vec{J}(\vec{r}\,')d\tau'\right)$$

Re-ordering sums

$$\vec{A}(r) = \frac{\mu_o}{4\pi}\sum_{n=0}^{\infty}\left(\frac{1}{r^{n+1}}\int r'^n P_n(\cos\theta')\vec{J}(\vec{r}\,')d\tau'\right)$$

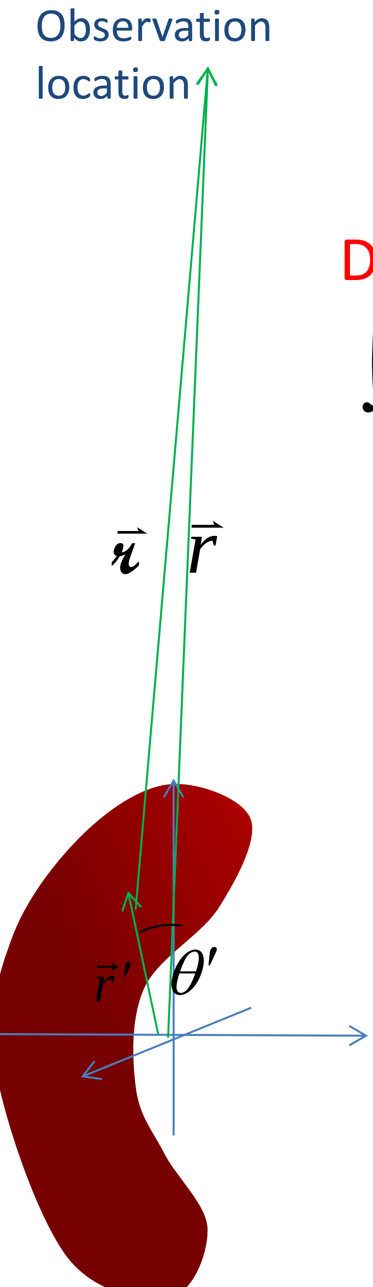
$$\left(\frac{\int \vec{J}(\vec{r}\,')d\tau'}{r} + \frac{\int r'\cos\theta'\vec{J}(\vec{r}\,')d\tau'}{r^2} + \frac{\int r'^2\left(3(\cos\theta')^2-1\right)\vec{J}(\vec{r}\,')d\tau'}{2r^3} + \ldots\right.$$

Monopole term

Dipole term

Recall Multi-pole Expansion of *Vector* Potential

Continuous *current* distribution



$$\vec{A}(r) = \frac{\mu_o}{4\pi}\int \frac{dq\vec{v}}{\mathscr{r}} = \frac{\mu_o}{4\pi}\left(\frac{\int r'\cos\theta' \vec{J}(\vec{r}')d\tau'}{r^2} + \ldots \right)$$

Dipole term

Dipole's integral

Steady current

$$\int r'\cos\theta' \vec{J}(\vec{r}')d\tau' = \int (\hat{r}\cdot\vec{r}')\vec{J}(\vec{r}')d\tau' = \oint (\hat{r}\cdot\vec{r}')Id\vec{l}\,' = I\oint (\hat{r}\cdot\vec{r}')d\vec{l}\,'$$

to mathland and back

$$= -I\int \hat{r}\times d\vec{a}' = -I\hat{r}\times\int d\vec{a}' = -I\hat{r}\times\vec{a}' = I\vec{a}'\times\hat{r}$$

