Hand out magnets!

Fri., 2/6	**17.1-3** Intro to Magnetic Field	**RE11**
Mon., 2/9 Tues., 2/10 Wed., 2/11 Thurs., 2/12 Fri., 2/13	**17.4-.6** Biot-Savart law for Currents **17.7-9** Magnetic Field For Distributions **Lab 5**: Biot-Savart – B fields of moving charges **17.10-11** Permanent Magnets	**RE12** **RE13, Exp** 20-22 **Exp** 23-29 (work together for 2 magnets)

Prep.

- Check WebAssign
- Load V for VPython

Equipment

- Magnetic attraction of parallel currents demo
  - Plank with long wires and power supply
- Magnetic Field direction due to a current demo
- Experiment Kits for early Magnetic experiments

Intro

Putting Magnetic Interaction in Intro Physics Context

- Where have been and where are we going?
- **Last Semester: Interactions & Motion.** So, last semester you looked at the impact of interactions on motion. Two of the main conceptual / mathematical tools we used were the Momentum Principle and the Energy Principle.
- **This Semester thus far:**
  - **Electric Interaction**
    - **Fields.** First we did so in the context of the Momentum Principle – we found that rather than thinking of Forces directly between two charged objects, it was useful to think of an intermediary – one object establishes a field which in turn communicates the force to the other object.
    - **Potentials / Voltages:** Next, we did so in the context of the Energy Principle – we found that rather than thinking of changing Potential Energies of interacting charged objects, we'd think of one charged object establishing an electric potential, and the other object's motion through it constitutes a change in Potential Energy.
    - **Conceptual Abstractions, worth it:** Electric Field and Electric Potential were conceptual abstractions, and we seldom make life more complicated for ourselves than we have to – in fact, both of these prove powerful and even necessary.

- **Now:**
    - **Magnetic Interactions.** Now we move on to another interaction, one that you *didn't* meet last semester, though you no doubt have some practical familiarity with it – the *magnetic* interaction. We'll see that it too is an interaction between charged particles, but its dependent not just on charge, but on *velocity* (both magnitude and direction). In that way, it is both related to and distinct from the Electric interaction.
        - **Magnetic Field.** The magnetic interaction is a little more peculiar than the electric, it is therefore a little more important that we conceptually break it down into bite-sized parts: in the context of the Momentum Principle, again, one could think of the interaction in terms of forces directly between two moving charged particles. But it is again powerful to consider an intermediary – a *magnetic* field that is generated by one moving charge and that influences the other.

Theoretical Framework

We'll start today building the theoretical basics of the Magnetic Interaction. Now, it's a much less intuitive push-pull kind of interaction than is the electric, so we'll then start working to *get familiar* with it.

Magnetic Interaction: The Magnetic Force

- **Magnetism as Relativistic Electric Interaction.**
    - **Stationary Charge & Current => no interaction.** Last time (Monday), we reasoned that if we had a current carrying wire and a charged particle just *sat* beside it, the charged particle would see the wire as net neutral, and wouldn't feel a thing.
    - **Moving Charge & Current => Interaction.** However, if the charge moved alongside the wire, while the wire still looks neutral to *us*, it *doesn't* look neutral to the moving charge (thank you Einstein.) That means that, while *we* wouldn't have expected the charge to feel a force, *it does.* We call it the magnetic force.
    - **E&M Unification.** Really, as we can conclude from the wire & charge example, the "Magnetic Interaction" and the "Electric Interaction" are two special cases of something more general: the Electro-Magnetic Interaction. Though this course won't get to it, these two have further been unified with the "Weak Interaction", making the "Electro(magnetic)-Weak Interaction."

Magnetic Interact as Distinct from Electric. While it is *conceptually* satisfying to understand the unification of electric and magnetic interactions, much of the time, it is *practically* convenient to treat them individually. That is what we'll do the vast majority of the time. So let's start thinking about the Magnetic Interaction in its own right.

Demo: Current carrying wires interacting

- **Intro. A new way to think about magnetism.** We're going to consider the interaction of two current carrying wires. Now, I expect that you already have some practical experience with magnetism – who among us *hasn't* used a magnet to post something to a refrigerator, or used a compass when hiking, or clipped together the cars of a child's train? Some of you may even have had some quantitative experience with magnetism in high-school physics. I urge you to put all that from your mind and get ready to start afresh. The essence of magnetism is found not by looking at fridge magnets, but at currents.

