Fri.	(C 16) 1.6, 2.3.1 -.3.3 Electric Potential	
Mon.	(C 16) 2.3.4-.3.5 Electric Potential	
Wed.	2.4.1-.4.2 Work & Energy in Electrostatics T3 Contour Plots	
Thurs		HW2

Motivating Electric Potential, Physically

Generally

$$W_{1\to2} \equiv \int_a^b \vec{F}_{1\to2} \cdot d\vec{\ell}$$

Object 2 is the "system", 1 is "external." Work done *by* object 1 when exerting force *on* object 2 which moves from *a* to *b*

$$\Delta P.E._{1,2} \equiv -\int_a^b \vec{F}_{1\to2} \cdot d\vec{\ell}$$

Objects 1 and 2 are the "system". Change in their potential as they interact while separating from *a* to *b*

Electrically

$$\vec{F}_{1\to2} = q_2 \vec{E}_1(\vec{r}_2)$$

Combining:

$$\Delta P.E._{1,2} = -\int_a^b q_2 \vec{E}_1(\vec{r}_2) \cdot d\vec{\ell} = -q_2 \int_a^b \vec{E}_1(\vec{r}_2) \cdot d\vec{\ell}$$

thus

$$\Delta V_1 \equiv \frac{\Delta P.E._{1,2}}{q_2} = -\int_a^b \vec{E}_1(\vec{r}_2) \cdot d\vec{\ell}$$

Motivating Electric Potential, Mathematically

In mathland, say you have scalar field *f.*

What's $\vec{\nabla}\times\left(\vec{\nabla}f\right)=?$ Well, $\vec{\nabla}f=\left(\frac{\partial}{\partial x}f\right)\hat{x}+\left(\frac{\partial}{\partial y}f\right)\hat{y}+\left(\frac{\partial}{\partial z}f\right)\hat{z}$

So,

$$\vec{\nabla}\times\left(\vec{\nabla}f\right)=\left(\frac{\partial}{\partial y}\left(\frac{\partial}{\partial z}f\right)-\frac{\partial}{\partial z}\left(\frac{\partial}{\partial y}f\right)\right)\hat{x}+(\ldots)\hat{y}+(\ldots)\hat{z}$$

$$\vec{\nabla}\times\left(\vec{\nabla}f\right)=\left(\frac{\partial^2}{\partial y\partial z}f-\frac{\partial^2}{\partial z\partial y}f\right)\hat{x}+(\ldots)\hat{y}+(\ldots)\hat{z}$$

$$\vec{\nabla}\times\left(\vec{\nabla}f\right)=0$$

Free to define

$$\vec{F}\equiv\vec{\nabla}f$$

So,

$$\vec{\nabla}\times\left(\vec{F}\right)=0$$

Phrased the other way around:

For any vector field $\vec{F}$ for which $\vec{\nabla}\times\vec{F}=0$
There is a scalar field f such that $\vec{F}\equiv\vec{\nabla}f$

$\vec{\nabla}\times\vec{E}=0$ so we can define a scalar field, call it $-V$,such that $\vec{E}=-\vec{\nabla}V$

Motivating Electric Potential, Mathematically

$$\vec{E} = -\vec{\nabla} V$$

The corresponding integral relation
(demonstrating the fundamental theorem of calc for gradients),

$$\int_a^b \vec{E} \cdot d\vec{l} = -\int_a^b \vec{\nabla} V \cdot d\vec{l}$$

$$\int_a^b \vec{E} \cdot d\vec{l} = -\int_a^b \left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z} \right) \cdot d\vec{l}$$

$$\int_a^b \vec{E} \cdot d\vec{l} = -\int_a^b \left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z} \right) \cdot \left(dx\hat{x} + dy\hat{y} + dz\hat{z} \right)$$

$$\int_a^b \vec{E} \cdot d\vec{l} = -\int_a^b \left(\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz \right)$$

