Physics 310
Lecture 1 – DC Circuits, more advanced

Mon. 1/18 Wed. 1/20 Thurs. 1/21 Fri. 1/22	**Ch 1.10-.13, App. A** pg. **A-1-A-4,** & *lightly* **Ch 6.1, .3, .4, .8, .10, .11** **Quiz** Ch 1 & 6**, Lab 1:** DC Circuits More of the same **Ch 2 & 6.6-7, App B-2:** Capacitors, Inductors, & Oscilloscopes	**HW1**: Ch1 Pr 4, 25; Ch 6 Pr 9*, 12 **Lab 1 Notebook**
Mon. 1/25 Wed. 1/27 Thurs. 1/28 Fri. 1/29	more of the same **Quiz** Ch 2 & 6**, Lab 2:** Oscilloscopes & Circuit Transients more of the same **Ch 3, 4.5, 6.5:** AC Circuits	**HW2**: Ch2 Pr 1,2,3,6* ,8,10 **Lab 2 Notebook**

Equipment
- Power supply
- D'Arsonval meter

<u>Handout</u>
- Thévenin's Equivalent Circuit (JR handout)

Announcements
- Office Hours
- Lab begins Wednesday.
  - Have done the pre-lab (will check at the beginning)
  - bring your notebook.
- Quiz Wednesday – see online equation sheet

Requests:
- Thevinin's Theorem – applying it
- Example pg 18Wheatstone (conditions for balance in pr. 12)
- "Open" vs. "Shorted" circuit
- "Short-circuited" voltage source?
- D'Arsonval meters susceptiple to external magnetic fields?
  - If constant, can be re-zeroed
- Four-wire resistance measurement
- Significance of "ideal" voltage source – application or just logical simplification for problems?

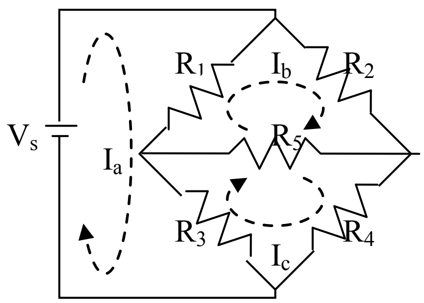
Last Time

Ch 1 : Direct Current Circuits

1-9 The Mesh Loop Method

1-9.1 Wheatstone Bridge

- This is an example where it's much easier to jump in with the Mesh Loop approach than applying the independent node and loop rules since we need only 3, rather than 5 distinct currents to consider.



$$V_s = R_1(I_a - I_b) + R_3(I_a - I_c)$$
$$0 = R_1(I_b - I_a) + R_2(I_b) + R_5(I_b - I_c)$$
$$0 = R_3(I_c - I_a) + R_4(I_c) + R_5(I_c - I_b)$$

Three equations, three unknowns – it's doable.

Now, the algebra mightn't be pretty, but you can slog through it. Alternatively, matrix math is great for handling coupled linear equations.

Rearranging as

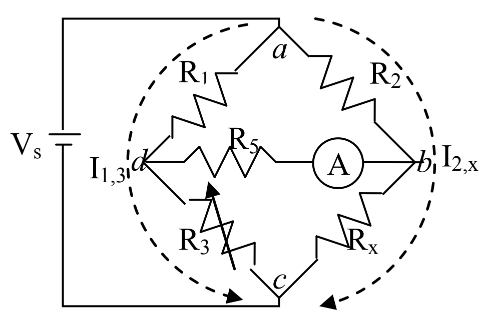
$$V_s = (R_1 + R_3)I_a - R_1 I_b - R_3 I_c$$
$$0 = -R_1 I_a + (R_1 + R_2 + R_5)I_b - R_5 I_c$$
$$0 = -R_3 I_a - R_5 I_b + (R_3 + R_4 + R_5)I_c$$

Appendix A: The Method of Determinants

This information could be rephrased in matrix form

$$\begin{bmatrix} V_s \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 + R_3 & -R_1 & -R_3 \\ -R_1 & R_1 + R_2 + R_5 & -R_5 \\ -R_3 & R_5 & R_3 + R_4 + R_5 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

1-9.1.1 / 6-8 What's the Wheatstone Bridge good for anyway? Historically, this peculiar configuration has been used as a very sensitive Ohm meter, i.e., to measure resistance. Put an ammeter in series with R5, say If R4 is unknown, but the other resistances are known, and one of the other resistors, say R3, is a precision potentiometer (so its resistance can be varied), then that resistance can be varied until the current through R5 is 0. So R2 would be varied until R5's current is 0, knowing the other three resistances, R4 can be solved for.