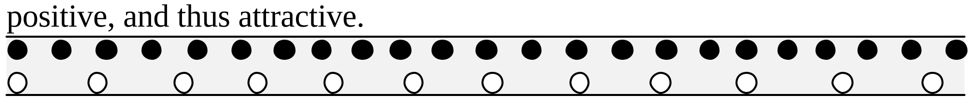
Q: I'm going to run current through these two wires. *What do you expect to see?*

Leading Questions

- What did we discuss Monday?
- If *we* see both wires as neutral, what does an electron flowing through one wire see in the other wire?
- So what force does it feel?
- Now, if *all* the moving electrons in the wire feel that, what should the wire do?
- **Do the demo:** Currents run parallel.

Q: *What's happening here?*

- **Charge flow:** In these two wires, charged particles are moving parallel. Mind you, they're moving over a backdrop of atomic ion cores, so the wires are neutral to us (except for a very miniscule charge build up).
- **Attraction of parallel:** They attract each other!
  - **Q:** *What does the charge distribution in the first wire look like from the perspective of the electrons moving in the second wire?*
    - **From the perspective of the moving charges:** This is exactly what we reasoned out on Monday, but instead of a wire and a moving charge, we have a wire and another wire's worth of moving charges. From the perspective of the electrons moving in the second wire, the first wire is net positive, and thus attractive.



- **Repulsion of Anti-parallel:** Guess what happens if I switch the direction of charge flow so their anti-parallel? Apparently charges moving in opposite directions repel each other.
  - **Q:** *What does the charge distribution in the first wire look like from the perspective of the electrons moving in the second wire?*
    - **From the perspective of the moving charges:** Apparently there's a net negative charge. That's because the positive atomic cores appear to be moving back yeay fast (so yeahy compressed separation) while the electrons appear to be moving back even faster (so even more compressed separation.)

- **Perpendicular force for Perpendicular wires:** It's not easy for me to show, but if I had one wire running up and the other coplanar but running perpendicularly across it, that second wire would be pushed as to spin into alignment.
- **Zero Force for encircling.** Finally, if the second wire encircled the first and the charges ran around it – there would be *no* push at all!

This is a most bazaar interaction!!

- **Magnetic Force Expression**
  - **Intro.** Monday, we actually derived the force equation for a charge interacting with a wire. The conceptually (if not mathematically) simpler case is the interaction of just two moving point charges.
  - **Electric.** Recall that Coulomb's law describes the electric force between two charged particles: $\vec{F}_{E.1\leftarrow 2} = \frac{1}{4\pi\epsilon_o}\frac{q_1 q_2}{r_{1-2}^2}\hat{r}_{1-2}$
  - **Magnetic.** Here's the Magnetic force for two, constant velocity charged particles:
    - $\vec{F}_{M.1\leftarrow 2} = \frac{\mu_o}{4\pi}\frac{q_1\vec{v}_1\times\left(q_2\vec{v}_2\times\hat{r}_{1-2}\right)}{r_{1-2}^2}\left\{\frac{1-\left(\frac{v_2}{c}\right)^2}{\left(1-\hat{r}_{1-2}\cdot\vec{v}_2/c\right)^3}\right\}$ Ack!
      - Of course, all particle-2 quantities would be evaluated at the retarded time (where was, and how fast was q_2 at the time the radiation was emitted that now reaches q_1, at time $t_r = t - cr_{1-2}$. (This expression follows from Griffiths' 10.66, or 10.67; since Griffiths makes the point that the "Biot-Savart law for a particle" is not exact, so, for the sake of consistency, it's worth noting this now).
    - **v<<c.** Fortunately, the term in squiggly brackets is negligible unless v is on order of c. So, for most cases we can approximate this as
      - $\vec{F}_{M.1\leftarrow 2} \approx \frac{\mu_o}{4\pi}\frac{q_1\vec{v}_1\times\left(q_2\vec{v}_1\times\hat{r}_{1-2}\right)}{r_{1-2}^2}$
        - Yes, these are cross-products, and yes, this is still kind of ugly.
    - **There's still an Electric interaction.** Mind you, just because the charges are moving doesn't mean that their *electric* interaction goes away –that's there too (though we have to deal with retarded times), but now there's an additional magnetic interaction.

