Fri. 1/13	**Ch 1.9-.10** Intermediate DC Circuits & **App. A** pg. **A-1-A-4**	
Mon. 1/16 Wed. 1/18 Thurs. 1/19 Fri. 1/20	**Ch 1.11-.13** & *lightly* **Ch 6.1, .3, .4, .8, .10, .11** **Quiz** Ch 1 & 6, **Lab 1:** DC Circuits More of the same **Ch 2.1-2.5:** Capacitors	**HW1**: Ch1 Pr 4, 25; Ch 6 Pr 9*, 12 **Lab 1 Notebook**

Today's Structure

- **Questions about previous readings**
- **Voltage Divider**
  - Derive relation,
  - Specific example with potentiometer – qualitative stereo speaker.
  - Consider *with* load
- **Mesh Loop rule**
  - **Introductory remarks –** Insightful vs Systematic
  - Conceptual / procedural difference from what you're familiar with
  - I do an example – *just* setting up equations
  - They do an example – *just* setting up equations
  - **Matrix methods**
    - I translate mine into matrix form and have Excel solve
    - They translate theirs into matrix form and have Excel solve
      - For the sake of practice getting to course website, they get it from there.

Requests from last reading:

1. **Ground**
   a. Safety hazard to not have the 3rd prong?
      i. Generally, if the device is not capable of producing high enough currents, then we don't need to worry about the zapp.
      ii. Note: some devices have a 3rd prong for *their* safety rather than ours – a very sensitive device needs a well establish ground so that the kind of static electric zapp that *we* can deliver doesn't harm it; or stray electric/magnetic fields could have noticeable effect on the signal so the ground line is connected to the case to shield its innards.
2. **Work & Charge motion**
   a. Charge moving through a potential difference, work is done – where does this come from, when does it apply? When it's in a resistor, what effect does that have?
      i. Let's step away from circuitry for a moment and briefly refresh our memory of charges, fields, and potentials. 2 Charges (one anchored, one free to move) interact and the kinetic energy of the free one changes; there's a change in potential energy associated with that; we define voltage difference in terms of that change in potential energy. Consider a

charge between charged capacitor plates. So we can see that energy is transferred to the free charge / work is done on it.

ii. Resistor's role – *without* resistance, the charge would speed up and speed up; with resistance, the charged particle collides a lot and thus energy is transferred a second time – to the atoms of the resistor which then jiggle more violently and so the resistor is warmed. Of course, the resistor itself transfers away that newly acquired energy – radiates and air conducts.

More General Questions

3. **Alternator**
   a. How does an alternator keep a car's battery from running down?
      i. Of the bat, I'm not sure where the name comes from, but in very broad strokes, the alternator is an electric generator. Thanks to the car's motor, a shaft spins, it probably has wires wrapped around it and that spinning shaft is encased in a doughnut of magnets. The electrons that naturally inhabit that wire, as the wire moves in the presence of the magnets, feel a force that pushes them along the wire, thus a current is driven along the wires. The wires are connected to the battery as to drive electrons *back* in the opposite direction from that which the batteries chemical processes would drive them. This *undoes* the chemical processes and thus 'recharges' the battery.
4. **Why AC vs. DC?**
   a. When do you choose which?
      i. Electricity gets used for two things – simply transmitting energy, and transmitting/manipulating information. AC has an advantage for the former.
         1. **Energy transmission**. Most devices run on DC; however, there's an advantage to *transmitting* energy electrically in AC instead. When current flows along a wire, resistive heating occurs. So energy is continually getting shed, thus, to keep the current flowing, somebody (the power plant) has got to keep expending energy. The rate of that loss goes like

            $$P_{loss} = I^2R.$$

            So, the greater the current, the more energy is lost. On the other hand, the rate at which you can get this electricity to do work for you wherever you deliver it is

            $$P_{use}=IV,$$

            where that V is the difference between the voltage at which the electricity is being transmitted and that to which you'll ultimately sink it (say, through a lightbulb with one end connected to the power line and the other to the ground.) So you can transmit the same amount of energy but with much less resistive loss along the way if you transmit it at *high* voltage and *low* current.
            Now, here's where AC becomes useful. Ultimatly, most devices we use want fairly *lower* voltages. So, we want to *transmit* from

