| Wed., 10/30 <br> Thurs. 10/31 <br> Fri., 11/1 | 7.1.1-7.1.3 Ohm's Law \& Emf 7.1.3-7.2.2 Emf \& Induction | HW7 |
| :---: | :---: | :---: |
| Mon. 11/4 <br> Wed., 11/6 <br> Fri., 11/8 | Exam 2 (Ch 3 \& 5) <br> 7.2.3-7.2.5 Inductance and Energy of B <br> 7.3.1-.3.3 Maxwell's Equations |  |
| Mon., 11/11 | 10.1-2.1 Potential Formulation | HW8 |

## Equipment

- Crank generator
- Magnet and swinging fins with and without fingers cut in


## Announcements:

- Test in 1 week.


## Summary

## Transition.

Stationary Charges \& Fields. In the first section of the book we considered constant electric fields produced by stationary charges; we tackled that head on by starting with Coulomb's Law (directly relating sources and their fields) and then developing a handful of related tools.

Steady charge flow \& Fields. In the next section (which Test 2 will be over) is focused on constant magnetic fields, produced by steady currents (charges with steady motion). Again, we approached this task head-on with the Biot-Savart Law which relates the source currents and their fields. We subsequently developed a handful of associated tools.

Unsteady currents and charge motion \& Fields. You wouldn't know it from section 7.1, but where we're going in this next section is to talk about the fields generated by non-steady currents and even accelerating charges. You were just presented with Coulomb's Law and the Biot-Savart Law right off the bat - no experimental motivation, just 'here's the way it is folks.' In that spirit I'll remind you that the general expression for the interaction between two charges (executing arbitrary motion) is

$$
\begin{equation*}
\vec{F}_{Q \leftarrow q}=\frac{q Q}{4 \pi \varepsilon_{o}} \frac{r}{\int \cdot \vec{u}^{\mathbf{u}}}\left\{\boldsymbol{r}^{2}-v^{2} \underline{\vec{u}}+\vec{\imath} \times \vec{a} \vec{\exists}+\frac{\vec{V}}{c} \times\left\{\times v^{2} \stackrel{\rightharpoonup}{\vec{u}}+\vec{\imath} \times \mathbf{a} \times \vec{a}\right\}\right. \tag{10.67}
\end{equation*}
$$

Broken out into terms that do and do not depend upon the sensing charge's velocity, we have

$$
\vec{F}_{Q \leftarrow q}=Q(\vec{E}+\vec{V} \times \vec{B})
$$

if we define

$$
\vec{E}=\frac{q}{4 \pi \varepsilon_{o}} \frac{r}{(\cdot \vec{u}\rangle} \boldsymbol{r}^{2}-v^{2} \overrightarrow{\vec{u}}+\vec{\imath} \times \mathbf{U} \times \vec{a}_{=}^{-}
$$

And

$$
\left.\vec{B}=\frac{q}{4 \pi c \varepsilon_{o}} \frac{r}{(\cdot \vec{u}\rangle} \right\rvert\, \times \mathbf{v}^{2}-v^{2} \overrightarrow{\vec{u}}+\vec{\imath} \times \mathbf{~} \times \vec{a} \bar{\beth}
$$

I could just plop that in your lap and say 'that's the way it is, now let's deal.' But, for better and for worse, that's not typically how it's taught. "For better" because those are pretty ugly expressions and some of the related tools are far simpler to use than those are, also it would be nice to see how those are logically connected to the special case ones that we've been using so far. "For worse" because when we follow the traditional pedagogical path it'll be easy to loose sight of one very important fact that's blatantly clear with these expressions - charges (and only charges) produce fields. Good physicists, perhaps the majority of physicists, get confused on that point because they followed the pedagogical path that you and I are about to follow without keeping these final equations in sight.

That warning issues, we're now going to obliquely, somewhat experimentally/historically approach the topic of fields generated by non-steady currents, time-varying currents. Section 7.1 may seem like quite the detour, and it partly is, but along the way, we're picking up one useful tool for our oblique - the relationship between magnetic flux and electro-motive force. Historically, this relationship played an important role in people figuring out how to deal with time-varying currents.