- **Basis of *all* Magnetism.**
    - Whether we're talking two moving charges, currents in wires, or even two magnets or the Earth and a compass – this interaction lies at the heart of *all* magnetism. (one might quibble about magnets and electron orbital angular momentum and spin, but at least at the semi-classical level, this is still the picture.)

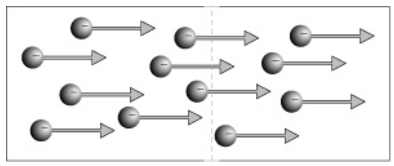
- **Magnetic Field**
    - **Electric Field.** Now, it was convenient to define the Electric field as essentially the Electric force without the particle 1 specific factor:
        - $\vec{E}_{1\leftarrow 2} = \frac{1}{4\pi\varepsilon_o}\frac{q_2}{r_{1-2}^2}\hat{r}_{1-2}$, so $\vec{F}_{E.1\leftarrow 2} = q_1\vec{E}_{E.1\leftarrow 2}$
    - **Magnetic Field.** Similarly, it will be convenient to define the *Magnetic* field as essentially the Magnetic force without the particle 1 specific factor:
        - $\vec{B}_{1\leftarrow 2} \approx \frac{\mu_o}{4\pi}\frac{\left(q_2\vec{v}_1 \times \hat{r}_{1-2}\right)}{r_{1-2}^2}$, so $\vec{F}_{M.1\leftarrow 2} = q_1\vec{v}_1 \times \vec{B}_{1\leftarrow 2}$.
            - **Units (T) Tesla = N/(C m/s)**
            - $\frac{\mu_0}{4\pi} = 1\times 10^{-7}\,\frac{\mathrm{T}\cdot\mathrm{m}^2}{\mathrm{C}\cdot\mathrm{m/s}}$
        - **Mathematically simpler.** Mathematically speaking, the magnetic field is at least one step nicer to look at than is the force (one fewer cross products). So, we will spend a lot of time getting familiar with this field.
        - **Conceptually Abstract.** One thing to notice is that, since the force is the *cross* product of the velocity and the field, unlike the Electric field, the Magnetic field does *not* point in the direction of the force. That is part of the price we pay for defining it so simply.

Getting Familiar with Magnetism.

- **Intro.** Now that we've laid down the theoretical framework, let's start getting familiar with it and the associated math. We'll return to this equation, but first we've got some getting ready to do.
- **Compasses and Currents.**
    - **Compasses and Magnetic Fields.** A compass needle points in the direction of the magnetic field at its location. Thus, in absence of any other magnetic sources, the compass points in the direction of the Earth's magnetic field. Heck, that's usually what they get used for.

Demo: Hand out boxes. Make sure your compass needle points north (red) – south (white).

- **Electron Current.**
    - **Intro.** It's kind of hard for us to experimentally work with individual point charges that move fast enough to significantly interact magnetically. But its' pretty easy for us to get a whole stream of them going as I did in the demo: an electrical current. So we'll use currents to explore magnetism.
    - **Electron Current.** $i = N_e / \Delta t$ (electrons per second)
        - Defined as the number of electrons per second pass through a cross section of a conductor.



Establishing Currents.

<u>Experiment 17.17</u>: First, let's get familiar with how to get a current going with your experiment kit.

students should learn

- A complete loop is required for current to flow
- The structure of the light bulb and the socket
- How to connect batteries in series

<u>Extra Observations</u>:

- Test that charged tapes have <u>no</u> effect on the compass needle
- The compass needle is attracted to steel objects (including the alligator clips on the wires), even if they're neutral

Magnetic Field of a Current

Now that you're a pro at establishing a current, let's investigate the magnetic field it generates.

