Phys 233 Practice Test 2 Ch 5-7

1. A 3-kg mass is attached to a spring with a spring constant of 2 N/m. Initially, the mass is released from rest at 0.5 m from its equilibrium. (30 pts.)

   (a) With what angular frequency would it oscillate if there were no damping?

   (b) Suppose there is a linear damping force, -*bv*, and, as a result, the mass oscillates at half the frequency it would if there were no damping. What must be the value of *b*?

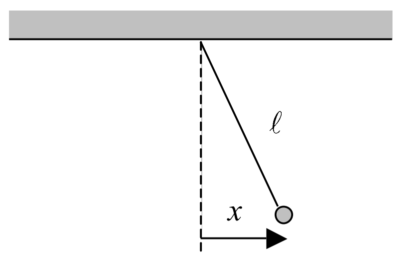
   (c) What equation (just in terms of symbols) describes the mass's displacement as a function of time?

   (d) Given the initial conditions, find numeric values for each constant in that equation.

   (e) What will be the mass's displacement 3 s after the initial time?

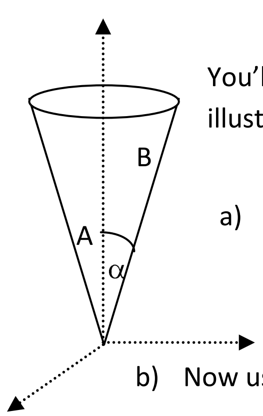
   (f) Say it was subjected to a driving force of $F(t) = (0.9N)\cos\left((\pi/2\text{ rad/s})\,t\right)$, what would be its position 20 **minutes** after the initial time?

2. A simple pendulum consists of a bob of mass *m* attached by a light string of length ℓ. Assume that there is no drag and that the pendulum swings in a plane. (15 pts.)



   (a) Find an expression for the potential energy as a function of *x*, the horizontal displacement of the bob.

   (b) Use the result from part (a) to determine the angular frequency, ω, for *small* oscillations.



You'll Find the equation of the shortest path along the surface of a cone with half-angle a (as illustrated) between points A and B.

a) First, if the path length is expressed as $S = \int_A^B ds = \int_A^B F(\phi', \phi, \rho)\,d\rho$, then find an expression for $F(\phi', \phi, \rho)$

b) Now use that to find an expression for $\phi(\rho)$; you don't need to solve for the integraction constants.

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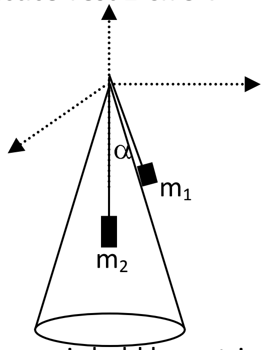

1. One mass is held by a string to rest on the surface of a cone (m_1); the string runs up to the cone's peak and then down through a hole so that a second mass (m_2) dangles from the other end of the string in the center of the cone. You'll find the rate at which the first mass can spin around the outside of the cone in order to hold the second mass suspended without rising or falling.

a) Write the Lagrangian of this system; Draw or carefully explain what you use for he generalized coordinates.

b) Find the equations of motion for this system.

c) For what rotational rate, $\dot{\phi}$, of mass 1 can mass 2 hang stationary?

d) What is the condition on m_1 and m_2 for this to even be possible (your equation for part c makes physical sense only if…)

2. Find the Lagrangian for this system of a puck (of mass *m*) resting on the surface of a cone while suspended from a spring (of spring constant *k* and equilibrium length r_o) mounted to the cone's peak. It should be in terms of ρ and ϕ. Note: since the cone is upside-down, $\rho = -z \tan\alpha$.