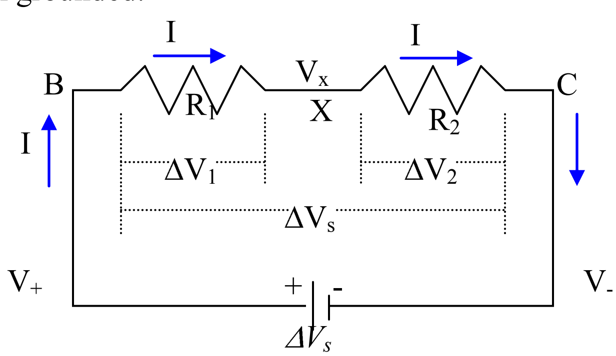
the power plant to the city & house at *high* voltage, but *use* at *low* voltage. In later chapters you'll learn about "transformers" which are devices that can "step up" or "step down" a signal's voltage (and compensatingly step down or up its current) without wasting a lot of its energy, but that works by magnetic induction – which only occurs with *varying* currents. So, we use AC currents so we can easily transform the voltages from transmission to useful levels.

5. **Voltage Divider** –
    a. Use, practical example – speaker's volume.
    b. Deriving the equation (1.17)
6. **Mesh Loop –**
    a. Example
    b. Wheatstone Bridge
7. **Matrix stuff**
    a. Refresher, please also go over the equations form A-10 onwards.

1-1 Voltage Dividers, Potentiometers, and Current Dividers

1-1.1 Voltage Dividers

- The book doesn't explicitly say this, but the voltages are relative to the voltage at C, i.e., the voltage of the negative terminal of the battery. In circuits, this is a common reference point, one that's often grounded.



$$\Delta V_s + \Delta V_1 + \Delta V_2 = 0$$

- $$\Delta V_s - I(R_1 + R_2) = 0 \Rightarrow I = \frac{\Delta V_s}{(R_1 + R_2)}$$
- $$\Delta V_2 = (V_- - V_x) = -IR_2 \Rightarrow (V_x - V_-) = IR_2$$
- Substituting in the relation for I gives

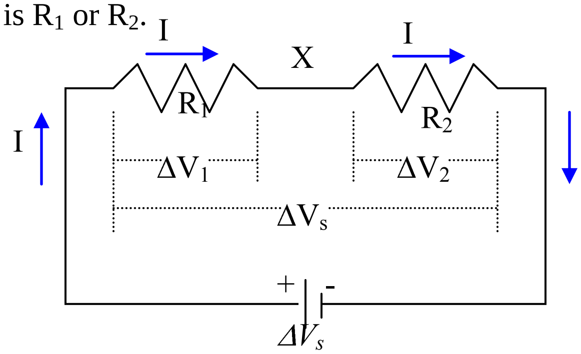
- $$(V_x - V_-) = \Delta V_s \frac{R_2}{(R_1 + R_2)} = \Delta V_s \frac{1}{\left(\frac{R_1}{R_2} + 1\right)}$$

So, the voltage at point X (relative to the - terminal) is a set fractoin of the whole voltage across the source.