<u>Experiment 17.18</u>:

(a) Compass needle deflects away from the direction of the wire

(b) Nothing happens (or very little) when the wire is perpendicular to the compass needle

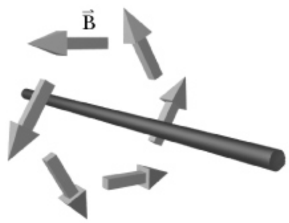
(c) The needle deflects the opposite direction when the direction of the current is reversed

(d) The needle deflects the opposite direction when the wire is below instead of above

(e) No effect due to the copper wire without current (be careful about the clips)

(f) More current results in a bigger deflection

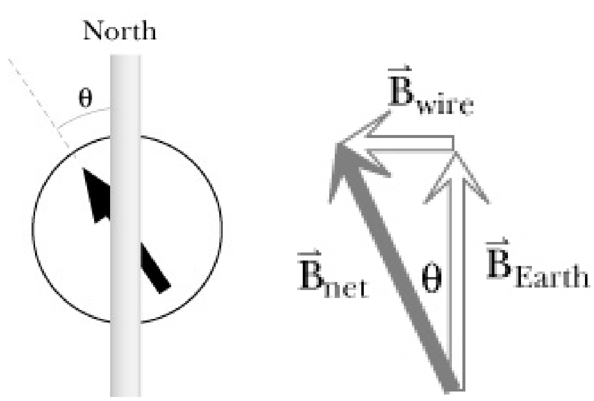
- **Conclusions**
  - the magnitude of the magnetic field produced depends on the size of the current
  - the direction of the magnetic field is perpendicular to the direction of the current
  - the direction of the magnetic field is in opposite directions on opposite sides of the wire
- Putting this all together: The magnetic field around a wire must look something like the diagram below.



Demo. 17_B_long_wire.py

Getting Quantitative.

- **Experimental: Compass Angle to Earth's field.**
  - We can use a compass to get a qualitative feel for the direction of the magnetic field. It may be surprising, but we can also use it to get a quantitative feel for the field's strength.
  - The horizontal component of the Earth's magnetic field has a magnitude of approximately $\left|\vec{B}_{\text{Earth}}\right| = 2\times10^{-5}$ T.
  - If the wire is in the North-South direction as shown below, the magnetic field due to the wire $\vec{B}_{\text{wire}}$ is perpendicular to the Earth's field. The compass needle will deflect in the direction of the net magnetic field $\vec{B}_{\text{net}} = \vec{B}_{\text{Earth}} + \vec{B}_{\text{wire}}$.
  - The deflection angle *q* can be used to calculate the magnitude of the magnetic field due to the wire in this case.



IMPORTANT NOTE: This technique will be used in several of the textbook experiments! It is important that the magnetic field to be measured is perpendicular to the Earth's magnetic field.

> **Example:** Suppose the wire is parallel to the compass needle when there is no current. When the circuit is connected, the needle deflects by 10°. What is the strength of the magnetic field due to the wire?
>
> Since $\vec{B}_{\text{Earth}}$ and $\vec{B}_{\text{wire}}$ are perpendicular, the deflection angle is (see the diagram above):
>
> $$\tan\theta = B_{\text{wire}} / B_{\text{Earth}}$$
>
> $$B_{\text{wire}} = B_{\text{Earth}} \tan\theta = \left(2\times10^{-5}\ \text{T}\right)\tan 10^{\circ} = 3.5\times10^{-6}\ \text{T}$$

Clicker Question 17.7d-g

- **Theoretical: Biot-Savart Law**
  - Now let's see how our observation of the magnetic field due to a current relates to our mathematical parameterization of it. The Biot-Savart Law that I've introduced speaks about the field due to a moving point charge, but what's a current other than a string of moving point charges (against a neutralizing backdrop of stationary ions). We'll derive the expression for a full current's field, but for now, let's return to the mathematical expression we have for the magnetic field due to a moving point charge.
    - $$\vec{B}_{1\leftarrow 2} \approx \frac{\mu_o}{4\pi}\frac{\left(q_2\vec{v}_1\times\hat{r}_{1-2}\right)}{r_{1-2}^2}$$
    - Okay, its strength depends on the charge and its velocity as well as the distance from the charge. One might have guessed as much. But what do we mean by crossing the velocity into the unit vector?
    - **Cross Product**

