Mon. 10/1 Tues. 10/2 Wed. 10/3 Fri. 10/5	**4.7-.8** Curvilinear 1-D, Central Force **5.1-.3 (2.6)** Hooke's Law, Simple Harmonic (Complex Sol'ns) ***What (research) I Did Last Summer***: *AHoN 116 @ 4pm* Review for Exam 1	 **HW4b** (4.C-.F)
Mon. 10/8 Wed. 10/10 Thurs 10/11	**Study Day** **Exam 1**(Ch 1-4)	 **HW5a** (5.2, 5.10)

Equipment

- Inclined plane with cart, mass, string, pully
- Disc & ring to roll down plane
- Air puck with belt around it and string to demonstrate v=rw
- Ppt and python of polar coordinates
- Plotting Tutorial handout

Coordinating for Wednesday evening: I'll get goodies, Alan and I will bribe the Freshmen and Sophomores, we'll be in AHoN 116 at 4pm.

Note on computational problem: I encourage you to first write the short code that's in this Plotting tutorial; once that works, you'll be ready to define your own functions to plot.

Time from energy considerations

You remember when we were nominally taking a 'force' approach, but we rephrased the 1-D version of Newton's 2nd as a relationship between velocity and position (using $\dot{v} = v\frac{dv}{dx}$) so we could say how speed depended on position without bothering with time. Similarly, now that we're nominally using an energy approach, we still rephrase it to pull out a time. Here's how that goes.

If energy is conserved, $E = T + U(x)$, then:

$$T = \tfrac{1}{2}m\dot{x}^2 = E - U(x),$$

which can be used to find the velocity as a function of position:

$$\dot{x}(x) = \pm\sqrt{2(E - U(x))/m}\,.$$

(note: this may look a little more familiar if I multiply both sides by m and recognize the left-hand-side as being momentum $p(x) = \pm\sqrt{2m(E - U(x))}$. If you're taking quantum right now, thanks to the relationship between p and wavelength, this comes in handy.)

The velocity is $\dot{x} = dx/dt$, so $dt = \frac{dx}{\dot{x}}$. This can be integrated to find the time for motion between two points:

$$t = \int_{x_0}^{x} \frac{dx'}{\dot{x}(x')} = \int_{x_0}^{x} \frac{dx'}{\sqrt{2\left(E - U(x')\right)/m}} \; .$$

The book goes through this; however, for many situations this can be difficult to calculate because the integrand goes to infinity as it approaches the turning point where $\dot{x} = 0$. Even for the simple pendulum, there is no analytical solution (see Prob. 4.38). Energy conservation is typically <u>not</u> a good way to get information about time.

That disclaimer issues, here *is* a tractable example.

<u>Example 3</u>: (2.10 of Fowles & Cassiday 5th ed.) A particle of mass m is released from rest at $x = b$ and its potential energy is $U(x) = -k/x$. (a) Find its velocity as a function of position. (b) How long does it take the particle to reach the origin?

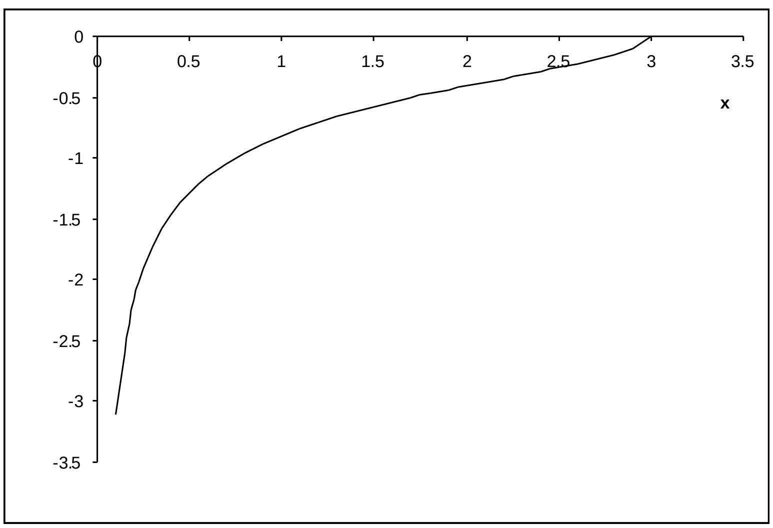
(a) At $x = b$, the kinetic energy is $T = 0$ so the total energy is $E = U(b) = -k/b$. Since energy is conserved:

$$E = -k/b = T + U(x) = \tfrac{1}{2} m\dot{x}^2(x) - k/x \, ,$$

so taking the <u>negative</u> root because the potential attracts the particle toward the origin:

$$\dot{x}(x) = -\sqrt{\frac{2k}{m}\left(\frac{1}{x} - \frac{1}{b}\right)} \, .$$

An example of this is shown below.



