Chapter 1

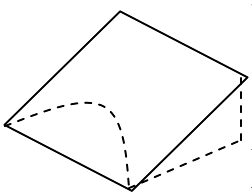
The text's 1.1, 1.11, 1.14, and 1.25 are suggested for practice, but will not be collected.

Section 1.5

1.A ★ Objects 1 slams into stationary object 2 and they travel off stuck together. In terms of their masses, m_1 and m_2, and the initial velocity, $\vec{v}_{1.i}$, what's the final shared velocity, $\vec{v}_{1\&2.f}$. How are the directions of these vectors related?

Section 1.6

1.B ★ On a bombing run on a moon[1] (thank heavens for no air resistance!), a pilot wants to release the bomb so it hits the target, taking the ship's height, h, and speed, v_s, into consideration. (a) Taking the target's location to be (0,0,0), and the ship's position at the drop instant to be (x_i, h, 0) derive equations for the bomb's x and y position components as a function of time, t after being dropped. (b) For the bomb to hit the target what must x_i be in terms of v_s, h, and the moon's 'g'?



1.C ★ Imagine you have a frictionless inclined plane sloping up from the level floor. If you put a hockey puck on the plane just resting in the seam where the plane meets the floor, and you kick it up so it arcs across the plane's surface (as illustrated), then how long until it returns to the floor? How far has it moved horizontally? Let's say the plane makes an angle θ with the floor; we'll call the origin where the puck starts, call $\hat{y}$ the direction straight up the plane, $\hat{x}$ the direction along the seam between the plane and the floor, and $\hat{z}$ the direction perpendicularly out of surface of the plane. Say that initial nudge, in this coordinate system, is $(v_{i.x}, v_{i.y}, 0)$. Make sure you understand the description of the coordinate system!

1.D ★★★ One early and highly valued application of mechanics was in aiming artillery (http://catalogue.museogalileo.it/room/RoomVI.html). Doing it right you can't neglect air resistance, but just to get the general flavor, try this out. Say a missile is launched from the origin with an initial velocity $(v_o \cos\theta, v_o \sin\theta, 0)$ from the horizontal ground (take up to by the y direction and over to be the x direction). What is the largest launch angel, θ_{max}, for which the projectile's distance from the origin, $r(t)$, i.e., the magnitude of its position vector, is ever increasing? That is, if you shot almost straight up (say θ = 89°), the projectile would start out going further and further away, and then start coming closer and closer again; what's the biggest launch angle for which that *doesn't* happen? To picture the scenario, draw one trajectory where r always increases and one where it eventually decreases.

Section 1.7

1.E ★ The book reasoned out equations 1.42 and 1.46 for the rates of change of the 2-D polar unit vectors $\hat{r}$ and $\hat{\phi}$; now you'll come up with another way. (a) First start from equation 1.32 (reduced to the x-y plane) and figure 1.10 to prove that $\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$. (b) Derive the similar relation for $\hat{\phi}$. (c) Okay, now if the angle, ϕ, varies with time, take the appropriate derivatives to derive equations 1.42 and 1.46.

[1] "That's no moon, it's a space station!"

1.F ★★ Show that in 1-D if the net force on a particle is a function of position only, $F(x)$, then Newton's second law can be soved to find v as a function of x $v^2 = v_o^2 + 2\int_{x_o}^{x} \frac{F(x')}{m} dx'$. To do this, it'll be handy to *prove* that $\dot{v} = v\frac{dv}{dx} = \frac{1}{2}\frac{d(v^2)}{dx}$. Don't just use it.

1.G=1.50 ★★★ **[computer]** Use Python, or Maple (Python is available on the computers in Appleton 101). Note that the differential equation is for the angle in radians, not degrees. How is the solution to the "exact" (numerical) solution different from the approximate solution? Note: in your old 1-D code, rather than updating a_x=F_x/m, v_x, and x, you're updating $\alpha = F_\theta/(mr)$, ω, and θ. Where $\omega \Leftarrow \omega + \alpha \cdot \Delta t$ and $\theta \Leftarrow \theta + \omega \cdot \Delta t$. A picture might help you to figure out an expression for the force component that points in the θ direction.