## First stop: Ohm's Law

## How does an electric field relate to a current inside a typical wire?

Absence of Field. First, let's think about what the charge carriers are doing inside the wire without the electric field. You might first guess 'just sitting there', but actually they're pretty free, so they're zipping all around, just like the air molecules in this room are doing. And, like the air molecules, they run into things a lot and bounce this way and that way and get nowhere in the end. So, at any given instant, a typical electron is going something like
$\nu_{\text {thermal }} \propto \sqrt{k T}$
But they collide frequently. At low enough temperatures, they mostly collide with impurities in the metal; at higher temperatures, they also collide with "phonons" - the jiggling of the atoms that make up the wire. Let's say, at some temperature, there's an average distance that they get before collision: $l_{\text {collision }}$ (and that distance will be shorter at higher temperatures since there are more phonons.) Then the time between collisions is something like

$$
\Delta t_{\text {collisions }}=\frac{l_{\text {collisions }}}{v_{\text {thermal }}}
$$

With Field. Now say we turn on an electric field. This has a similar effect to that of turning on a fan for air molecules - it superimposes on the random motion a slight drift. For an individual charge carrier, this drift is much smaller than the thermal velocity;
$\vec{v}=\vec{v}_{\text {drift }}+\vec{v}_{\text {thermal }} \approx \vec{v}_{\text {thermal }}$, so, for example, the time between collisions is still about the same

$$
\Delta t_{\text {collisions }} \approx \frac{l_{\text {collisions }}}{v_{\text {thermal }}} .
$$

However, looking at the whole population, the random thermal motions all cancel out, and the only net motion is this common drift.

(average displacement to the left due to the field)

Now, let's get more specific about this drift. Between collisions the electric field's the only thing acting upon a charge carrier, so

$$
\begin{aligned}
& \vec{F}=m \vec{a} \\
& q \vec{E}=m \vec{a} \\
& \frac{q \vec{E}}{m}=\vec{a}
\end{aligned}
$$

Of course, with a constant force, the average velocity is just

$$
\vec{v}_{\text {ave }}=\frac{\vec{v}_{i}+\vec{v}_{f}}{2}=\frac{\vec{v}_{i}+\vec{a} \Delta t}{2}=\frac{\vec{v}_{i}+\frac{q}{m} \vec{E} \Delta t}{2}
$$

Where the initial velocity is just its random thermal motion. Averaged over all the charge carriers, this comes to 0 .

$$
\left\langle\vec{v}_{\text {ave }}\right\rangle=\frac{q}{2 m} \vec{E} \Delta t
$$

Now, it's a bit of a cartoon, but it gives us the right flavor if we imagine that the charge carrier looses all it's drift each time it collides, and that happens every

$$
\Delta t_{\text {collisions }} \approx \frac{l_{\text {collisions }}}{v_{\text {thermal }}} . \text { So, }
$$

So, $\vec{v}_{\text {drift }}=\left\langle\vec{v}_{\text {ave }}\right\rangle=\frac{q}{2 m} \frac{l_{\text {collisions }}}{v_{\text {thermal }}} \vec{E}$
If that's the velocity with which they drift in response to the field, then we can easily say what the current density is:
$\vec{J}=n_{\text {carriers }} q \vec{v}_{\text {drift }}=n_{\text {molecules }} f_{\text {carriers/molecule }} q \vec{v}_{\text {drift }}=\left(n_{\text {molecules }} f_{\text {carriers/molecule }} \frac{q^{2}}{2 m} \frac{l_{\text {collisions }}}{v_{\text {thermal }}}\right) \vec{E}$
$\vec{J}=\sigma_{\text {conductivity }} \vec{E}$
where $\sigma$ is the conductivity (not the charge per area!) and the conductivity is $\rho=1 / \sigma$.

Ohm's Law - the current density is proportional to the field
This is often written in terms of Current and Voltage as

$$
I=\frac{V}{R}
$$

Warning: not all materials are "ohmic", for example, an object may have some capacity to store charge (rather than just passing them along) so it is capacitive.