In problem 9, you're asked to derive equation 6-8, i.e., show that, when the Ammeter reads 0 current, i.e., when the bridge is "balanced", the four resistances are related to each other by

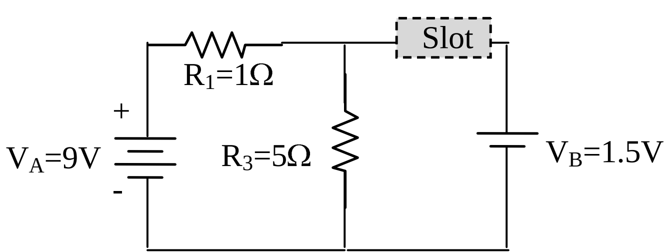
$$\frac{R_1}{R_2} = \frac{R_3}{R_x} \Rightarrow R_x = R_3 \frac{R_2}{R_1}$$

So the unknown resistance can be determined. We'll actually go most of the way there later today.

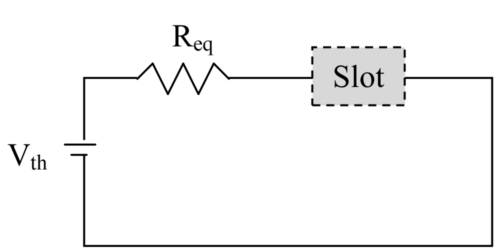
1-10 Thevenin's Equivalent Circuit Theorem

- As we explored last time, a complex network of resistors, from an outside perspective (say that of a battery wired up to them) is equivalent to a single resistor – the same amount of current gets drawn from the battery whether there's the whole mess, or just the right single resistor. What if you had a whole mess of resistors *and* batteries? Does that have a simple equivalent from a given outside perspective? Yes.
- Say you had a complex circuit with a slot / plug where you're keen on changing a component frequently – perhaps the load on a voltage divider or one of the resistors in the Wheatstone bridge. Rather than having to re-analyze the whole circuit from scratch for each new resistance value, it would be handy to break the problem up – those things that *don't* depend on what goes in the slot and those things that do. Thevenin's Theorem says that, from that slot's perspective, no matter how complicated the rest of the circuit is, it behaves like a simple voltage supply in series with an impedance (if we're just dealing with resistors, a resistor.)

- That is to say, if you had, say, this circuit



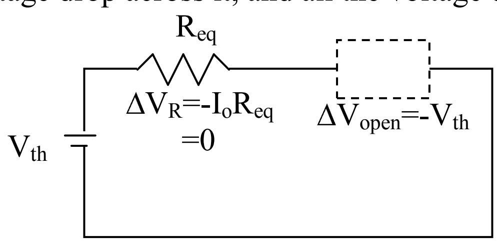
- from the *slot's* perspective, the circuit looks like



If that's correct, then here's how we can figure out what the equivalent voltage supply and resistor values should be.

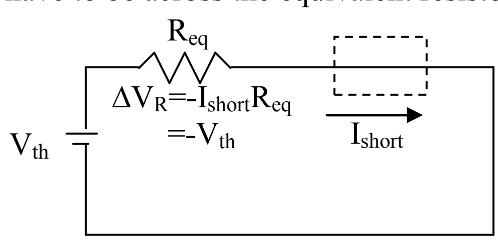
Note: the book gives a recipe but no particular logic behind it; I'll give a slightly different approach whose logic is more apparent.

- **Leave the slot empty.** Then no current flows through the adjacent resistor, so there's no voltage drop across it, and all the voltage drop must be across the empty slot.



  So, if the equivalence is true, you should be able to **determine V_{th}** by simply removing any components from that slot in the real circuit and determining the voltage drop across it.