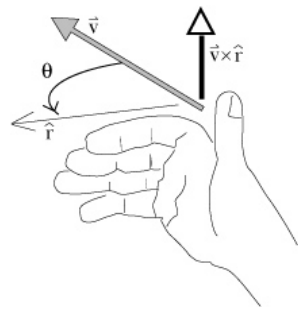
$$\vec{A}\times\vec{B} \equiv \det\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \left\langle \left(A_yB_z - A_zB_y\right), \left(A_zB_x - A_xB_z\right), \left(A_xB_y - A_yB_x\right) \right\rangle$$

magnitude: $\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$

- where θ is the angle between $\vec{A}$ and $\vec{B}$

direction: determined with the "right-hand rule" (RHR)

- point fingers of right hand in direction of first vector
- rotate wrist so you can curl the fingers to the second vector's direction
- your thumb will point in the direction of the cross product



The direction of the cross product is also perpendicular to the two vectors being multiplied. If the vectors are parallel, the cross product is zero.

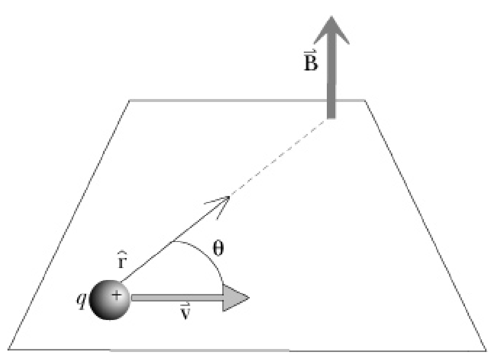
Demo: 17_Crossproduct.py

Here's a visual to go along with the math. See how the product's direction and magnitude varies with the magnitudes and relative angles of the multiplied vectors.

Clicker Questions 17.3a-c

- **Back to Biot-Savart**
  - Now that we 'get' the cross-product, let's return to looking at the Biot-Savart expression for a moving charge's magnetic field.
  - The magnetic field at a location $\vec{r}$ relative to a charge q moving a velocity $\vec{v}$ is:

$$\vec{B}=\frac{m_0}{4p}\frac{q\,\vec{v}\times\hat{r}}{r^2},$$



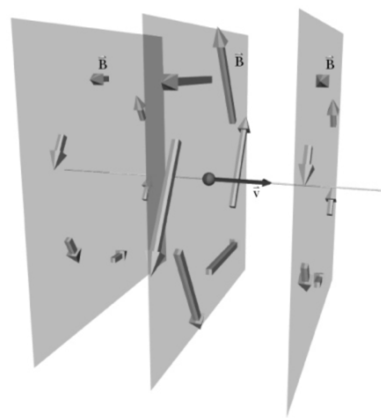
Clicker Questions 17.3d-g

Demo: **17_Bparticle_1loc_PRIVATE.py**

See how the direction and magnitude vary according to the $1/r^2$ and the v×r.

Demo: 17_Bproton_PRIVATE.py

In 3-D, the magnetic field looks like the following (use VPython to demonstrate)

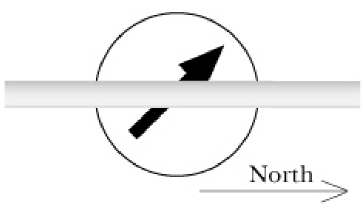


For a whole string of charges, i.e., a current, it looks then like this

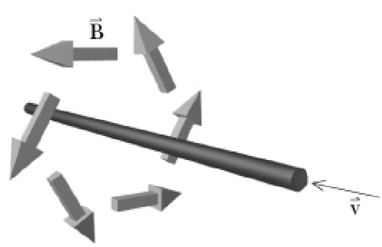
Demo: 17_long_wire.py

For a positive charge, $\vec{B}$ is in the direction of $\vec{v}\times\hat{r}$, but for a negative charge $\vec{B}$ is in the opposite direction of $\vec{v}\times\hat{r}$ (the minus sign reverses the direction).

<u>Exercise</u>: Suppose a current carrying wire is above a compass as shown below. If the needle deflects to the west, what is the direction of the electron current in the wire?



The magnetic field around a wire with negative charges moving is shown below.



The electrons must be flowing to the left in the wire.