1-1.1.1 Example 1.4

Using two resistors, one which has a 10kΩ resistance and a 12 V battery, desing a circuit from which an 8 V potential (relative to the negative terminal) can be derived

So, the real question is, what's the appropriate value of the second resistor? We have our choice of whether the 10 kΩ is R_1 or R_2.



$$(V_x - V_-) = \Delta V_s \frac{1}{\left(\frac{R_1}{R_2} + 1\right)}$$

Let's say that R_1 is the 10 k, then, we want to solve for the R_2 that gives us the desired 8V drop across R_2.

$$\left(\frac{R_1}{R_2} + 1\right) = \Delta V_s \frac{1}{(V_x - V_-)}$$

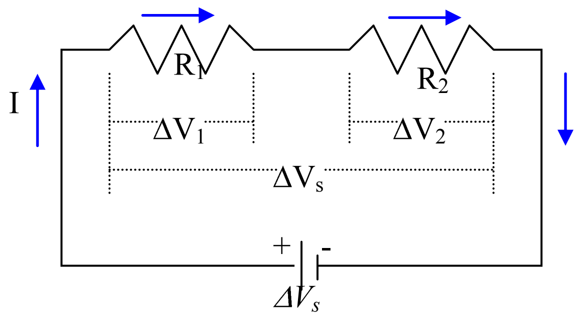
$$\frac{R_1}{R_2} = \frac{\Delta V_s}{(V_x - V_-)} - 1$$

$$\frac{R_1}{\frac{\Delta V_s}{(V_x - V_-)} - 1} = R_2$$

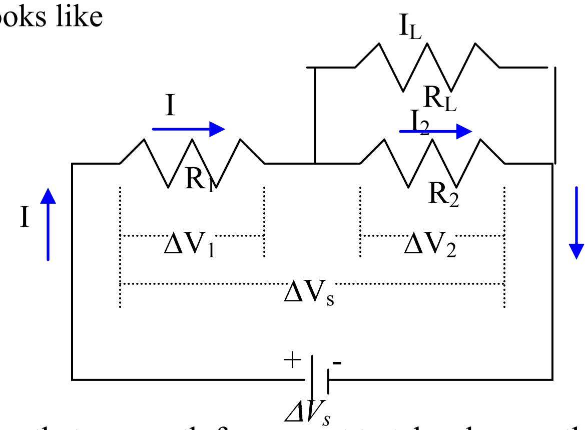
$$\frac{10k\Omega}{\frac{12V}{10V} - 1} = 50k\Omega = R_2$$

- **With a Load**
    - A voltage divider is a very common chunck of circuitry, though in the context of a divice, it's usually just one small piece. Maybe V_+ is obtained not from a battery

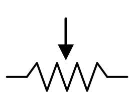
but from a transducer, perhaps a pressure sensor, and its value is proportional to the pressure you're measuring. Perhaps you need to divide the signal by 10 before you can input it to your computer interface, so you select R_1 and R_2 so that $V_x = V_+/10$. Then V_x is "passed" to the interface.

- 

- But you've got to be careful. The interface is composed of circuitry too, so, it has some resistance between the two leads that connect to your divider, we'll just call that R_L, L for "load." So, when the divider's "driving a load", the composite circuit looks like



- So, adding that new path for current to take changes the voltage. How? Well, R_L and R_2 are in parallel, so this new circuit is equivalent to the voltage divider with R_2 replaced by the equivalent resistor, $\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_L}$.
- Then $(V_x - V_-) = \Delta V_s \frac{1}{\left(\frac{R_1}{R_2} + 1\right)}$ changes to $(V_x - V_-) = \Delta V_s \frac{1}{\left(R_1\left(\frac{1}{R_2} + \frac{1}{R_L}\right) + 1\right)}$
- As you might imagine, if the load resistance is much larger than R_2, it has little effect on the circuit since $1/R_L << 1/R_2$.
- That's why the internal resistance of most voltage measuring devices is quite large.

1-1.2 Potentiometer 

- Variable resistor / voltage divider

- Imagine you had a long carbon bar that you wanted to use as a resistor in your circuit. Now, you could clip the wires onto the two far ends, and that would give you one resistance, or you could clip one wire on one end, and the other somewhere near the middle, that would give you a smaller resistance. That's an example of a very crude potentiometer – you can choose where along the length of the resistive element to make your connection, and thus through how much of that material the current must flow / how much resistance it contributes to the circuit. For that matter, you can use the potentiometer like two resistors in series – one connection at one end, one at the other, and one somewhere in the middle then you've got a "voltage divider" arrangement, just like in the diagram on the previous page.