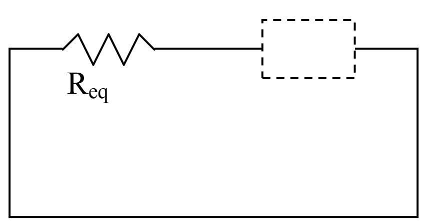
- **Short across the slot.** Then again, if you short across the slot, then the entire voltage drop will have to be across the equivalent resistor.



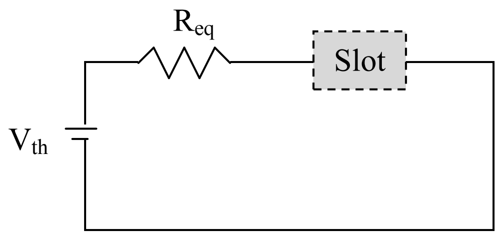
  So, if you short (insert a wire across it) the slot in the *real* circuit and determine the current flowing across it, I_{short}, then armed with that and V_{th}, you can **determine R_{th}**.

$$R_{eq} = -\frac{\Delta V_{th}}{I_{short}}$$

  - **Alternatively:** It is often simpler to do as the book describes, you short all voltage sources and you open all current sources so all you're left with are the resistors, then it's easy to calculate their equivalent resistance.

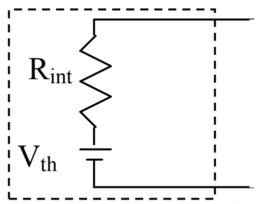


- **Example (part of 1-13): Real Voltage Supply**
  - Look at this real power supply; inside is a *whole mess* of circuitry, but Thevenin's theorem says that, form the perspective of anything plugged into this slot, it looks like



(more generally, it's an equivalent impedance)

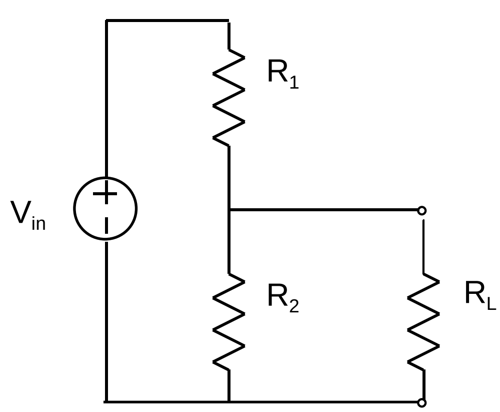
So, a *real* voltage source interacts with a circuit that you plug into it simply as an *ideal* voltage source (that fictitious perfect battery) with some resistor in series. Wrapping my wires around a bit and changing the notation a tad, a real voltage source is represented as:



The equivalent resistance (or complex impedance) is sometimes referred to as the supply's "internal resistance", or as it's "output impedance." The former name is self explanatory; the latter comes form the fact that any current this supply *puts out* must flow through this resistor / impedance on the way.

Experimental Determination. For theoretical calculations, you can imagine shorting the slot; however, that's not always practical – it might be bad for the power supply; so you can instead deduce the internal resistance and voltage by *assuming* this form, then measuring the voltage across the slot for a few different resistors that you plug in – Plotting the curve, you could determine the two parameters – V_{th} and R_{int}.

Thevenin Example: Voltage divider with a load



The open circuit voltage (R_L disconnected) is:

$$V_{TH} = V_{in} \frac{R_2}{R_1 + R_2} \quad (3)$$

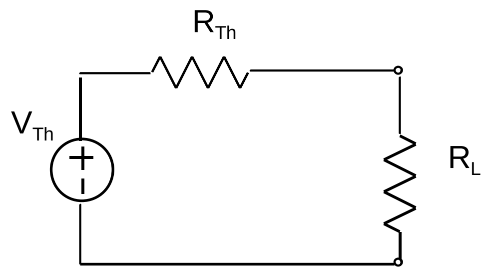
The short circuit current in the circuit is:

$$I_{short-circuit} = \frac{V_{in}}{R_1}. \quad (4)$$

So:

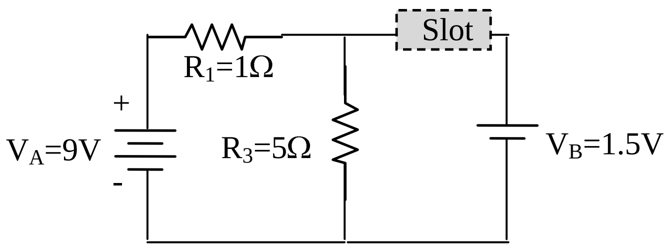
$$R_{Th} = \frac{V_{Th}}{I_{s-c}} = V_{Th} \frac{R_1}{V_{in}} = \left(V_{in} \frac{R_2}{R_1 + R_2} \right) \frac{R_1}{V_{in}} = \frac{R_1 R_2}{R_1 + R_2} \quad (5)$$

For calculations of what the current through or voltage across the load resistor would be, the original circuit can be replaced by the following Thévenin equivalent.