(b) Since $\dot{x} = dx/dt$, the time required to move from b to 0 is:

$$\int_0^t dt = t = \int_b^0 \frac{dx}{\dot{x}(x)} = -\sqrt{\frac{m}{2k}} \int_b^0 \frac{dx}{\sqrt{1/x - 1/b}} = +\sqrt{\frac{mb}{2k}} \int_0^b \frac{\sqrt{x}\, dx}{\sqrt{b - x}}$$

Use the integral (from the front cover of the text):

$$\int \frac{\sqrt{y}\, dy}{\sqrt{1-y}} = \sin^{-1}\left(\sqrt{y}\right) - \sqrt{y(1-y)}$$

with the change of variables $x = by$ and $dx = b\, dy$. The integral for the time becomes:

$$t = \sqrt{\frac{mb}{2k}} \int_0^1 \frac{\sqrt{by}\, b\, dy}{\sqrt{b-by}} = \sqrt{\frac{mb^3}{2k}} \int_0^1 \frac{\sqrt{y}\, dy}{\sqrt{1-y}} = \sqrt{\frac{mb^3}{2k}} \left[\sin^{-1}\left(\sqrt{y}\right) - \sqrt{y(1-y)}\right]_0^1$$

$$t = \sqrt{\frac{mb^3}{2k}} \sin^{-1}(1) = \sqrt{\frac{mb^3}{2k}} \left(\frac{\pi}{2}\right)$$

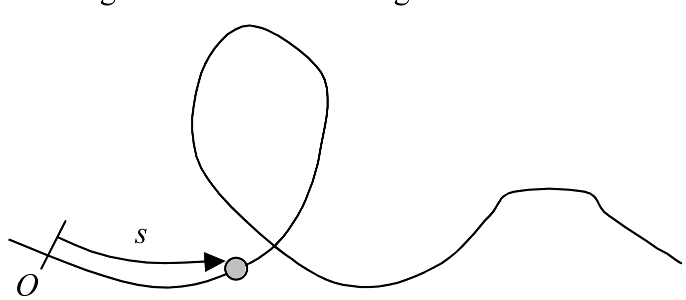
$$t = \pi \sqrt{\frac{mb^3}{8k}}$$

Curvilinear One-Dimensional Systems:

With *forces* and *momentum,* you were dealing with *vectors,* so it was particularly important to define a good, orthogonal, coordinate system in order to keep track of the components. But with *energy,* you're dealing with *scalars;* you don't care about the *direction* of the velocity, just its magnitude. So it becomes less important, sometimes unnecessary or even inconvenient to think strictly in terms of orthogonal coordinates. Another difference between a force approach and an energy approach is that with energy, potential energy in particular, we're generally more interested in the *change* than in any specific value.

What this adds up to is that we can be rather cavalier in describing our system's energy in terms of whatever variables are convenient.

Many systems can be describe by one variable, or "coordinate" (a distance, angle, etc.) even if the motion is not along a line! An example of such a 1-D system is a bead sliding along a curved rigid wire. If the distance along the curve from an origin *O*.



The speed of the bead is $\dot{s}$ and the kinetic energy is:

$$T = \tfrac{1}{2} m\dot{s}^2 .$$

The force along the wire (tangential) is:

$$F_{\text{tang}} = m\ddot{s}\,.$$

Taking it as a given that the bead stays on the string, than any forces acting upon it, even if they depend upon, say the bead's *elevation,* should be able to be rephrased in terms of the bead's location along the wire (since, even *elevation* is a function of how far the bead is along the wire.) So

$$F_{\text{tang}}(s) = m\ddot{s}$$

If all of the tangential forces are conservative, a potential energy can be defined as:

$$U(s) = -\int_0^s F_{\text{tang}}(s')\,ds'$$

Of course, the total energy of the system depends on the kinetic and this potential:

$$E = T + U(s).$$

Phrasing the energy-force relationship the other way,

$$F_{\text{tang}} = -\frac{\partial U}{\partial s}.$$

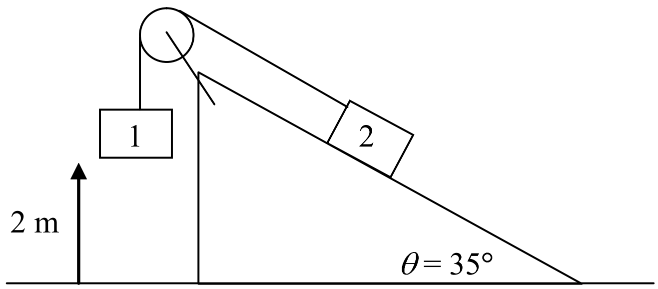
If $dU/ds = 0$, the bead is at a point of equilibrium. If $U(s)$ is a minimum (maximum), the equilibrium is stable (unstable).