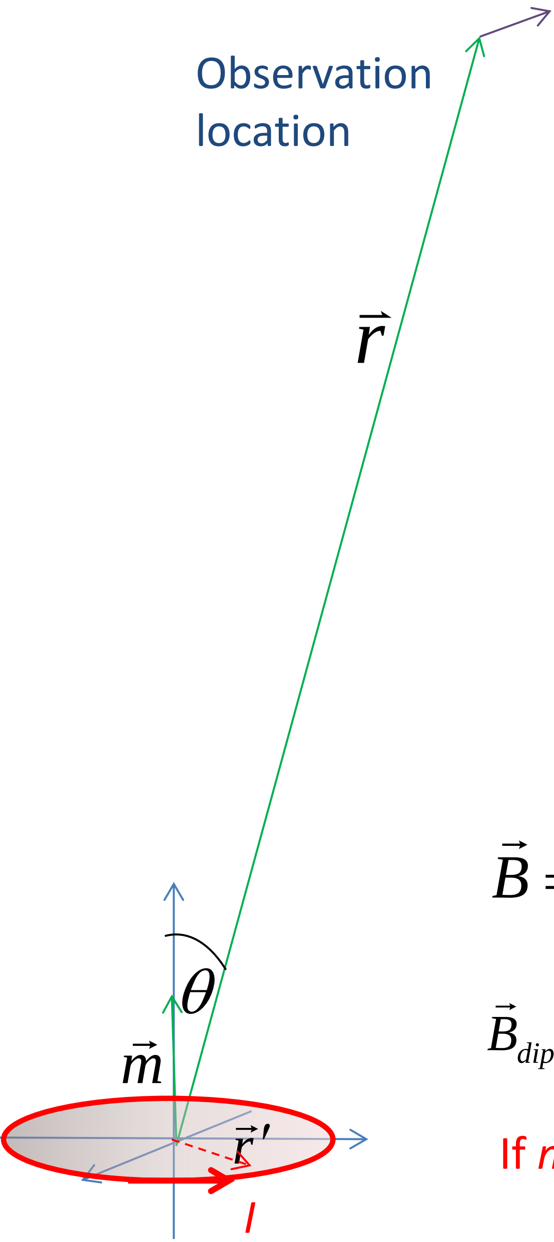
$$\vec{A}(r) = \frac{\mu_o}{4\pi}\int \frac{dq\vec{v}}{\mathscr{r}} = \frac{\mu_o}{4\pi}\left(\frac{I\vec{a}'\times\hat{r}}{r^2} + \ldots \right)$$

Magnetic Dipole Moment

$$\vec{m} \equiv I\vec{a}'$$

$$\vec{A}(r) = \frac{\mu_o}{4\pi}\left(\frac{\vec{m}\times\hat{r}}{r^2} + \ldots \right)$$

Dipole term for a loop



$$\vec{A}(r) = \frac{\mu_o}{4\pi}\left(\frac{\vec{m}\times\hat{r}}{r^2} + \ldots\right)$$

Magnetic Dipole Moment

$$\vec{m} \equiv I\vec{a}'$$

$$\vec{m} = I\left(\pi R^2 \hat{z}\right)$$

$$\vec{A}(r) = \frac{\mu_o}{4\pi}\left(\frac{I\pi R^2 \hat{z}\times\hat{r}}{r^2} + \ldots\right) = \frac{\mu_o}{4\pi}\left(\frac{I\pi R^2 \sin\theta}{r^2}\hat{\phi} + \ldots\right)$$

Same direction as current

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{B}_{dip}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{3(\vec{m}\cdot\hat{\mathscr{r}})\hat{\mathscr{r}} - \vec{m}}{\mathscr{r}^3}$$

If *m* at origin and pointing up

$$\vec{B}_{dip}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{m}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

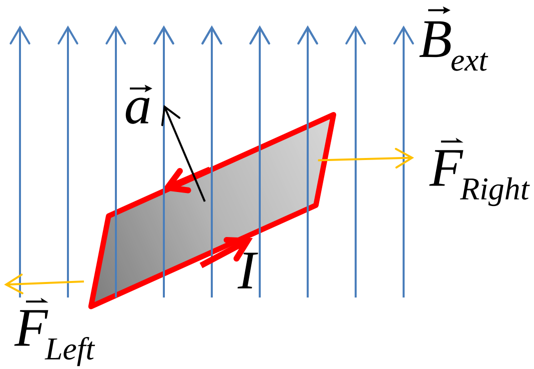
(yes, same form as *E* for *p*)

Magnetic Dipoles

$$\vec{m} \equiv I\vec{a}' \qquad \vec{A}(r) = \frac{\mu_o}{4\pi}\left(\frac{\vec{m}\times\hat{r}}{r^2} + \ldots\right) \qquad \vec{B}_{dip}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{m}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

From a distance much greater than the current distribution’s size, the dipole term dominates

Recall **Torque** for real current loop



$$N = ILB_{ext} w \sin\theta$$

$$\vec{N} = I\vec{a} \times \vec{B}_{ext}$$

In terms of dipole moment

$$\vec{N} = \vec{m} \times \vec{B}_{ext}$$

Like for *electric* dipoles

$$\vec{N} = \vec{p} \times \vec{E}_{ext}$$

Change in **energy**

$$\Delta U = -\int \vec{F}_R \cdot d\vec{l}_R - \int \vec{F}_L \cdot d\vec{l}_L$$
$$= -2\int \vec{F}_{right} \cdot d\vec{l}_{right}$$
$$= -2\int (Ib)\vec{B} \cdot \tfrac{a}{2}\, d\vec{\theta}$$
$$= -2\int (Ib)B\sin\theta\, \tfrac{a}{2}\, d\theta$$
$$= -(Iba)B\cos\theta\Big|_{\theta_i}^{\theta_f}$$