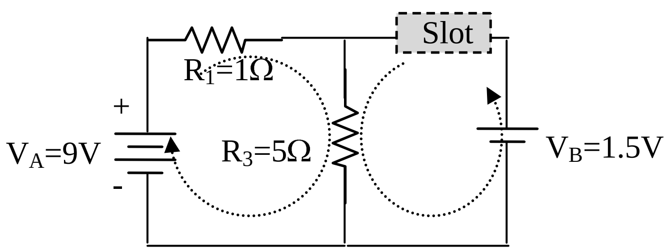


Why bother: Again, the real virtue of breaking up a problem like this is *not* that it's less work the first time around, but that it saves you work in the long run if you're often going to changing what load you plug into the slot. That is particularly common if the 'rest of the circuit' is a real power supply.

Example 2

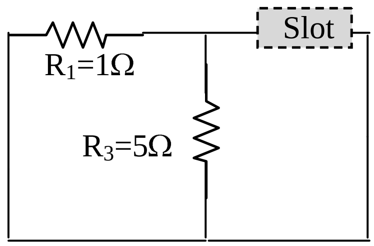


Open Slot. What's the voltage across it?
Kirchoff's loop rule.



$$\left.\begin{array}{l} V_A = I_A R_1 + I_A R_3 \Rightarrow I_A = \dfrac{V_A}{R_1 + R_3} \\ V_B + V_{Th} = I_A R_3 \end{array}\right\} \Rightarrow V_B + V_{Th} = \frac{V_A}{R_1 + R_3} R_3 = \frac{V_A}{\frac{R_1}{R_3} + 1} \Rightarrow \boxed{V_{Th} = \frac{V_A}{\frac{R_1}{R_3} + 1} - V_B}$$

Shorted Slot. What's the current through it? Or **Short all supplies.** What's the resistance across it?

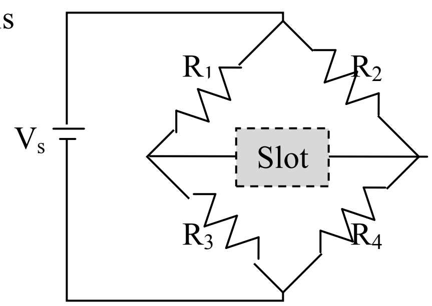


It's pretty clear that, from the slot's perspective, there are just two resistors in parallel to it, and the equivalent resistance is then

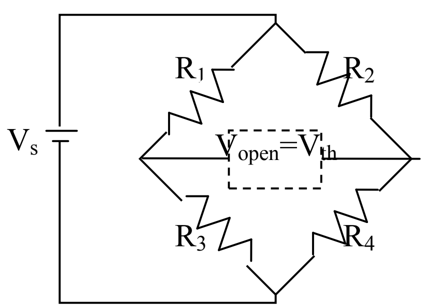
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Group Problem.ppt #2

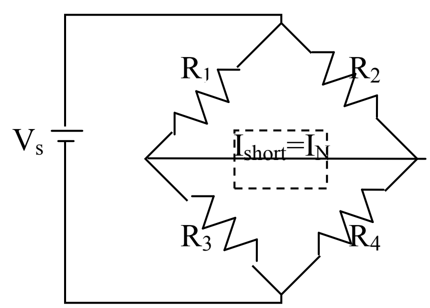
Let's look at the Wheatstone example. Whichever equivalence you're after, the *real* circuit is