Generally, if a system depends on only one coordinate *s,* then (as long as we're in an inertial reference frame), there's an equilibrium at $dU/ds = 0$. (Counter example: can't do this for a pendulum on an accelerating train.)

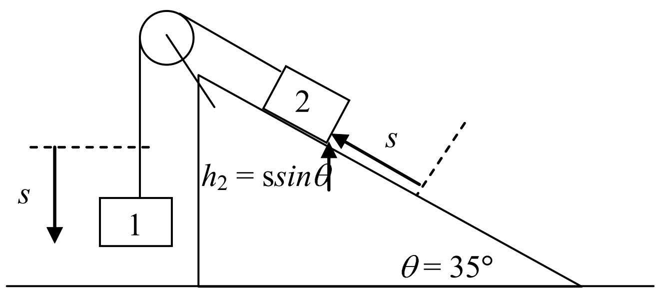
If two objects within your system interact in such a way that their separation measured along this coordinate is constant, then there's no net work done by that interaction, (block A exerts force forward on block B while they move forward, we'll block B exerts force backward on block A while they move forward – no net work.) This does not affect the total mechanical energy, so conservation of energy can still be used if all other forces are conservative.

Do on CHALK board so I can tuck it away and then return to it.

Example 1 (I do most of): Suppose the masses $m_1 = 6\,\text{kg}$ and $m_2 = 4\,\text{kg}$ are initially at rest. Ignore friction and assume that the mass of the pulley is small. What will the speed of m_2 be when it hits the ground?



We could use a nice cartesian coordinate system to describe the motion of the two blocks, or maybe even two different coordinate systems, one for each block. Then again, we could simply call the distance they move *s* (which is the same for both.) They are tied together, so they move the same amount (until 2 hits the floor).



The total mechanical energy of the system is:

(walk them through this process)

Q: First in very general terms:

$$E = T_1 + T_2 + U_1 + U_2$$

Q: next, in terms of speeds and heights:

$$E = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 + m_1 g h_1 + m_2 g h_2$$

Q: now in terms of the common distance they move, *s* and common speed $\dot{s}$:

$$E = \tfrac{1}{2} m_1 \dot{s}^2 + \tfrac{1}{2} m_2 \dot{s}^2 - m_1 g s + m_2 g s \sin\theta$$

Check signs: observe that m_1's potential becomes more negative with s while m_2's becomes more positive.

Okay, now that we've expressed the energy in terms of this variable, we'll use this expression to find the final speed.

The initial condition of the system is $s(0) = 0$ and $\dot{s}(0) = 0$, so $E_o = 0$. Conservation of mechanical energy gives:

$$0 = \tfrac{1}{2}\left(m_1 + m_2\right)\dot{s}^2 - m_1 g s + m_2 g s \sin\theta \,.$$

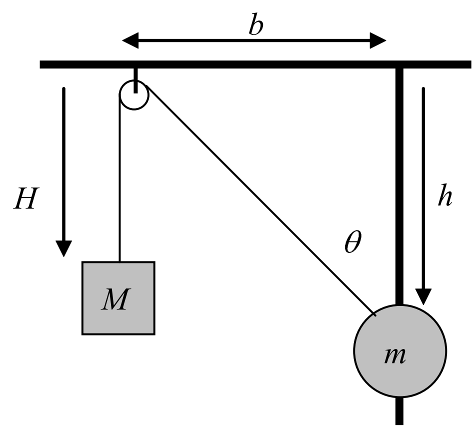
Solving for the speed and putting in the final condition $s = 2$ m gives:

$$\dot{s} = \sqrt{\frac{2gs\left(m_1 - m_2 \sin\theta\right)}{m_1 + m_2}} = \sqrt{\frac{2\left(9.8\,\mathrm{m/s^2}\right)\left(2\,\mathrm{m}\right)\left[6\,\mathrm{kg} - \left(4\,\mathrm{kg}\right)\sin 35^\circ\right]}{4\,\mathrm{kg} + 6\,\mathrm{kg}}} = 3.8\,\mathrm{m/s}$$

<u>**Example 2 / Exercise**</u>: (Prob. 4.36) The ball (mass *m*) has a hole through it and slides on a frictionless vertical rod. A light string of length *l* passes over a small frictionless pulley and attaches to another mass *M*. The positions of the objects can be specified by the angle *θ*.