Chapter 2

Section 2.2

2.A ★ What a drag! A drag force is generally dependent on the speed of the object, not explicitly its position or the time. In an essentially 1-D situation, you can then rewrite $F(v) = m\frac{dv}{dt} \Rightarrow dt = m\frac{dv}{F(v)} \Rightarrow \Delta t = m\int_{v_o}^{v}\frac{dv'}{F(v')}$. So, say that an object has some initial speed v_o at $t = 0$ and it experiences a non-linear drag force $F_{drag}(v) = -cv^{3/2}$. If that's the only force on the object, then do the integral to be able to find v as a function of t.

Section 2.4

2.B = 2.36 ★★ Galileo vs. Aristotle. The series expansion for *cosh* is useful for finding the $cosh^{-1}$ of numbers close to one. (It can also be done exactly.)

2.C ★★ Projectile motion with & without drag. As mentioned in Chapter 1's homework, for artillery soldiers to aim their weapons correctly, they really needed to take drag into consideration. To prove the point, here's the simplest case (which you wouldn't want to do in practice!) shooting straight up. Say the projectile is launched straight up, $(0,v_o,0)$, from the origin and it experiences the expected quadratic drag force, $\vec{f}_d(v) = -cv^2\hat{v}$, as well as the usual gravitational force down. (a) Rewrite its equation of motion in terms of the terminal speed, v_{ter}, for the equivalent of equation 2.54. Note: 2.54 was derived for a *dropped* object, so gravitational force was down and drag force was up; also the y-direction was taken to be down. In this problem, gravitational and drag forces *both* are down and take the y-direction to be *up.* (b) use $\dot{v} = v\frac{dv}{dy}$ (which you proved while working on problem 1.F) so the equation is in terms of speed and position, then separate variables (get all v's on one side and y on the other) and integrate both sides to get y in terms of v (or vice versa). (c) Use this to show that the projectile's maximum height is $y_{\max} = \frac{v_{ter}^2}{2g}\ln\left[1+\left(\frac{v_o}{v_{ter}}\right)^2\right]$. Compare this with what you'd get assuming no drag.

2.D ★★ Watch out below! Now for the projectile's return journey, again start by writing down the equation of motion, but this time the drag force is up while the gravitational force is down. From there, follow the same steps as in the previous problem to find the relation between the projectile's height and speed. Taking the its initial height to be y_{max} from the previous problem, show that it will land with final speed $v_f = v_o \Big/ \sqrt{1+(v_o/v_{ter})^2}$ (note: without drag, $v_{ter} = \infty$ and you get the regular result).

2.E ★★ The calculation of the maximum height reached by a projectile is similar to the calculation of the range, except the result is analytical even with linear air resistance. Suppose a projectile is launched with an initial velocity $\vec{v}_o = (v_{xo}, v_{yo})$. (a) Find the maximum height it would reach in vacuum. (Hint: What is the v_y at the peak?) (b) Repeat the calculation of the maximum height with a linear air resistance in terms of the time parameter τ_l and the initial conditions. Be very careful about the direction of the vertical axis. An iterative solution is not necessary! (c) Taylor expand your result from the previous part to find the first two terms of an approximate expression for the maximum height when the air resistance is small. (Hint: The first term should be the answer from the first part and the second term will involve v_{yo}^3.)

2.F ★★★ **[Computer]** Determine the angle at which a basketball should be shot from a height of 2 m with an initial speed of 8.5 m/s to make a free throw. Account for drag using the data about the ball in Problem (2.31). The hoop is 10 feet high and its center is 15 feet from the free throw line. Assume that the ball must go directly through the center of the hoop on the way down (no bank shots). Turn in a graph of the path of the shoot and report the angle of the shoot above horizontal. (Hint: To check your program, you can you it to solve Problem (2.43) and compare with the answer in the back of the book.)