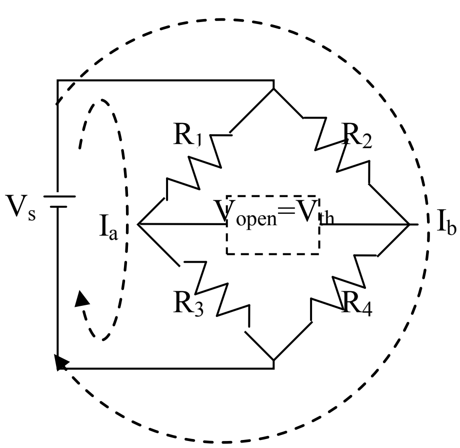
And the two versions you're going to analyze are



And



Looking at the open-slot version: Applying the loop rule around three closed loops gives three equations.



$$I_a = \frac{V_s}{(R_1 + R_3)}$$

$$V_s - I_a(R_1 + R_3) = 0$$
$$V_s - I_b(R_2 + R_4) = 0 \Rightarrow I_b = \frac{V_s}{(R_2 + R_4)}$$
$$V_{th} - I_b R_4 + I_a R_3 = 0$$

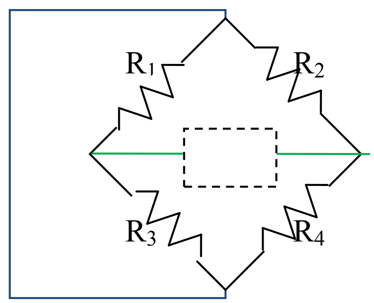
$$V_{th} = I_b R_4 - I_a R_3$$

$$\boxed{V_{th} = V_s\left(\frac{R_4}{(R_2 + R_4)} - \frac{R_3}{(R_1 + R_3)}\right)}$$

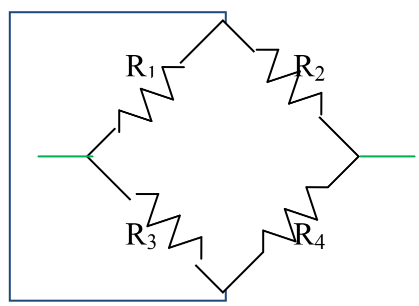
We'll come back to this in a bit.

Resistance. Now you can take either one of two approaches: short the slot and determine the current going through it in order to, with Vth, determine the resistance that would be in series with it or short the supply and determine the resistance between the two terminals. The book did the latter; let's walk through its reasoning first.

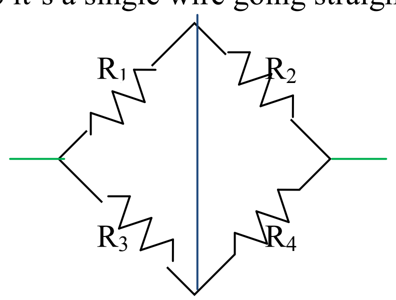
Short supply & find resistance. Figure 1.20 – figure it out.



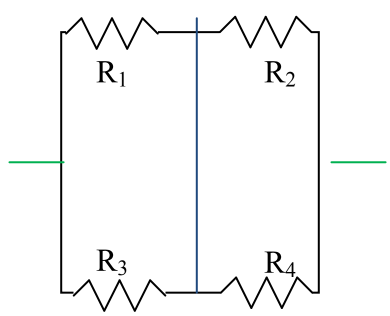
Step 1 Grab nodes and bend them to point *out* of the inner diamond



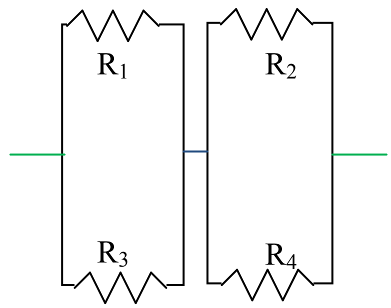
Step 2 Grab the big wire loop on the far left and twist it into the middle of the diamond so it's a single wire going straight up



Step 3 square out the diamond so the resistors lie horizontal rather than diagonal



Step 4 pinch the waist of the resulting circuit.