$$\int_a^b \vec{E} \cdot d\vec{l} = -\int_a^b (dV)$$

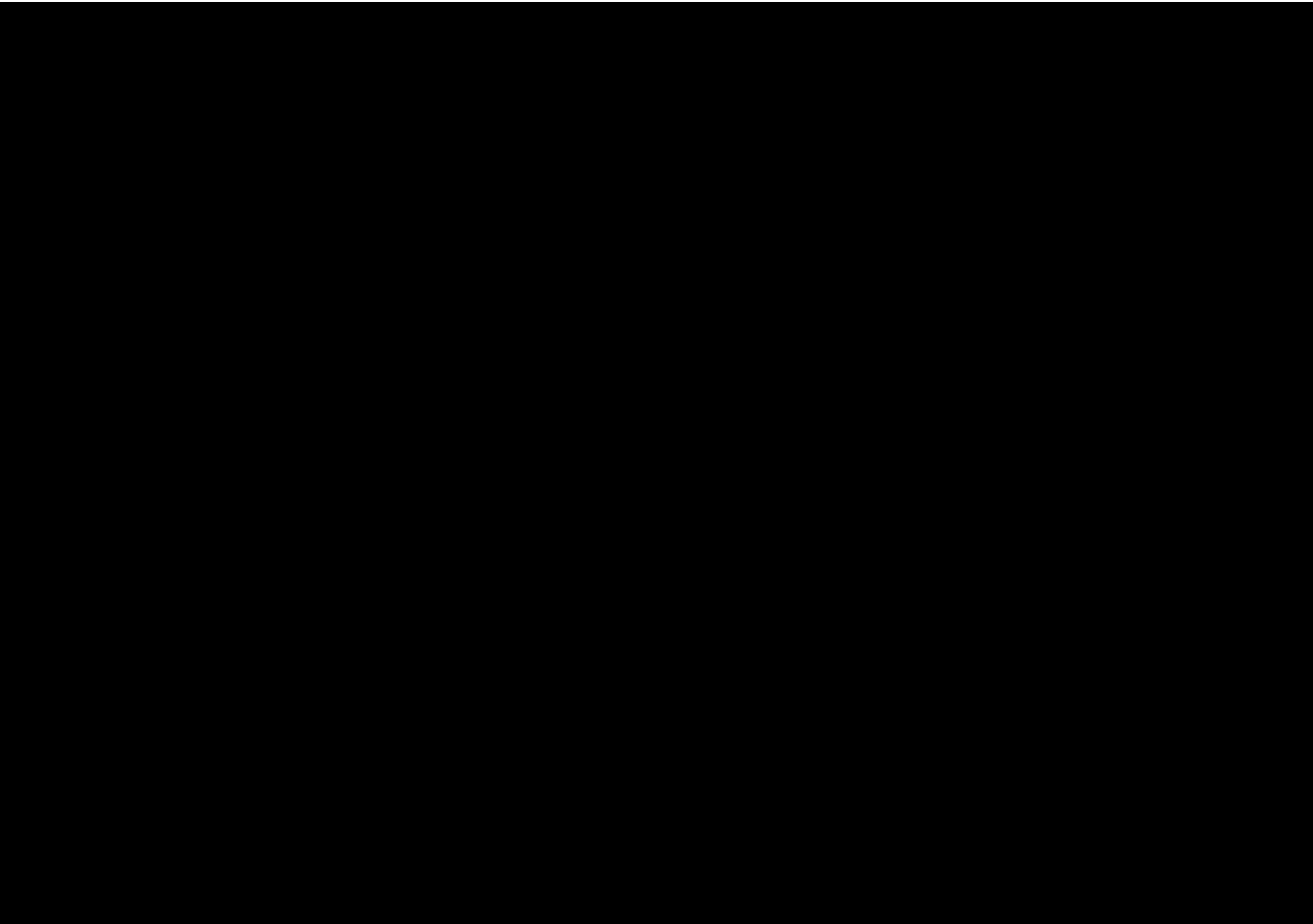
$$\int_a^b \vec{E} \cdot d\vec{l} = -(V_b - V_a)$$

<u>Exercise a</u>:

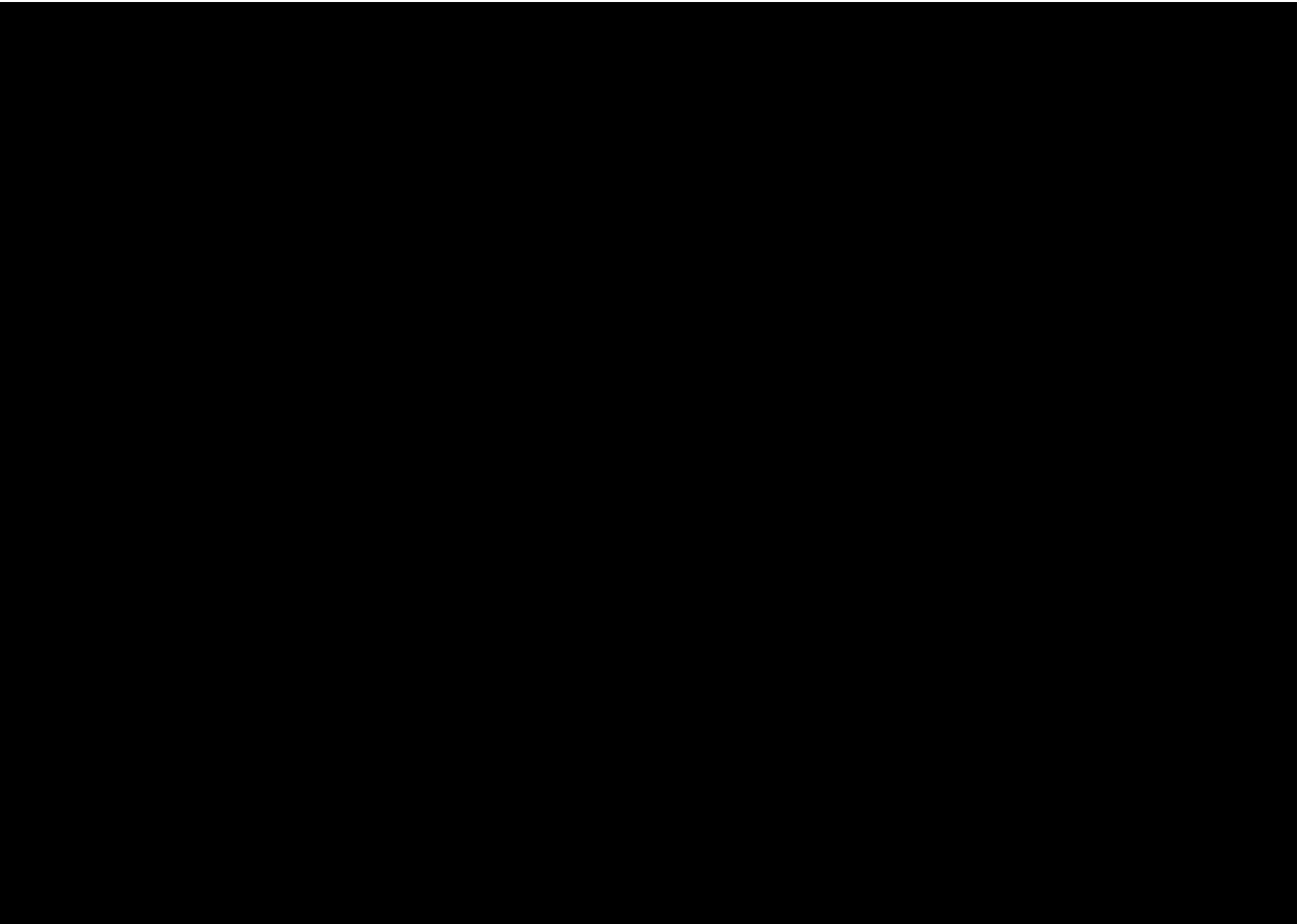
Check that $\vec{E} = ky\hat{x} + kx\hat{y}$ is a valid (curl-less) electric field.

[redacted]

<u>Example</u>: Potential of a Point Charge at a Distance r



Example: Prob. 2.22 - Potential of a Uniform Line Charge



Exercise b:

Find the electric potential at point (x,y,z) relative to (0,0,0) if

$$\vec{E} = ky\hat{x} + kx\hat{y}$$

[redacted]

Exercise c:Check that $\vec{\nabla} V = -\vec{E}$ for your answer to (b).

[redacted]

Poisson's Equation and Laplace's Equation

From last time: $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$ And now: $\vec{E} = -\vec{\nabla} V$

so: $\vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot \left(-\vec{\nabla} V\right) = -\nabla^2 V = \frac{\rho}{\varepsilon_0}$

where:

$$\vec{\nabla} \cdot \left(\vec{\nabla} V\right) = \vec{\nabla} \cdot \left(\left(\frac{\partial}{\partial x} V\right)\hat{x} + \left(\frac{\partial}{\partial y} V\right)\hat{y} + \left(\frac{\partial}{\partial z} V\right)\hat{z}\right)$$

Poisson's where there *are* charges: $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$

Laplace's where there *aren't*: $\nabla^2 V = 0$

Ch. 3