(a) Write an expression for the potential energy $U(\theta)$.

(b) Find whether or not the system has an equilibrium position and for what values of *m* and *M*. Are any equilibrium positions stable?

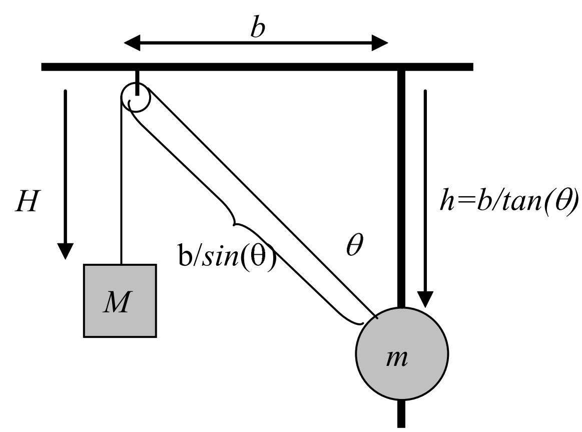


(a) Find an expression for the potential.

First, purely in terms of how far the masses are down from the ceiling, h and H, what's the potential?

$$U = -mgh - MgH$$

Now that we've got that expression, rephrase it in terms of the angle θ.



The length of the string from the pulley to the ball is $b/\sin\theta$. The heights are $h = b/\tan\theta$ and $H = l - b/\sin\theta$ (calling the total length of the string l). Since these distances are measured <u>below</u> a reference point, the PE is:

$$U = -mgh - MgH$$
$$U = -mg\left(b/\tan\theta\right) - Mg\left(l - b/\sin\theta\right)$$

$$\boxed{U(\theta) = gb\left(\frac{M}{\sin\theta} - \frac{m}{\tan\theta}\right) - Mgl = \frac{gb}{\sin\theta}(M - m\cos\theta) - Mgl}.$$

The last term is just a constant that will have no bearing on whether a particular angle is an equilibrium.

(b) Okay, what about equilibria? The derivative of U is:

$$\frac{dU}{d\theta} = -\frac{gb}{\sin^2\theta}\cos\theta\left(M - m\cos\theta\right) + \frac{gb}{\sin\theta}m\sin\theta$$

It'll be convenient to put this all over a common denominator, so rather than canceling off the sin's, I'll actually multiply be another factor of sin on the last term

$$\frac{dU}{d\theta} = -\frac{gb}{\sin^2\theta}\left(M\cos\theta - m\cos^2\theta\right) + \frac{gb}{\sin^2\theta}m\sin^2\theta = -\frac{gb}{\sin^2\theta}\left(M\cos\theta - m\left(\cos^2\theta + \sin^2\theta\right)\right)$$
$$-\frac{gb}{\sin^2\theta}\left(M\cos\theta - m\right) = \frac{gb}{\sin^2\theta}\left(m - M\cos\theta\right)$$

Okay, at an equilibrium angle,

$$\left.\frac{dU}{d\theta}\right|_{\theta_{eq}} = \frac{gb}{\sin^2\theta_{eq}}\left(m - M\cos\theta_{eq}\right) = 0$$

So it must be that

$$\left(m - M\cos\theta_{eq}\right) = 0$$

or

$$\cos\theta_{eq} = \frac{m}{M}.$$

This only has solutions if $m \le M$. When $m = M$, the answer is $\theta = 0$, which is not possible for a finite length of string. Therefore, if $m < M$ there is an equilibrium point at:

$$\theta_{\mathrm{eq}} = \cos^{-1}\left(\frac{m}{M}\right).$$

The requirement that $m < M$ makes sense because in equilibrium the tension of the string must be equal to Mg, but upward force on m is only a fraction of the tension. Therefore, m must be smaller if they are to both be in equilibrium.

Take the second derivative of U to check the stability:

$$\frac{d^2U}{d\theta^2} = \frac{gb}{\sin^4\theta}\left\{\sin^2\theta(M\sin\theta) - [m - M\cos\theta](2\sin\theta\cos\theta)\right\}$$

We already figured out that at the equilibrium point, the term in square brackets is zero, so we're left with:

$$\left.\frac{d^2U}{d\theta^2}\right|_{\theta_{\text{eq}}} = \frac{gb}{\sin^4\theta_{\text{eq}}}\sin^2\theta_{\text{eq}}M\sin\theta_{\text{eq}} = \frac{gb}{\sin\theta_{\text{eq}}}M$$

Since we know $0 \le \theta < 90^\circ$, $\sin\theta_o$ is positive. Therefore, $\left(d^2U/d\theta^2\right)_{\theta_o} > 0$ and the equilibrium is stable. (we could go a step further and use that $\sin\theta_{\text{eq}} = \sqrt{1-\cos^2\theta_{\text{eq}}} = \sqrt{1-(m/M)^2}$ to get an expression for this second derivative, but we've already figured out what we need to: that it's positive.