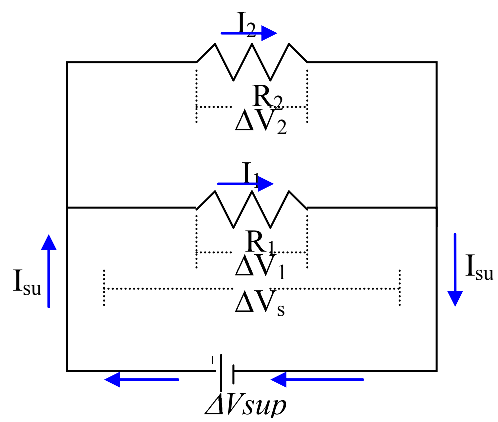
1-1.3 Current Divider

$\Delta V\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=I_s$ while $\Delta V = I_1 R_1$

So

$I_1 R_1\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=I_s$

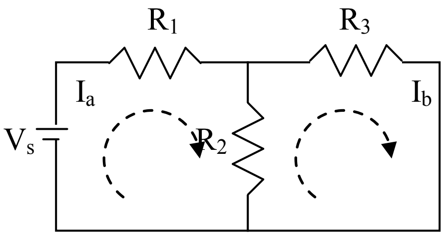
or flipping it around $I_1 R_1\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=I_s\left(\frac{1}{1+\frac{R_1}{R_2}}\right)$



1-2 The Mesh Loop Method

- Insightful vs. Systematic

- The Mesh Loop Method conceptualizes the circuit as composed of adjoining loops. Each loop has its own current, and components shared by more than one loop have concurrent currents flowing through them.
    - Note that senses of circulation need not agree, so the voltage drops across shared elements may be evaluated in opposite directions.
    - The book does this one, so I'll only reproduce the set up



Loop a)

$$\Delta V_s + \Delta V_1 + \Delta V_{2.a} = 0$$
$$\Delta V_s = -\Delta V_1 - \Delta V_{2.a}$$
$$\boxed{\Delta V_s = I_a R_1 + (I_a - I_b)R_2}$$

Loop b)

$$\Delta V_3 + \Delta V_{2.b} = 0$$
$$-I_b R_3 - (I_b - I_a)R_2 = 0$$
$$\boxed{I_b R_3 + (I_b - I_a)R_2 = 0}$$

These two equations can be combined to solve for the two unknowns of the currents and the current through resistor 2 can be found as (I_a-I_b). By the convention of loop a, if this is positive, the current goes down the page.

Rephrase in Matrix Form

$$\Delta V_s = I_a R_1 + (I_a - I_b)R_2 \qquad \Delta V_s = I_a(R_1 - R_2) + I_b(-R_2)$$
$$0 = I_b R_3 + (I_b - I_a)R_2 \qquad 0 = I_a(-R_2) + I_b(R_3 + R_2)$$

$$\begin{bmatrix} \Delta V_s \\ 0 \end{bmatrix} = \begin{bmatrix} (R_1 - R_2) & -R_2 \\ -R_2 & (R_3 + R_2) \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix}$$

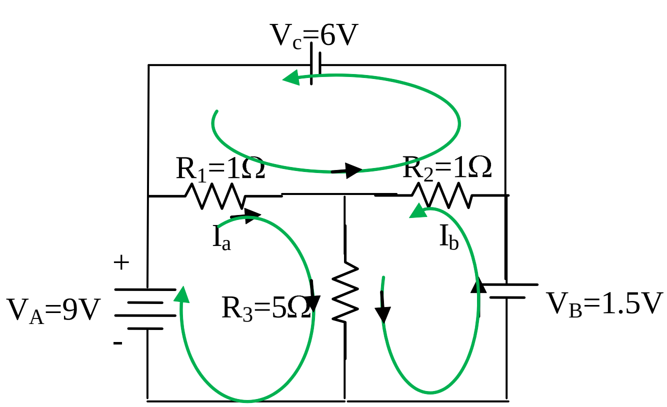
$$V = RI$$

$$VR^{-1} = I$$

We'll get to in a moment.