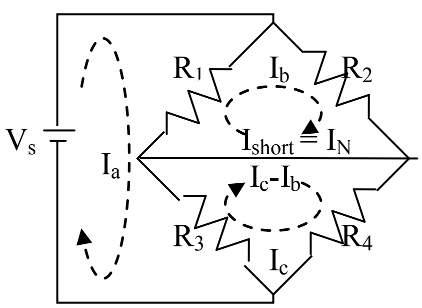
Finally, it's easy to see that R1 and R3 are in parallel with each other, ditto for R2 and R4, and then those sets are in series. Thus

$$R_{th} = \left(\frac{R_1 R_3}{R_1 + R_3}\right) + \left(\frac{R_2 R_4}{R_2 + R_4}\right)$$

Short slot and find current (and then resistance)

Now, a start on the other approach.

Looking at the short-slot version: Applying the mesh-loop approach around three closed loops gives three equations. (only set up – too tedious to go through)



$$V_s - I_b R_2 - I_c R_4 = 0$$
$$-(I_b - I_a)R_1 - I_b R_2 = 0$$
$$-I_c R_4 - (I_c - I_a)R_3 = 0$$

$$\Rightarrow$$

$$V_s = I_b R_2 + I_c R_4$$
$$\frac{I_a}{\left(\frac{R_2}{R_1} + 1\right)} = I_b$$
$$\frac{I_a}{\left(\frac{R_4}{R_3} + 1\right)} = I_c$$

From here the algebra's a little tedious, but the result is

$$V_s = \frac{I_a}{\left(\frac{R_2}{R_1}+1\right)} R_2 + \frac{I_a}{\left(\frac{R_4}{R_3}+1\right)} R_4 = I_a \left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} + \frac{1}{\frac{1}{R_3}+\frac{1}{R_4}} \right) = I_a \left(\frac{R_1 R_2}{R_1+R_2} + \frac{R_3 R_4}{R_3+R_4} \right)$$

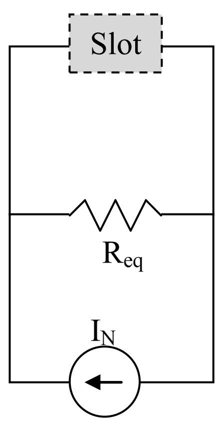
$$I_a = \frac{V_s}{\left(\frac{R_1 R_2}{R_1+R_2} + \frac{R_3 R_4}{R_3+R_4} \right)}$$

$$I_N = I_c - I_b = \frac{I_a}{\left(\frac{R_4}{R_3}+1\right)} - \frac{I_a}{\left(\frac{R_2}{R_1}+1\right)} = \frac{V_s}{\left(\frac{R_1 R_2}{R_1+R_2} + \frac{R_3 R_4}{R_3+R_4} \right)} \left(\frac{1}{\left(\frac{R_4}{R_3}+1\right)} - \frac{1}{\left(\frac{R_2}{R_1}+1\right)} \right)$$

$$I_N = I_c - I_b = \frac{V_s (R_2 R_3 - R_4 R_1)}{R_1 R_2 R_3 + R_1 R_3 R_4 + R_1 R_2 R_4 + R_2 R_3 R_4}$$

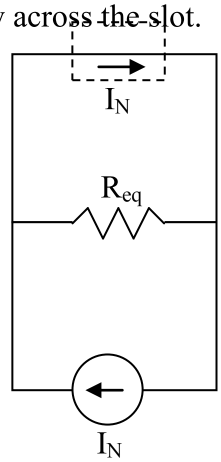
1-11 Norton Equivalence

Similarly, Norton's Theorem says that, from the perspective of such a slot, the rest of a real circuit is equivalent to a single *current* source in *parallel* with a single impedance. That is to say, from the slot's perspective, the real circuit is equivalent to



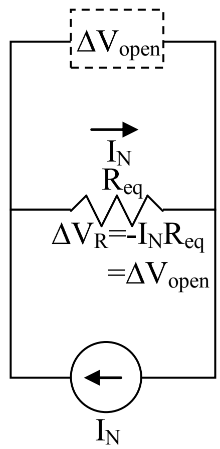
Similar to the way that we could determine the Thevenin equivalent voltage and resistance, we can determine the Norton equivalent current and resistance.

- **Short the slot,** then none of the current will chose to flow through Req, and it'll all flow across the slot.



Thus, short the slot in the real circuit and determine the current flowing across it to find the **Norton current, I_N**.

- **Open the slot,** then all of the current will flow through R_{eq} and the voltage across it and across the gap will relate to the current via Ohm's law.