Chapter 3

Section 3.1

3.A ★ A fire cracker shot straight up in the air splits into two separate, equal-mass, fire crackers which go their own ways before individually exploding. If the system's velocity just before exploding is $(0, v_o, 0)$, and just after the split one of the pieces goes straight off horizontally ($+x$ direction) at that same speed, what is the velocity of the other piece just after the split. Note: this inverse-collision happens so quickly, that you don't have to worry about the effect of gravity during the brief instant.

Section 3.2

3.B ★ A rocket of total initial mass m_o, starts off with some fraction, λ of its mass as fuel, $m_{ofuel} = \lambda m_o$. If it accelerates through space with an acceleration of g (to make things feel nice and homey inside) by shooting out its fuel at v_{ex} relative to the rocket, how long can it keep accelerating before it's out of fuel? Hint: At any moment, the thrust of the rocket equals what the rocket's weight would be on earth (since it's maintaining $a = g$); if you write down that condition and separate variables t and m, then you can integrate to find mass as a function of time, and so the time when all the fuel is lost. (b) What if $v_{ex} = 3000$ m/s and $\lambda = 0.1$?

3.C ★★ Two Stage or not Two Stage, that is the question. As you're no-doubt aware, rockets are constructed in stages so that they don't have to carry around the dead weight of fuel emptied fuel tanks. How big is the advantage? Say you've got two rockets with the same initial mass, m_o, and for both 60% of that mass is fuel ($m_{ofuel} = 0.6m_o$) Rocket A is a single-stager and rocket B is a two-stager which drops it's empty lower stage (with mass $0.1m_o$) when it's spent half its fuel ($0.3m_o$). (a) As a multiple of v_{ex}, what's the one-stage rocket's final speed? (b) As a multiple of v_{ex}, what's the two-stage rocket's final speed?

Section 3.3

3.D ★★ Find the Center of Mass of a uniform solid hemisphere of radius R whose flat face lies in the x-y plane with its center at the origin. It may be tempting to do this in spherical coordinates, but it turns out being easier to do in Cartesian.

Section 3.5

3.E ★ Say a proto-planet at one time has radius R_o and is spinning on its axis with ω_0. Over time, it slowly attracts more and more matter which is, on average, initially not rotating. Eventually the planet has radius R. Assuming it has the same density the whole time, what's the new ω ($I = \frac{1}{2}MR^2$ for a sphere spinning about its axis)? Say $R = 2\,R_o$, so what's ω?

3.F ★★ What's the moment of inertial of a uniform disc of mass M and radius R rotating about its central axis like a record on a record player? Don't just quote a result, start with the sum $I = \sum_i m_i r_i^2$ and develop and evaluate the appropriate integral.

3.G ★★★ **[Computer]** The total mass of a rocket is 2x10^6 kg and 90% of its mass is fuel. It burns fuel at a constant rate of 18,000 kg/s and the exhaust speed is 3000 m/s. The rocket is launched vertically and the rotation of the earth can be ignored. **(a)** Ignore air resistance and assume that g is constant. Calculate the maximum altitude reached by the rocket. (This part could be solved analytically, but you may find the answer with a computer.) **(b)** The size of g varies with the altitude above the earth's surface as:

$$g(y) = \frac{9.8\ \text{m/s}^2}{(1 + y/R_e)^2}$$

where $R_e = 6.38 \times 10^6$ m is the radius of the earth. What is the maximum altitude when this is taken into account? (Including air resistance would make a smaller correction.)

- There is no thrust after all of the fuel has burned, but the rocket continues to go upward for a while.
- The mass of the rocket is <u>not</u> constant because of the fuel being ejected.
- If you use the Euler-Cromer method, a time step of about 0.25 s is short enough.

Chapter 4

Section 4.1

4.A = 4.2 ★★

Section 4.2

4.B ★★ Here's a classic: Imagine you place a (perfectly frictionless) penny on the top of a bowling ball of radius R and give it an infinitesimal nudge so it starts sliding down the side. How much altitude will it loose (Δy) before it separates from the ball's surface and free falls? Hint: conservation of energy gives the penny's speed as a function of height and Newton's 2nd relates it's weight and normal force to the rate of change of that speed – what's the normal force when the penny goes free? Polar coordinates will be handy for the Newton's 2nd Law.