Now, how do we tackle this one?

They Do – just the setup:

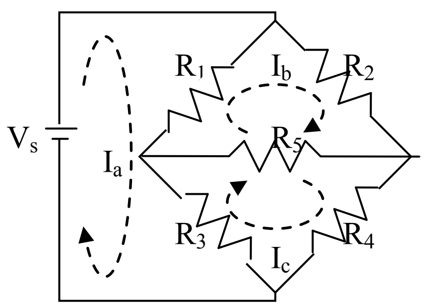


We should analyze each loop in the circulation we've represented and be careful about whether each voltage step is up (positive) or down (negative) following that path.

Excel

1-2.1 Wheatstone Bridge

- This is an example where it's much easier to jump in with the Mesh Loop approach than applying the independent node and loop rules since we need only 3, rather than 5 distinct currents to consider.



$$V_s = R_1(I_a - I_b) + R_3(I_a - I_c)$$
$$0 = R_1(I_b - I_a) + R_2(I_b) + R_5(I_b - I_c)$$
$$0 = R_3(I_c - I_a) + R_4(I_c) + R_5(I_c - I_b)$$

Three equations, three unknowns – it's doable.

Now, the algebra mightn't be pretty, but you can slog through it. Alternatively, matrix math is great for handling coupled linear equations.

Rearranging as

$$V_s = (R_1 + R_3)I_a - R_1 I_b - R_3 I_c$$
$$0 = -R_1 I_a + (R_1 + R_2 + R_5)I_b - R_5 I_c$$
$$0 = -R_3 I_a - R_5 I_b + (R_3 + R_4 + R_5)I_c$$

Appendix A: The Method of Determinants

This information could be rephrased in matrix form

$$\begin{bmatrix} V_s \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 + R_3 & -R_1 & -R_3 \\ -R_1 & R_1 + R_2 + R_5 & -R_5 \\ -R_3 & R_5 & R_3 + R_4 + R_5 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Appendix A gives a recipe for solving for the currents. It's a little tedious. The good news is that Excel's good at tedious things. I've posted on the course website a document that explains how to use Excel to deal with a matrix relation like this. Here's the basic idea.

The basic point is that the mathematical operation of multiplying by a matrix has an inverse, just like multiplying and dividing, or cosine and arc-cosine. If you multiply both sides of an equation by a matrix's inverse, the matrix is removed from one side, and its inverse is left on the other.

Writing the above equation as $\vec{V} = \hat{R}\vec{I}$ where $\hat{R}$ is the resistance matrix. Then, its inverse would be represented as $\hat{R}^{-1}$ (which doesn't literally mean that the matrix is upside-down). So,

$\hat{R}^{-1}\vec{V} = \hat{R}^{-1}\hat{R}\vec{I} = \hat{1}\vec{I}$. The 1 is the identity matrix $\hat{1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ which is trivial to multiply by our "current array."

As I said, Excel will handle this for you, so check out the website and look at today's entry on the schedule.

1-2.1.1 What's the Wheatstone Bridge good for anyway? Historically, this peculiar configuration has been used as a very sensitive Ohm meter, i.e., to measure resistance. If R4 is unknown, but the other resistances are known, and one of the other resistors, say R2, is a precision potentiometer (so its resistance can be varied), then that resistance can be varied until R1 + R2 = R3 + R4, at which point the current through R5 = 0. So R2 would be varied until R5's current is 0, then R1 + R2 = R3 + R4 must be true and, knowing the other three resistances, R4 can be solved for.