Problem: 7.4 Two long, coaxial metal cylinders separated by a material with conductivity $s(s)=k / s$. What is the resistance, $R$ ?
Note: in the ppt, I derive $R \equiv-\frac{d I}{d V}=\int \frac{1}{\text { to find R for this problem. }} \frac{\sigma_{\text {cond }} d a_{\perp}}{} d l_{\|}$and use then use this relationship
The starting point is $V=-\int \vec{E} \cdot d \vec{l}$ and $I=\oint \vec{J} \cdot d \vec{a}=\oint \sigma_{\text {cond }} \vec{E} \cdot d \vec{a}$ where $\sigma_{\text {cond }}=\frac{k}{s}$.
If we choose a cylindrical surface, symmetry tells us that E is constant over it,

$$
\begin{aligned}
& I=E \oint \sigma_{\text {cond }} d a \\
& \text { Now, } \sigma_{\text {cond }}=\frac{k}{s} \text { so } \oint_{S}^{k} d a=\frac{k}{s} 2 \pi \Delta L=2 \pi L k
\end{aligned}
$$

Then

$$
I=E 2 \pi L k \Rightarrow E=\frac{I}{2 \pi L k} \text { which, rather surprisingly, is independent of } \mathrm{s} .
$$

Now,
$V=-\int_{a}^{b} \vec{E} \cdot d \vec{l}=-\int_{a}^{b} \frac{I}{2 \pi L k} d l=-\frac{I}{2 \pi L k}-a_{=}=-\left(\frac{\boldsymbol{\varphi}_{-}-a}{2 \pi L k}\right) I$
Apparently, $R=\left(\frac{(-a)}{2 \pi L k}\right)$

## Pr. 7.3

Generally, the place to start with these problems is

$$
I=\oint \vec{J} \cdot d \vec{a}=\oint \sigma_{\text {cond }} \vec{E} \cdot d \vec{a} \text { and } V=-\int \vec{E} \cdot d \vec{l}
$$

$R C=\frac{V / I}{V / Q}=\frac{Q}{I}$ so want to find $\mathrm{Q} / \mathrm{I}$.
$I=\oint \vec{J} \cdot d \vec{a}=\oint \sigma_{\text {cond }} \vec{E} \cdot d \vec{a}=\sigma_{\text {cond }} \oint \vec{E} \cdot d \vec{a}$
But Gauss's Law tells us that this last integral is
$\oint \vec{E} \cdot d \vec{a}=\frac{Q_{\text {encl }}}{\varepsilon_{o}}$
So, $I=\sigma_{\text {cond }} \frac{Q_{\text {encl }}}{\varepsilon_{o}}$
So, $\frac{Q}{I}=\frac{\varepsilon_{o}}{\sigma_{\text {cond }}}$

Power Disipation - We've previously seen that the work done in moving a charge across a potential difference is

$$
W_{\rightarrow q}=q V
$$

The rate at which this work is done is then

$$
P=\frac{d W_{\rightarrow q}}{d t}=\frac{d q}{d t} V=I V
$$

So that's the rate at which the electric field is investing energy in the charges as they move through the potential difference. Meanwhile though, they are not accelerating because they keep running into the impurities and phonons and slowing back down. Through these collisions the energy must be getting dissipated at the same rate as it's
getting invested. We can more explicitly phrase this in terms of the collisions, or at least the associated resistance through Ohm's Law

$$
P=\frac{V^{2}}{R}=I^{2} R
$$

## EMF

We define "electro motive force" as the path integral of force per charge:

$$
\varepsilon \equiv \oint \vec{f} \cdot \overrightarrow{d \ell}=\oint \vec{f} \cdot \vec{d} \cdot \overrightarrow{\ell \ell}
$$

Why do this? Because, within a conductor, over the region that the force is applied a charge separation will be established. That charge separation, of course, will generate an electric field, E , which will itself exert a force on the charges in that region, qE . In equilibrium,
$\vec{F}+q \vec{E}=0$
$\vec{F} / q+\vec{E}=0$
$\vec{f}+\vec{E}=0$
$\vec{f}=-\vec{E}$
$\int \vec{f} \bullet \vec{\ell}=-\int \vec{E} \bullet d \vec{\ell}$
$e m f=V$
Now, while this force may be confined to a small region, the electric field is not, and the voltage difference between the two ends of that region is the same whether you cross inside the region or outside it (that's what it means to be path independent). Of course, $\mathrm{V}=\mathrm{IR}$ in a resistive element.