Section 4.4

4.C ★★ Which of the following forces is conservative? (a) $\vec{F} = k(x, 2y, 3z)$ where k is a constant. (b) $\vec{F} = k(y, x, 0)$. (c) $\vec{F} = k(-y, x, 0)$. For those which are conservative, find the corresponding potential energy *U,* and verify by direct differentiation that $\vec{F} = -\nabla U$. Note that, although the integral of $\vec{F} \cdot d\vec{r}$ does not depend on the path for a conservative force, you still must pick *some* path to do the integration. Explicitly describe or sketch the path you use to find the potential energy for the conservative force.

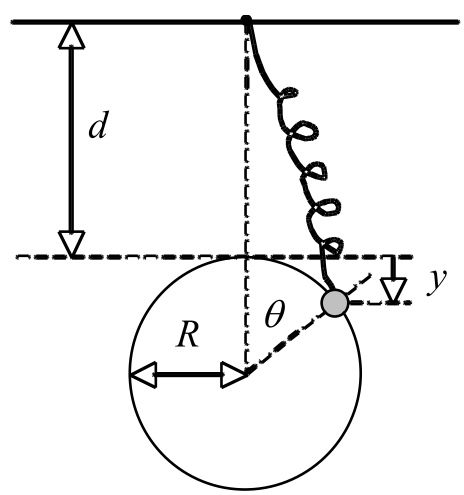
Section 4.6

4.D=4.28 ★★

Section 4.7

4.E =4.34 ★★ Be sure to measure the height from the equilibrium position. In part (b), what does the derivative of the energy equal?

4.F ★★★ **[Computer]** A 1-kg mass is constrained to move around a circular track with no friction. A spring is attached to the mass and a fixed point above the track (the diagram below is a side view). It has a spring constant k and is unstretched when the mass is at the top of the circle. **(a)** Write expressions for the distance y below the position at $\theta = 0°$ and the length s of the spring as functions of the angle θ (measured from vertical) and the dimensions d and R. **(b)** Write an expression for the total potential energy in terms of y and s. Define the potential energy to be zero where $\theta = 0°$. Don't forget that the potential energy of the spring depends on how much it's stretched, not its overall length. **(c)** Suppose the spring constant k = 25 N/m and the dimensions are d = 1 m and R = 0.5 m. Use a computer to make a graph of the total potential energy as a function of θ between -180° and 180°. **(d)** Use a computer to make graphs of the speed of the mass as a function of θ between -180° and 180° for total energies of E = 2 J and 3 J. Also, describe the motion for each case.



Chapter 5

Section 5.1

5.2 ★ Find the point of equilibrium, then expand the exponential around that point. Keep linear and quadratic terms in the potential energy. This approach is much easier than directly expanding the whole function.

Section 5.2

5.10 ★

Section 5.4

5.26 ★★

5.28 ★★ First, find the system's natural frequency.

Section 5.5

5.36 ★★ **[computer]** First, try to reproduce Figure 5.15(b) from Example 5.3.

Section 5.6

5.43 ★★ In part b, note that there are *two* spring in parallel attached to one 50-kg axel assembly.

Section 5.7

5.46 ★

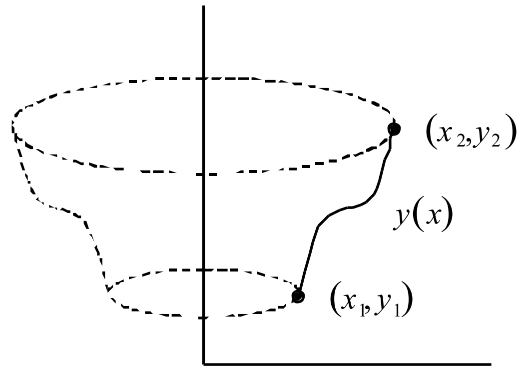
Section 5.8

5.51 ★★ Do this by plugging $z_n = C_n e^{in\omega t}$ and $g_n = f_n e^{in\omega t}$ into equation 5.60 and showing that it leads to the predicted C_n relation. You do not need to resolve z_n into its real and imaginary parts.