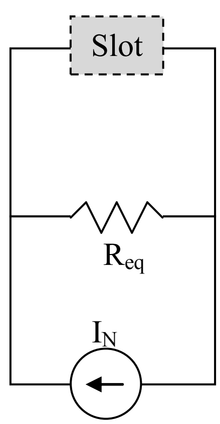


Thus the **equivalent resistance, Req**, can be found by applying Ohm's Law:

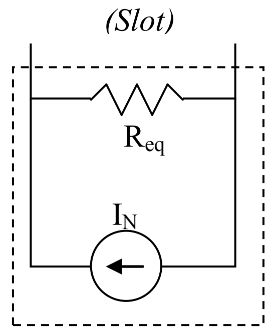
$$R_{eq} = -\frac{\Delta V_{open}}{I_N}$$

These two equivalencies are more equivalent to each other than you might first guess. In the Thevenin scheme, the open circuit voltage we called V_{thev}, it's the same open circuit voltage here in the Norton scheme. In the Norton scheme, the short-circuited current was called I_N, that's the same short circuit current as in the Thevanin scheme. Therefore their ratio is the same in both schemes – that is, R_{eq} is the same.

- **Example (part of 1-13): Real Current Supply**
  - Look at this real power supply; inside is a whole mess of circuitry, but Thevenin's theorem says that, form the prespective of anything plugged into this slot, it looks like



So, a *real* current source interacts with a circuit that you plug into it simply as an *ideal* current source with some resistor in parallel. Sometimes a Real current source is then represented as



Ch 6 Test Equipment and Measurement

6-1 Introduction

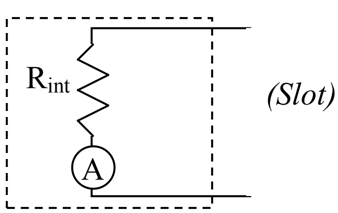
The point is for you to get a peak inside the common meters that you'll be using so you can understand why they're used the way they are and some of their limitations.

Kind of like real Current Supplies and real Voltage Supplies, real meters may be a mess of circuitry inside, but from the perspective of the open 'slot', they behave as an *ideal* ammeter with a resistor in series (and sometimes one in parallel too).

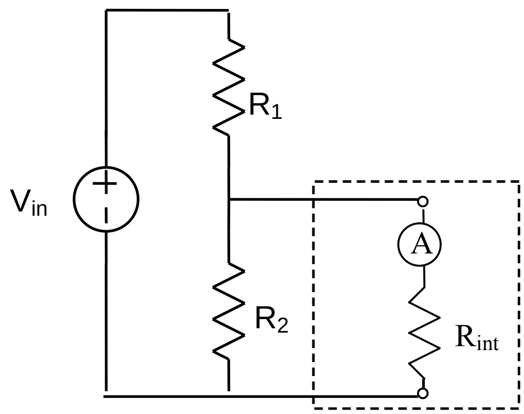
6-3 Input Impedance

This internal resistance or impedance is sometimes referred to as the meter's "input impedance" since whatever current flows into the device must pass through it.

6-3.1 Voltmeter



Say you have this really simple circuit and you want to measure the voltage drop across R_2. The idea is that you want to divert a *little* bit of current to go through the meter instead of R_2, and then measure that current. Knowing the meter's internal resistance, the device can report what the voltage drop must be.



Clearly, putting R_{int} in parallel with R_2 will *change* the effective resistance of the lower half of the circuit and thus *change* the voltage across R_2. To minimize this effect, we need $R_{int} >> R_2$.

On the other hand, if *too* small a current flows through the ammeter, it'll have a hard time measuring such a small signal, so R_{int} must be large enough, but no larger than necessary.

One way of keeping the current through the ammeter small enough is providing an internal shunt resistor that diverts a set fraction of the current.

Example 6.3

6-3.2 Ammeter

Example 6.4

6-4 Digital Meters—

Voltage: Regardless of whether they're being used to determine a current, a resistance, or a voltage, the final step is that a voltage is measured and processed through an Analog to Digital converter. We'll get to that step later, but it's easy to think how a current or resistance is determined:

Current: the current being measured passes through the meter and, in particular, through an internal resistor of known resistance (which needs to be very small as not to significantly impede and affect the current being measured.) This internal circuitry is fused in case the current that would pass through it would be enough to damage the small internal resistor (or other internal components.) Then the voltage across that resistor is measured, and from it the current through it is determined. For different voltage ranges, different internal resistors should be used – sometimes ranging is done automatically, sometimes the user must do it.