In a battery. To make this all concrete, imagine a battery wired up to an external resistor. The way a battery works is chemical processes drive positive and negative ions opposite directions through a solution (though, for our purposes, it might as well be fairies running a conveyor belt on which they load charged particles.) This produces a charge separation which, in turn, generates an electric field that opposes further charge separation. Eventually, the charge separation gets large enough that the force that's driving the charges is perfectly opposed by the electric field. Now, that leads to the chain of math/logic shown above. In the end, there's a voltage established between the two battery terminals, and that's equal to the emf of the generating a charge separation. Now, if that's the voltage difference between the two terminal on the inside, then, well, that's the voltage difference between the two terminals on the outside. That is, from the perspective of the resistor wired to the two terminals, that's the voltage drop across it. So that's what drives a current through it.

One more point, Griffiths notes that, since this driving force is likely confined to a specific region (in this example, inside the battery), we're free to extend the integral
to be a closed loop - since the integrand is 0 everywhere outside the region (battery), the integral out there contributes nothing.

$$
\varepsilon \equiv \oint \vec{f} \cdot \overrightarrow{d \ell}
$$

## Motional emf

This idea holds regardless of what force is driving the charge separation. One popular force is magnetic. Suppose a metal bar of length $L$ is moved through a magnetic field with a magnitude $B$ into the page at a speed $v$.


There is a magnetic force on each electron in the wire:

$$
\begin{gathered}
\vec{F}_{\mathrm{mag}}=(-e) \vec{v} \times \vec{B} \\
F_{\mathrm{mag}}=e v B \quad \text { downward }
\end{gathered}
$$

There is also a force on each proton, but they are not free to move. Electrons will move from the top of the bar to its bottom, polarizing the bar. This will produce a downward electric field and upward force $\vec{F}_{\mathrm{e}}=-e \vec{E}$ on each electron in the bar. Equilibrium will be reached when the magnetic and electric forces are the same size:

$$
e v B=e E
$$

In terms of Voltage and Emf:

$$
\begin{aligned}
& e m f=V \\
& \int \vec{f} \bullet d \vec{\ell}=-\int \vec{E} \bullet d \vec{\ell} \\
& \int-v B \hat{y} \bullet d \vec{y}=-\int \vec{E} \bullet d \vec{y} \\
& -v B L=-E L
\end{aligned}
$$

The potential difference between the ends of the bar is:

$$
|\Delta V|=E L=v B L
$$

The bar acts like a battery because a charge separation is maintained by a non-Coulomb force (in this case the magnetic force). Suppose the bar is pulled along two frictionless metal rails which are connected by a resistor (as shown below).


Then the emf separates charge and that charge separation drives current through the resistor.

$$
I R=|\Delta V|=e m f=v B L
$$

As the bar moves at a constant speed and a steady current flow, the resistor dissipates energy to its surroundings. Where does this energy come from? Work must be done on the bar to keep it moving.


If there is a current $I$ flowing upward through the bar, the magnetic force on it is $I L B$ to the left on it. Someone or something must apply the same size force to the right on the bar to keep it moving at a constant speed. Of course, once we've got a current flowing up the wire, there's a force exerted on it back to the left to the tune of

$$
\vec{F}_{\text {mag }}=\int I d \vec{l} \times \vec{B}=-I L B \hat{x}
$$

This is the force that would have to be opposed in order to maintain a constant speed.

