Goals

1. To learn about uncertainty and the distribution of measurements for a repeated counting experiment.
2. Investigate the Boltzmann distribution, which describes the probability of finding a system at different energies.
3. To get additional practice writing a lab report.

Equipment:

Squiggle ball, ping-pong ball, calibrated ramp

Reading:

- Chapter 11 (Exponential Curve Fitting) of the lab reference manual
- Review chapter T6
- If you are measuring the number of events occurring during a time interval for a random phenomenon, often only a single number is measured. For that reason, standard deviation typically isn't found in the usual way for this type of measurement.

  Suppose the number of events was measured for several trials. If the average number of events measured was μ, the probability of measuring n events in a single trial is

  $$P(n) = \mu^n e^{-\mu} / n!\,,$$

  which is a Poisson distribution. Note that the number of events can be any non-negative integer (including zero). Since the total probability of measuring some value of n is one, we know that

  $$\sum_{n=0}^{\infty} P(n) = 1\,.$$

  For a large number of trials, the standard deviation in the number of events measured for this type of distribution will be $s \approx \sqrt{\mu}$. For more details, see Philip R. Bevington, *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, New York, 1969), pp. 36-43.

  When making a single measurement, assume that the standard deviation in the number of events (N) is approximately the square root of the number of events ($\sqrt{N}$). Since the number of events measured in a single time interval is an integer, its uncertainty should also be reported as an integer. For example, if you measure 50 radioactive decays in 1 second, you should present the result as 50±7 decays per second.

- Review chapter 3 (How to Write a Lab Report) of the lab reference manual

Pre-Lab Problems (to be done in WebAssign; not required in notebook):

1. You'll use *LinReg* to determine the exponential decay constant and initial population, including uncertainties, for a colony of bacteria.
2. To check the activity of a radioactive sample, an inspector places the sample in a device to count the number of decays.

   a. If the device measures 33 counts in a minute, how should the number of decays per minute be reported (including the uncertainty)?
   b. Suppose, instead, the sample was monitored for 25 minutes and 907 counts are measured. How would the number of decays per minute be reported?

   The advantage of running the counting experiment for a longer time should be evident.

Lab Procedure:

- Determine the masses of the ping-pong ball and the Squiggle Ball™.
- When the battery-powered Squiggle Ball™ is turned on, it randomly moves around the area caged in by rubber bands, serving as a model of a constant-temperature "reservoir." Mechanical energy can flow across the flexible boundary to or from the ping-pong ball, so it remains in "thermal equilibrium" with the Squiggle Ball™.
- In order to test that the Poisson distribution applies to a counting experiment, perform one hundred 30-second trials in which you count how many times the ping-pong ball reaches the 2nd level (but *not* higher). We will say the ping-pong ball has reached a level if its trailing edge gets to that level. Be sure to include counts of zero.
- In order to investigate the Boltzmann distribution, do the following:
  - Tally the number of times the ping-pong ball makes it each gravitational "energy level" on the inclined plane for about 500 bounces. You should use the same method as above for determining what level the ball reaches. This will require you to pay careful attention to the experiment for 45-50 minutes. It will probably work best to have one person call out the level reached and another person keep a tally of them. For the extra credit part of the post-lab, it will be helpful to keep track of how long the large number of bounces takes.
  - Determine the slope of the ramp, the spacing of the levels, and the difference in the ping-pong ball's energy between levels.
  - Place the Squiggle Ball™ on the ground and take measurements to determine its approximate speed when it's traveling in a nearly straight line. Be sure to record what you did in your lab notebook.
- Make any additional measurements that might help with the extra credit part of the post-lab.

Post-Lab Assignment:

1. For the 100 short trials, compare your measurements to a Poison distribution:
   a. Calculate the average number of times the ping-pong ball reaches the 3rd level in a trial. Since you calculating the average from many trials, do *not* round it to an integer.
   b. Directly calculate the standard deviation in the number of counts (do *not* use the approximation mentioned in the reading). Since you calculating the standard deviation from many trials, do *not* round it to an integer. How does this compare to what you expect it to be?

c. Make a histogram of the *probability* of various outcomes (numbers of counts). Be sure to include the number of times that there were zero counts. Recall that this is a *height-normalized histogram* because the heights of all of the bins add up to one. It can be directly compared to the Poisson distribution, which also has this property.

d. For each number of counts, calculate the value of the Poisson distribution using the average found above. At the middle of each bin horizontally, plot these points, which are the theoretical probability for each outcome. There is no need to draw a smooth curve through the theoretical points since the number of counts can only be an integer.

Note: Since you only made 100 measurements, the distribution of your data may not exactly match your expectations.

2. Analyze the approximately 500 consecutive measurements of the level reached assuming that they obey a Boltzmann distribution:
   a. Make a plot of the natural logarithm of the number of counts vs. the energy of the level reached. Do not include any level for which you had less than ten counts. Be sure to include error bars. Use the *twice* the standard deviation of the number of counts as the uncertainty, which is the 95% confidence level. You will have to determine the energy associated with each level. You can define the energy to be zero anywhere that you want.
   b. Suppose the Squiggle Ball™ acts as a constant temperature reservoir and that the ping-pong ball is thermal equilibrium with it. Using linear regression, determine the *effective temperature* of the Squiggle Ball™. (Hint: How do you expect the number of counts to depend on the energy?)
   c. Find the rms speed of the Squiggle Ball™. Based on your measurement of the speed of the Squiggle Ball™ by itself, is your result reasonable? Explain any substantial discrepancy. (Hint: Do you expect the rms speed to be smaller, larger, or the same as the speed when the ball is moving across the floor?)
3. Write a *complete* lab report that discusses *both* parts of the experiment. It should be typed. Follow the instructions given in chapter 3 the laboratory manual. Be sure to include pictures or diagrams of your setup and derivations of any equations used. <u>This report is due Tues, December 6</u>. You’re getting extra time to write this report, but you won’t get an opportunity to rewrite it.
4. **Extra Credit**: Estimate how long you would have to wait for the ping-pong ball to go off the end of the ramp. Be sure to explain your work!