Handling systems with <u>multiple</u> "coordinates" (not necessarily Cartesian, polar, etc.) will be easier using Lagrange's approach (Ch. 7)

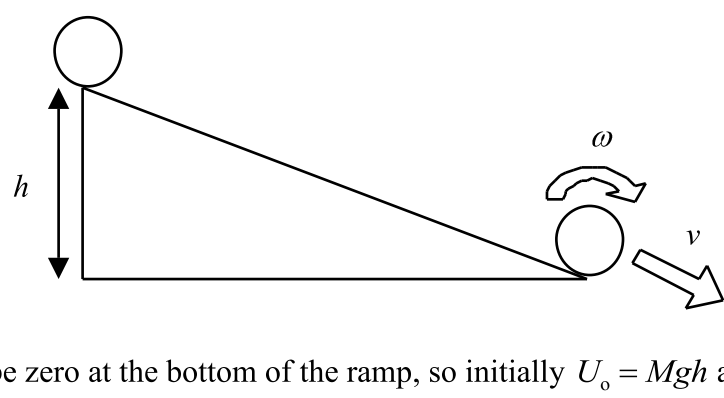
<u>**Rigid Bodies**</u>**:** Before doing another one-coordinate example, I want to remind you about how to handle rigid bodies with an energy approach. The end of the chapter discusses multi-particle systems. One common situation is if they define a rigid object. Then the total kinetic energy can be rephrased as the kinetic energy of the center of mass + the kinetic energy of all the parts *about* the center of mass:

$$\sum \tfrac{1}{2} m_i v_{i.cm}^2 = \tfrac{1}{2}\sum m_i \left(r_{i-cm}\dot{\phi}\right)^2 = \tfrac{1}{2}\sum m_i \left(r_{i-cm}\right)^2\dot{\phi}^2 = \tfrac{1}{2} I\dot{\phi}^2$$

So, for a rigid body rotating about an axis in a fixed direction (we generalize this in Ch. 10):

$$T = T_{CM} + T_{rot} = \tfrac{1}{2}MV^2 + \tfrac{1}{2}I\omega^2.$$

<u>**Example 1**</u>: (related to Ex. 4.9) If they start from rest, which will make it to the end of a ramp faster, a cylinder (disk) or a thin ring? Do the masses or radii matter?



Define the PE to be zero at the bottom of the ramp, so initially $U_o = Mgh$ and finally $U_f = 0$. The initial KE is $T_o = 0$ and the final KE is

$$T_f = T_{CM} + T_{rot} = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2.$$

For an object that is rolling without slipping,

$\omega = v/R$,

Pause and demonstrate this with puck with string. Δx = 2πR while Δθ = 2π

where R is the radius.

The moment of inertia for a cylinder is $I_{cyl} = \tfrac{1}{2}MR^2$, so

$$T_{f.cyl} = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 = \tfrac{3}{4}Mv^2$$

and for a thin ring it is $I_{ring} = MR^2$, so

$$T_{f.ring} = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}\left(MR^2\right)\left(\frac{v}{R}\right)^2 = Mv^2$$

$$\begin{array}{lll} \text{cylinder:} & \begin{array}{l}\Delta T = -\Delta U \\ \tfrac{3}{4}Mv^2 = Mgh\end{array} & v_c = \sqrt{4gh/3} \\ \text{ring:} & Mv^2 = Mgh & v_r = \sqrt{gh} \end{array}$$

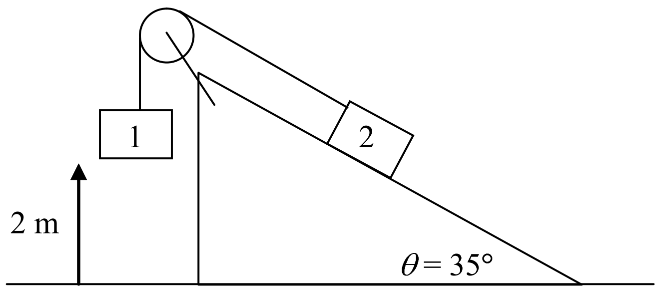
The cylinder will be going faster at <u>any</u> point along the ramp, so it will reach the bottom first. This result does not depend on the mass or radius, just how the moment of inertia depends on the shape. For any round object, it will be $I = (\text{shape factor})MR^2$.