Example: Pr. 7.7. This problem asks us to consider the situation above and, first, to express this magnetic force in terms of the resistance rather than the current

$$
\vec{F}_{m a g}=-I L B \hat{x}=-\left(\frac{v B L}{R}\right) L B \hat{x}=-\left(\frac{B L^{2}}{R}\right) \vec{v}
$$

Next, if this force went unopposed, assuming some initial velocity, $\mathrm{v}_{\mathrm{o}}$, what would be the velocity as a function of time?
$m \frac{d \vec{v}}{d t}=\vec{F}_{\text {mag }}=-\left(\frac{\left(L^{\boldsymbol{Z}}\right.}{R}\right) \vec{v}$
$\frac{d \vec{v}}{d t}=-\left(\frac{\left(\left\langle L^{\hat{2}}\right.\right.}{m R}\right) \vec{v}$
Apparently,

$$
\vec{v}=\vec{v}_{o} e^{-\left(\frac{\beta L^{2}}{m R}\right) t}
$$

Next, it asks to show that, eventually, all the bar's kinetic energy gets dissipated through the resistor.

$$
\begin{aligned}
& P=I^{2} R \\
& \frac{d E}{d t}=I^{2} R
\end{aligned}
$$

$$
\Delta E=\int_{0}^{t} I^{2} R d t=\int_{0}^{t}\left(\frac{v B L}{R}\right)^{2} R d t=\int_{0}^{t}\left(\frac{\vec{v}_{o} e^{-\left(\frac{\beta L^{2}}{m R}\right) t} B L}{R}\right)^{2} R d t=\int_{0}^{t} \frac{\sigma_{o} B L^{2}}{R} e^{-2\left(\frac{\left.B L^{-2}\right)}{m R}\right) t} d t
$$

$$
\Delta E=\frac{\boldsymbol{\sigma}_{o} B L^{\mathcal{Z}}}{R} \frac{m R}{2 \boldsymbol{B} L_{-}^{\mathfrak{Z}}}\left(e^{-2\left(\frac{\left.\Delta L^{\mathcal{Z}}\right) t}{m R}\right)}-1\right)=\frac{1}{2} m \vec{v}_{o}^{2}\left(e^{-2\left(\frac{\boldsymbol{\sigma} L^{\mathcal{Z}}}{m R}\right) t}-1\right)
$$

as $t$ goes to infinity, this goes to
$\Delta E=-\frac{1}{2} m \vec{v}_{o}{ }^{2}$
so it loses all its initial kinetic energy.

## Example 7.4 - Faraday's Disk

A metal disk of radius $a$ rotates with an angular frequency $\omega$ (counterclockwise viewed from above) about an axis parallel to a uniform magnetic field. A circuit is made by a sliding contact. What is the current through the resistor $R$ ?

Note: This problem cannot be solved using $\varepsilon=-d \Phi / d t$.

Find the $e m f$ by calculating the line integral of the force per charge from the center to the contact point. The speed of a point at a distance $s$ from the center is $v=\omega s$, so the force per charge is $\vec{f}_{\text {mag }}=\vec{v} \times \vec{B}=\operatorname{\omega s} B \hat{s}$. The emf is

$$
\varepsilon=\int \vec{f}_{m a g} \cdot d \vec{\ell}=\int_{0}^{a} f_{m a g} d s=\omega B \int_{0}^{a} s d s=\frac{\omega B a^{2}}{2} .
$$

The current found using Ohm's law is

$$
I=\frac{\varepsilon}{R}=\frac{\omega B a^{2}}{2 R} .
$$

By the RHR, it flows from the center to the outer edge of the disk.

## Phrasing in terms of Magnetic Flux

Let's return to the original result for the emf. It can be rephrased a bit. Here, we are changing the area of a loop through which the field is flowing.
$e m f=B v L=B \frac{d A}{d t}=\frac{d(\beta A)}{d t}=\frac{d \Phi_{B}}{d t}$
If we impose the Right Hand Rule for sign conventions, we'd have

$$
e m f=-\frac{d \Phi_{B}}{d t}
$$

## Problem 7.11

A square loop is cut out of a thick sheet of aluminum. It is placed so that the top portion is in a uniform, horizontal magnetic field of 1 T into the page (as shown below) and allowed to fall under gravity. The shading indicates the field region. What is the terminal velocity of the loop? How long does it take to reach $90 \%$ of the terminal velocity?