In terms of dipole moment

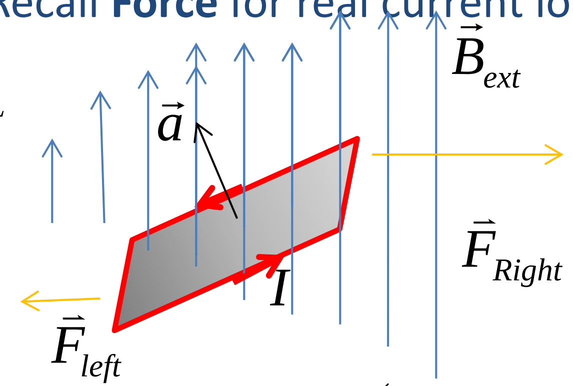
$$\Delta U = -mB\cos\theta\Big|_{\theta_i}^{\theta_f}$$

$$\Delta U = -\Delta\left(\vec{m} \cdot \vec{B}\right)$$

Like for *electric* dipoles

$$\Delta U = -\Delta\left(\vec{p} \cdot \vec{E}\right)$$

Recall **Force** for real current loop



$$\vec{F}_{net} = \vec{F}_{Right} + \vec{F}_{left} = \left(ILB_{Right} - ILB_{Left}\right)\hat{x}$$

In terms of dipole moment

$$\vec{F} = \left(\vec{m} \cdot \vec{\nabla}\right)\vec{B}_{ext}$$

If m is constant, the same as

$$\vec{F} = \vec{\nabla}\left(\vec{m} \cdot \vec{B}_{ext}\right)$$

Like for *electric* dipoles

$$\vec{F} = \left(\vec{p} \cdot \vec{\nabla}\right)\vec{E}_{ext}$$

Magnetic Dipoles

$$\vec{m} \equiv I\vec{a}' \qquad \vec{A}(r) = \frac{\mu_o}{4\pi}\left(\frac{\vec{m}\times\hat{r}}{r^2} + \ldots\right) \qquad \vec{B}_{dip}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{m}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

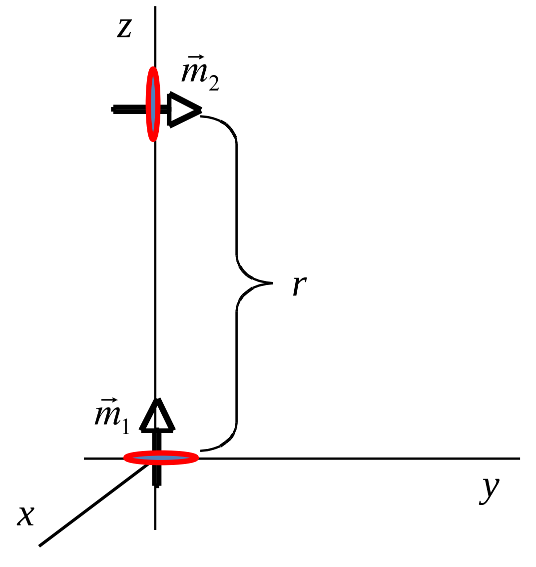
From a distance much greater than the current distribution's size, the dipole term dominates

$$\vec{N} = \vec{m}\times\vec{B}_{ext} \qquad \Delta U = -\Delta\left(\vec{m}\cdot\vec{B}\right) \qquad \vec{F} = \left(\vec{m}\cdot\vec{\nabla}\right)\vec{B}_{ext}$$

If m is constant, the same as

$$\vec{F} = \vec{\nabla}\left(\vec{m}\cdot\vec{B}_{ext}\right)$$

Exercise: you have two *magnetic* dipoles; find the torque m_1 applies on m_2.