Question: How would a solid sphere compare to the other two shapes?

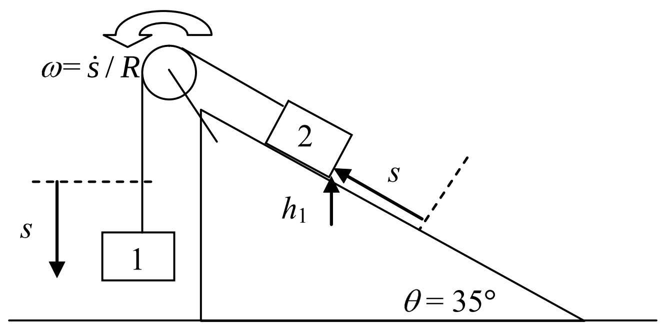
Answer: It would beat both because a larger fraction of its mass is near the axis of rotation. (Its moment of inertia is $I = \tfrac{2}{5}MR^2$, but you don't need to know that.)

Single-coordinate problems. Okay, earlier we'd looked at this problem but with a *massless* pully; now let's say it has mass.

<u>Example 2</u>: Suppose the masses $m_1 = 6$ kg and $m_2 = 4$ kg are initially at rest. Ignore friction, but assume the pulley is a cylinder of mass $m_p = 1$ kg and radius $R = 0.2$ m. Also, the rope does not slip over the pulley. What will the speed of m_2 be when it hits the ground?



This system is described by one parameter, *s*, the distance that the masses have moved. They are tied together, so they move the same amount (until 2 hits the floor).



The angular speed of the pulley is related to the speed of the rope ($\dot{s}$) by $\omega = \dot{s}/R$. The moment of inertia of the cylinder is $I = \frac{1}{2} m_p R^2$. The total mechanical energy of the system is:

$$E = T_1 + T_2 + U_1 + U_2 + T_p = \tfrac{1}{2}(m_1 + m_2)\dot{s}^2 + \tfrac{1}{2} I\omega^2 + (-m_1 g s \sin\theta + m_2 g s)$$

$$E = \tfrac{1}{2}(m_1 + m_2)\dot{s}^2 + \tfrac{1}{2}\left(\tfrac{1}{2} m_p R^2\right)(\dot{s}/R)^2 + (-m_1 g s \sin\theta + m_2 g s)$$

$$E = \tfrac{1}{2}\left(m_1 + m_2 + \tfrac{1}{2} m_p\right)\dot{s}^2 - m_1 g s \sin\theta + m_2 g s$$

The initial condition of the system is $s(0) = 0$ and $\dot{s}(0) = 0$, so $E_o = 0$. Conservation of mechanical energy gives:

$$0 = \tfrac{1}{2}\left(m_1 + m_2 + \tfrac{1}{2} m_p\right)\dot{s}^2 - m_1 g s \sin\theta + m_2 g s.$$

Solving for the speed and putting in the final condition $s = 2$ m gives:

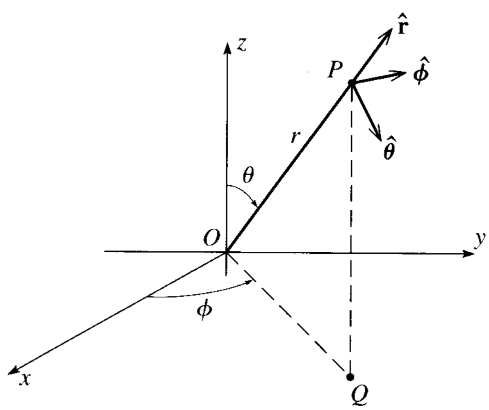
$$\dot{s} = \sqrt{\frac{2gs(m_1 \sin\theta - m_2)}{m_1 + m_2 + \frac{1}{2} m_p}} = \sqrt{\frac{2(9.8\,\text{m/s}^2)(2\,\text{m})\left[(6\,\text{kg})\sin 35^\circ - (4\,\text{kg})\right]}{4\,\text{kg} + 6\,\text{kg} + \frac{1}{2}([illegible]\,\text{kg})}} = 3.7\,\text{m/s}$$

which is slight smaller than what we found for a massless pulley.

Central Forces:

Speaking of ‘other’ coordinates, in this chapter the book introduces (though doesn’t yet do much with) spherical polar. If a force is always directed toward or away from a fixed point (“force center”), it is natural to take that point as the origin and to describe the force in spherical polar coordinates. These are shown below.

PPT.



Note that the definitions of θ and ϕ are usually reversed in math textbooks!