Use $x$ for the distance from the bottom of the field region. The magnetic flux is $\Phi=B \ell x$, so the size of the $e m f$ is

$$
|\varepsilon|=\frac{d \Phi}{d t}=B \ell v .
$$

By Ohm's law, the size of the current is $I=\varepsilon / R=B \ell v / R$. By the RHR, the current flows in the direction of $\vec{v} \times \vec{B}$ (for the top segment), which is to the right.
The magnetic field is perpendicular to the current in the loop so the force is

$$
|F|=I\left|\int \overrightarrow{d \ell} \times \vec{B}\right|=I \ell B=\frac{B^{2} \ell^{2} v}{R},
$$

where $\ell$ is the length of a side. By the RHR, the direction of $\overrightarrow{d \ell} \times \vec{B}$ and the force on the loop is to the upward. This is a 1-D problem. Using downward as positive, Newton's second law is

$$
m g-\frac{B^{2} \ell^{2}}{R} v=m a=m \frac{d v}{d t}
$$

Terminal "velocity" is reached when the acceleration is zero, so

$$
m g-\frac{B^{2} \ell^{2}}{R} v=0 \Rightarrow v_{t}=\frac{m g R}{B^{2} \ell^{2}}
$$

The equation of motion can be written as

$$
g-\frac{B^{2} \ell^{2}}{m R} v=\left(1-\frac{v}{v_{t}}\right) g=\frac{d v}{d t}
$$

This can be integrated to get (starts from rest):

$$
\begin{gathered}
\frac{d v}{\left(v_{t}-v\right)}=\frac{g}{v_{t}} d t \\
\int_{0}^{v(t)} \frac{d v}{\left(v_{t}-v\right)}=\frac{g}{v_{t}} \int_{0}^{t} d t \\
{\left[-\ln \left(v_{t}-v\right)\right]_{0}^{v}=-\ln \left(\frac{v_{t}-v}{v_{t}}\right)=\frac{g t}{v_{t}}} \\
\frac{v_{t}-v}{v_{t}}=e^{-g t / v_{t}} \Rightarrow v=v_{t}\left(1-e^{-g t / v_{t}}\right) .
\end{gathered}
$$

At $90 \%$ of terminal velocity,

$$
\begin{gathered}
\frac{v}{v_{t}}=0.9=\left(1-e^{-g t / v_{t}}\right) \Rightarrow e^{-g t / v_{t}}=0.1, \\
-g t / v_{t}=\ln (1 / 10) \Rightarrow t=\frac{v_{t}}{g} \ln (10) .
\end{gathered}
$$

Suppose the cross sectional area is $A$. The mass is $m=4 \eta(A \ell)$, where $\eta=2.7 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$ is the mass density of aluminum. The resistance of the loop is $R=4(\ell / A \sigma)=4 \ell \rho / A$, where $\rho=2.8 \times 10^{-8} \Omega m$ is the resistivity of aluminum. The terminal velocity is

$$
v_{t}=\frac{m g R}{B^{2} \ell^{2}}=\frac{(4 \eta A \ell) g(4 \ell \rho / A)}{B^{2} \ell^{2}}=\frac{16 g \eta \rho}{B^{2}}
$$

so the time to reach $90 \%$ of terminal velocity is

$$
t=\frac{v_{t}}{g} \ln (10)=\frac{16 \eta \rho}{B^{2}} \ln (10)=\frac{16\left(2.7 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(2.8 \times 10^{-8} \Omega m\right)}{(1 \mathrm{~T})^{2}}=2.8 \times 10^{-3} \mathrm{~s}=2.8 \mathrm{~ms}
$$

The units work because ohm $=\mathrm{V} / \mathrm{A}$ and $\mathrm{T}=\mathrm{N} /(\mathrm{Am})$.