Resistance: the meter itself produces a known current and then the voltage across the object is measured – from those two, the object's resistance is determined. So the voltage is of sufficient size to be well measured, different currents get used for different resistance ranges – again, this ranging could be automatic or manual.

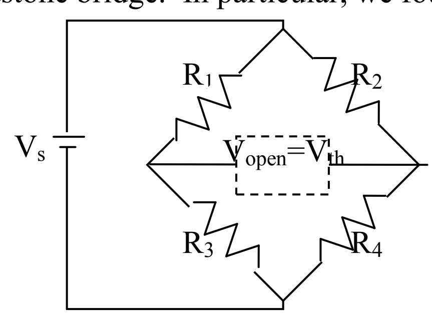
- **Qualities of a Digital Multi Meter**

- **Range**: how small a value can it measure and how large a value can it measure.
- **Number of Digits**
- **Resolution / Precision**: On a given scale, what's the least significant digit? For example, a 6.5 digit display measuring on the 200 Volts scale would display down to 200.0000 V, or a resolution of tenths of milivolts. This is akin to 'precision.'
- **Accuracy / Uncertainty**: Of course, just because it can display so many digits doesn't mean they're all *correct.* A meter's specs will give a relation for determining the Accuracy / uncertainty. It's often in terms of (X% of reading + Y % of full scale), though it may be (X% of reading + z in the last digit(s)).
- **Input Impedance**: Regardless of how accurate the meter is at reporting the voltage across it or the current through it, inserting the meter into a circuit changes the voltages and currents.
  - **Voltage measurements**. Ideally – infinite input impedance. the voltmeter has a finite impedance between its two terminals, usually on order of 1M to 10MΩ; still, when you're making a measurement, that resistance is parallel to whatever element you're measuring across, so the over-all resistance between the two probe points will be reduced by the presence of the meter, and thus the voltage will too.
  - **Current measurements**. Ideally – zero input impedance. For a current measurement, you want the impedance presented by the meter to be much smaller than the resistances already present in the circuit being investigated, so it does not affect the amount of current flowing. Still, it is finite, and will have a significant affect if on-par with the resistances in the circuit.

6-8 Wheatstone Bridge

Let's look back at our work analyzing the Wheatstone bridge. In particular, we found that the voltage across the central slot is

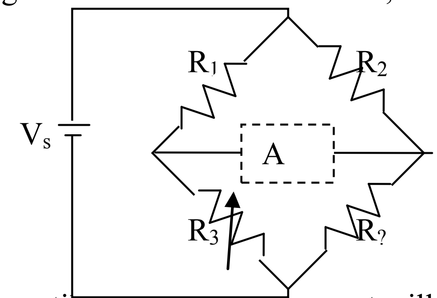
$$V_{th} = V_s\left(\frac{R_4}{(R_2 + R_4)} - \frac{R_3}{(R_1 + R_3)}\right)$$



So, how could the resistances be related so that there is *no* voltage across the slot?

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}.$$

A very sensitive resistance meter is based upon this. Say two of the resistors have well known values, a third is a finely calibrated variable resistor, and the fourth is the resistor you want to learn about. Then placing a current meter in the center,



and adjusting the variable resistor until you register no current will imply that

$$\frac{R_1}{R_2} = \frac{R_3}{R_?}$$

Or

$$R_? = \frac{R_2 R_3}{R_1}$$

6-10 High-Voltage Measurement

Main Points: Don't hurt yourself, and don't hurt your volt meter.

Can use a simple voltage divider so you measure only a fraction of the voltage drop across the two points (but know the relation between what you've actually measured and what you're interested in), but you need the voltage divider's over-all resistance to be much higher than the resistance of the elements across which you're measuring – if not, you'll inadvertently draw much *more* current and reduce the voltage.

6-11 Four-Wire Resistance Measurement

To determine a resistance, we often measure a voltage and the corresponding current. However, using the same leads to pass the current and across which to measure the voltage means the resistance you measure includes that of the leads themselves.