A central force can be written as:

$$\vec{F}(\vec{r}) = F(\vec{r})\hat{r}.$$

If the force only depends on the distance from the origin r and not on θ and ϕ, it is *spherically symmetric* or *rotationally invariant*. A central force is conservative if and only if it is spherically symmetric. In that case, the potential energy only depends on r:

$$U(r) = -\int_{r_0}^{r} F(r')\,dr',$$

or flipping the relationship around:

$$\vec{F}(\vec{r}) = -\left(\frac{\partial U}{\partial r}\right)\hat{r}.$$

Now, even if we *don't* have a central force, it may still be convenient to work in spherical coordinates. In that more general case, we have

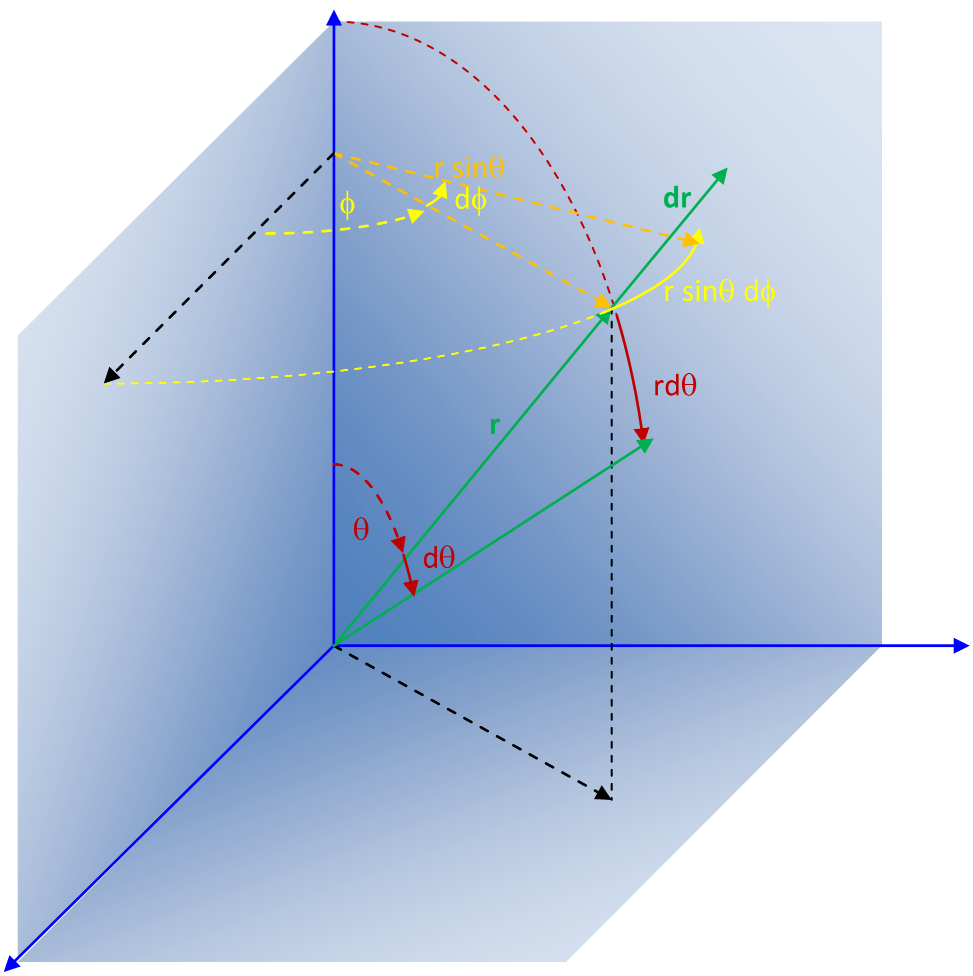
$$U(r) = -\int_{r_o}^{r} \vec{F}(r') \cdot d\vec{r}'$$

Which begs the question of 'what is a differentially small step in spherical coordinates? Rather than saying

$$d\vec{r} = dx\hat{x} + dy\hat{y} + dz\hat{z}$$

(baby steps in three orthogonal *Cartesian* directions)

We say, in terms of small changes in r, ϕ, and θ, is $d\vec{r} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\hat{\phi}$. The image below helps illustrate that each term represents a baby step in each of the three orthogonal directions.



How do we express the potential energy – force relation in these coordinates? What's the operator $\vec{\nabla}$ expressed in spherical coordinates?