## Proof of Generality

Say you have a fairly elastic and mobile wire loop in the presence of a non-uniform magnetic field (steady in time, but varying from one location to another.) Say you flex and move this wire. Here's a picture representing the old wire configuration and the new one.


The green region represents the change in area. That can be described in terms of each little point on the loop having its own velocity such that it gets to the new location in time dt.
Note that the area swept out by moving our little line segment dl from the inner curve position to the outer curve position is $d \vec{a}=\vec{v} d t \times d \vec{l}$

So, the little bit of flux gained by moving $d l$ out is
$\vec{B} \cdot d \vec{a}=\vec{B} \cdot(d t \times d \vec{l}=(\beta \times \vec{v} d t) d \vec{l}=-(d t \times \vec{B}) d \vec{l}$
Making use of Vector Identity (1): $\vec{A} \cdot \vec{C}=\vec{C} \times \vec{C}$ and the fact that flipping the order of a cross-product flips signs.
Then the total gain in flux from expanding the loop is gotten by summing over the whole loop.

$$
d \Phi_{B}=-\oint(d t \times \vec{B} ; d \vec{l}
$$

Finally, divide by the dt

$$
\begin{aligned}
& \frac{d \Phi_{B}}{d t}=-\oint\left(\vec{B} ; d \vec{l}=-\oint f_{m a g} \cdot d \vec{l}=-E m f_{\operatorname{mag}}\right. \\
& \frac{d \Phi_{B}}{d t}=-E m f_{m a g}
\end{aligned}
$$

## Example 7.10 Motors:

The commutator switches the direction of the current as the loop spins so that it is always moving in the same direction on each side of the axis.


Generators: Suppose a loop rotates at an angular speed $\omega$.
As the loop spins, there is a motional emf on each side with length $w$, but in opposite directions. That leads to a conventional current around the loop.


The emf is largest when the angle $\theta$ is $90^{\circ}$, because the wires are moving the fastest in the direction perpendicular to the magnetic field. The size of the emf on the left wire depends on the component of the velocity perpendicular to the magnetic field:

$$
\mathrm{emf}_{\text {left }}=-\frac{d \Phi}{d t}=-\frac{d \boldsymbol{\beta} \cdot \overleftarrow{A}^{-}}{d t}=-\frac{d \boldsymbol{\beta} A \cos \theta}{d t}=B A \text { in } \theta \frac{-d \theta}{d t}=B A \text { in } \theta \varrho .
$$

"The book talks about how emf is the integral of a force per unit charge. What does this quantity even mean conceptually?" Casey P, - other (non-electrical) work done on a charged object per charge

The way Griffiths is describing it, seems like the Emf is the pushing power outside of the source (the battery), that keeps the charges moving when they are away from the battery itself. Is this right, and if so, does this answer your question? Freeman,
"Can we go over griffith's derivation of Vave for eqn 7.6?" Jessica
"In equation 7.10 and above, Griffiths shows that $f$ (the source) and the electric field E are essentially one in the same, and I am not necessarily following this. I think some sort of vector diagram might help." Rachael Hach in equilibrium rather, they must balance
"Also I didn't follow the proof of eqn 7.13, can we go over this?"
Jessica we will, but on Friday
I'd also like to see the derivation of 7.13 . Where does the negative sign come from and what does it mean?

## Spencer

Yes, and I don't quite understand what Griffith's is trying to illustrate in Figure 7.13?

## Casey McGrath

"For figure 7.8 In what circumstances would you have current flowing in greater than it is out I thought It was just the movement of the electrons already in the wire." Antwain only for that split second while the current is getting turned on; soon there-after, a steady current is established (unless we continually vary the source - AC)
"Section 7.1.3 gave me a little trouble connecting the math to the physical system (in this case a generator). Could we talk about how a generator is able to create a constantly changing flux in a loop?"
Ben Kid - next time
"Can we briefly talk about that "flux rule paradox" Griffith's illustrates in Figure 7.14?" Casey McGrath - in the derivation, the rate of change in area, da/dt corresponded to the rate of charged particles moving, vL. The Flux rule only relates such changes in area to emf.