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Magnetic Dipoles

$$\vec{m} \equiv I\vec{a}' \qquad \vec{A}(r) = \frac{\mu_o}{4\pi}\left(\frac{\vec{m}\times\hat{r}}{r^2} + \ldots\right) \qquad \vec{B}_{dip}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{m}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

From a distance much greater than the current distribution's size, the dipole term dominates

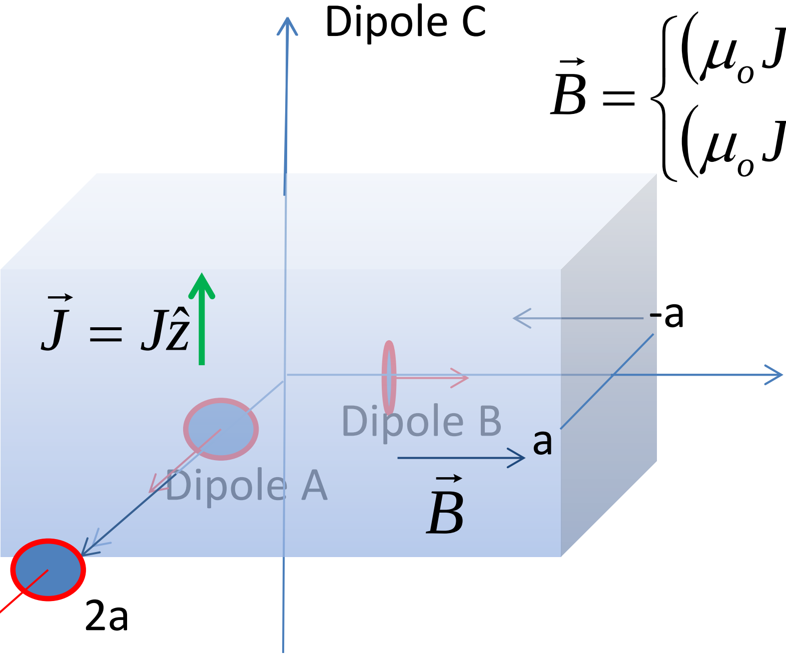
$$\vec{N} = \vec{m}\times\vec{B}_{ext} \qquad \Delta U = -\Delta\left(\vec{m}\cdot\vec{B}\right) \qquad \vec{F} = \left(\vec{m}\cdot\vec{\nabla}\right)\vec{B}_{ext}$$

If m is constant, the same as

$$\vec{F} = \vec{\nabla}\left(\vec{m}\cdot\vec{B}_{ext}\right)$$

Exercise: find the force on dipoles A, B, C in and near a slab of uniform current

$$\vec{B} = \begin{cases}(\mu_o Jx)\hat{y} & \text{inside}\\ (\mu_o Ja)\hat{y} & \text{infront}\end{cases}$$



Magnetic Dipoles

$$\vec{m} \equiv I\vec{a}' \qquad \vec{A}(r) = \frac{\mu_o}{4\pi}\left(\frac{\vec{m}\times\hat{r}}{r^2} + \ldots\right) \qquad \vec{B}_{dip}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{m}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

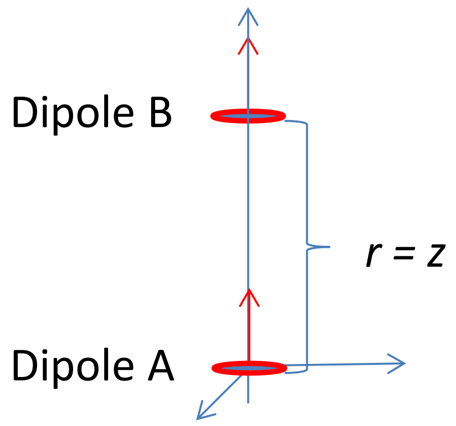
From a distance much greater than the current distribution's size, the dipole term dominates

$$\vec{N} = \vec{m}\times\vec{B}_{ext} \qquad \Delta U = -\Delta\left(\vec{m}\cdot\vec{B}\right) \qquad \vec{F} = \left(\vec{m}\cdot\vec{\nabla}\right)\vec{B}_{ext}$$

If m is constant, the same as

$$\vec{F} = \vec{\nabla}\left(\vec{m}\cdot\vec{B}_{ext}\right)$$

Exercise: Force between two dipoles (read, "bar magnets"). What's the force A exerts on B?



Considering 'real' dipoles (with real radii), roughly sketch the field and resulting forces on the current loops

Effect of Magnetic Field on Dipoles

Para-magnetic: Rotate the loop (torque)

Dia-magnetic: Stretch the loop; changing field(s) – Faraday-effect: impede the current

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