5.52 ★★★ **[computer]**

Chapter 6

6.A ★★ Suppose a surface is generated by revolving a curve $y(x)$ that passes through (x_1, y_1) and (x_2, y_2) about the y axis.



(a) Find an integral for the area of the surface for a function $y(x)$. Start by finding the area related to a short segment of the curve.
(b) Find the equation for the curve such that the surface will have the smallest possible area. There will be two constants in the answer, which you do not have to find because the endpoints are not specified.

6.B ★★ In terms of the cylindrical coordinates, a helix is a path on a cylinder where z changes at a constant rate with respect to ϕ, or $\frac{dz}{d\phi}$=constant.

(a) Given the relations between Cartesian and cylindrical polar coordinates:

$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$z = z$$

Write the element of distance on a cylinder, $ds = \sqrt{dx^2 + dy^2 + dz^2}$, in terms of cylindrical coordinates.
(b) Set up an integral with respect to ϕ for the length of a path on a cylinder. Hint: consider z to be a function of ϕ and define $z'(\phi) = dz/d\phi$.
(c) Show that for a cylinder centered on the z axis (ρ=constant), the shortest path between any two points on the cylinder is a segment of a helix (i.e. show that the condition explained above is met).

6.C ★★★ **[Computer]** Problems in geometrical optics can be solved using Fermat's principle which states that light will follow the path which will result in the shortest transit time between two points. The speed of light is $v = c/n$, where n is the index of refraction. The time required to travel a short distance, $ds = \sqrt{dx^2 + dy^2}$, is ds/v.

(a) Suppose the index of refraction is a function of position, $n(x)$. Write down the integral for the time to be minimized. It should be an integral over x which involves an unknown function $y(x)$ and its derivative $y' = dy/dx$.

(b) Suppose the index of refraction varies as $n(x) = e^x$, what differential equation does Euler's equation give? Hint: the result should be a first-order differential equation in the form $dy/dx = g(x)$.

(c) What path $x(y)$ will light take from $x_1 = 1,\ y_1 = -1$ to $x_2 = 1,\ y_2 = 1$? Hint: The differential equation from part (b) can be integrated with the change of variables $q = e^{2x}$. Note that the solution should be in the form $x(y)$, not $y(x)$ which is not single valued. There are two constants which must be determined using the endpoints.

(d) Use a computer to plot the path found in part (c). Does it go where you expect? Explain in terms of Fermat's principle.

Chapter 7

Section 7.1

7.2 ★

7.4 ★

Section 7.3

7.10 ★ Note: you're *just* writing x, y, and z in terms of ρ and ϕ (and vice versa); in problem "53" you'll go the rest of the way and write out the Lagrangian and find the equations for $\ddot{\rho}$ and $\ddot{\phi}$.

Section 7.4

7.14 ★ How does the acceleration of the yo-yo compare with free fall?

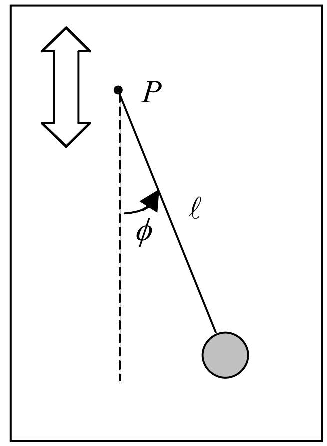
Section 7.5

7.20 ★

7.30 ★★ Start by writing the position and velocity of the bob relative to an inertial observer on the ground. Don't forget that $\cos^2 + \sin^2 = 1$, and I don't think you'll find any use for the book's hint about combining cos and sin terms like Eq'n 5.11.