As we've already seen in this chapter,

$$dU = -\vec{F} \cdot d\vec{r}$$

Then substituting in our new expression, $d\vec{r} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\hat{\phi}$, and performing the dot product gives us

$$dU = -\vec{F} \cdot d\vec{r} = -\left(F_r dr + F_\theta r d\theta + F_\phi (r\sin\theta) d\phi\right).$$

Now, we can turn this relationship around, express F in terms of a derivative of U just as we did when dealing with Cartesian components. Here we go:

The little bit of work done while the object just moves radially (dθ = dϕ = 0) is

$$dU\big|_{\theta,\varphi=const} = -F_r dr$$

and so we'd say that the partial derivative of U with respect to dr is

$$\frac{\partial U}{\partial r} = -F_r$$

Saying that relation the other way around, we see that the *r* component of the force is

$$F_r = -\frac{\partial U}{\partial r}$$

Similarly, imagining moving the object by just swinging down a little further with θ, we see that

$$\frac{\partial U}{\partial \theta} = -F_\theta r \Rightarrow F_\theta = -\frac{1}{r}\frac{\partial U}{\partial \theta}$$

$$\frac{\partial U}{\partial \phi} = -F_\phi (r\sin\theta) \Rightarrow F_\phi = -\frac{1}{(r\sin\theta)}\frac{\partial U}{\partial \phi}$$

Or sweeping around a little further with ϕ, we see that

$$\frac{\partial U}{\partial \phi} = -F_\phi (r\sin\theta) \Rightarrow F_\phi = -\frac{1}{(r\sin\theta)}\frac{\partial U}{\partial \phi}.$$

Then expressing the force in terms of these relations for its components gives

$$\vec{F} = F_r\hat{r} + F_\theta\hat{\theta} + F_\phi\hat{\phi} = -\left(\frac{\partial U}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial U}{\partial \theta}\hat{\theta} + \frac{1}{(r\sin\theta)}\frac{\partial U}{\partial \phi}\hat{\phi}\right).$$

Just as we did for Cartesian coordinates, we identify the operation we're performing on U as $\vec{\nabla}$:

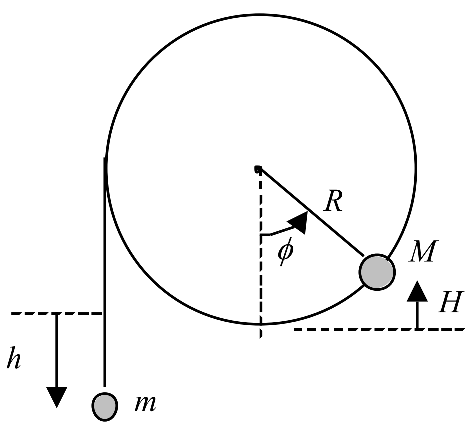
$$\vec{\nabla} = \left(\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta} + \frac{1}{(r\sin\theta)}\frac{\partial}{\partial \phi}\hat{\phi}\right)$$

Then we again have

$\vec{F} = -\vec{\nabla} U$

Again, this may seem a little out of place in chapter 4, but we'll make use of all this soon enough.

<u>Return to this example if there's time</u> **<u>Example 3</u>**: (Prob. 4.37) A massless (or very light) wheel of radius R is mounts on a horizontal axis. A mass M is attached to the rim of the wheel and a mass m is hung by a string wrapped around the rim. (a) Write an expression for the total PE as a function of the angle ϕ. Choose $U = 0$ when $\phi = 0$. (b) Find any positions of equilibrium and discuss their stability. (c) Suppose the system start at rest at $\phi = 0$. For what values of the ratio m/M will the system oscillate?



(a) As the wheel turns through an angle ϕ, mass M rises by $H = R(1-\cos\phi)$ (this works for any angle!) and mass m descends by $h = R\phi$ (the arclength unwound). The total PE is:

$$U(\phi) = MgH - mgh = MgR(1-\cos\phi) - mgR\phi.$$

(b) The condition for stability is:

$$0 = dU/d\phi = MgR\sin\phi - mgR$$

$$\sin\phi = m/M.$$

This only has solutions if $m \le M$. If $m = M$, there is one solution at $\phi = \pi/2$. If $m < M$, there are two solutions, one with $\phi < \pi/2$ (M below the axis) and one with $\phi > \pi/2$ (M above the axis). The stability is determined from the second derivative:

$$d^2U/d\phi^2 = MgR\cos\phi.$$

This is positive (negative) and the equilibrium is stable (unstable) for $\phi < \pi/2$ ($\phi > \pi/2$). For equal masses, the equilibrium at $\phi = \pi/2$ is a saddle point because $d^2U/d\phi^2 = 0$

(c) For the given initial conditions, the total energy of the system is $E = 0$. Plot the potential $U(\phi)$. The system will oscillate if there are turning points on both sides of the equilibrium. If $m/M < 0.725$, this condition is met.