7."53" ★★ (a) Write down the Lagrangian for a small mass moving on the inside surface of a frictionless circular cone with its axis on the z axis, vertex at the origin (pointing down), and half-angle α. The answer should be in terms of the generalized coordinates ρ and ϕ from problem 7.10. (b) Find the two Lagrange equations. Do <u>not</u> attempt to solve the differential equations!

7."54" ★★★ **[Computer]** A simple pendulum of mass m and length ℓ is attached to a point of support P that is forced to oscillate vertically. P moves with an angular frequency ω and an amplitude R, so its position is given by $Y(t)=R\cos\omega t$, where positive Y is upward. Assume that the mass is attached by a very light, rigid rod. (a) Write down the Lagrangian for the system and find the equation of motion for the angle ϕ. (Hint: Start by writing expressions for the position and velocity of the pendulum bob in Cartesian coordinates.) Check that your answer makes sense in the limit that $\omega=0$. (b) Suppose $\ell=1\text{ m}$ and the support oscillates with a small amplitude $R=0.2\text{ m}$ and a high angular frequency $\omega=25\text{ radians/s}$. Use the Euler-Cromer method (as in the Vpython code) to plot $\phi(t)$ vs. t for 50 seconds, given the initial conditions $\phi(0)=3$ radians (the pendulum starts pointing almost directly upward) and $\dot{\phi}(0)=0$. What is surprising about the result?



Chapter 8

Section 8.3

8.8 ★★

Section 8.4

8.12 ★★ Answers may be expressed in terms of $\gamma=Gm_1m_2$. Be sure to compare the period of the oscillations with the planet's orbital period.

Section 8.6

8.18 ★★

8.25 ★★★ **[computer]** Part (b) can be solved by making successive guesses. In part (c), two initial conditions are required – the values of $u(\phi=0)$ and $u'(\phi=0)=(du/d\phi)\big|_{\phi=0}$. Show that the second of those is zero (if you get stuck, just use that value in the computational solution). After solving the differential equation for $u(\phi)$ in part (c), calculate Cartesian coordinates (x,y) corresponding to each pair (u,ϕ) so that you can plot the path.

Section 8.7

8.30 ★★

8."36" ★★★ Data about Haley's comet is given in Example 8.4. (a) How often does the comet come near the sun (and earth) in units of years? How many opportunities will most people get to see this comet? (b) What is the comet's speed at perihelion and at aphelion in units of m/s? (Hint: what is the reduce mass for a comet?)

Chapter 9

Section 9.1

9.2 ★ Find the angular frequency in revolutions per minute (rpm).

Section 9.5

9.8 ★ Draw the diagrams needed to determine the directions of the forces. You will <u>not</u> get much credit for just stating the answers.

Section 9.6

9.14 ★★ Start by finding the potential energy associated with the centrifugal force. At the surface of the water, the total potential energy (gravitational and centrifugal) is constant.

Section 9.7

9.24 ★★★ **[computer]** Use the results of Problem 9.20 (done in class). Use a computer to make plots of the motion for 6 seconds showing x and y between –4 and +4.

Section 9.8

9.25 ★ Explain what perspective you are describing the plumb line in the train from.

9.27 ★★ Explain what perspective your sketches are drawn from. For each part, give an explanation of the motion relevant to the reference frame used.

Chapter 10

Section 10.1

10.6 ★★ This problem is easiest in spherical polar coordinates. A volume element in those coordinates is $dV = r^2 dr \sin\theta\, d\theta\, d\phi$. Do part (c) for <u>extra credit</u>. Let $b = a + \varepsilon$ where ε is small and expand the result for part (a).

Section 10.2

10.14 ★★ Use the answer for Problem 10.11 from the back of the book. Assume that the flywheel is a uniform disk and that it reaches the final angular speed very quickly. Be extra careful of units in part (b)!

10.15 ★★

Section 10.3

10.22 ★★

Section 10.5

10.36 ★★ I am willing to check your result for part (a) before you do part (b).

Section 10.6

10.39 ★ Some of the necessary information is in examples. Report your answer in revolutions per minute (rpm).