Fri., 11/ 2	**20.2,5** Current and Motional Emf	**RE20**, **Exp** 23-25
Mon., 11/5 Tues., 11/6 Wed., 11/7 Thurs., 11/8 Fri., 11/9	**20.6-8** Reference Frames and Relativity, Torque **Review** **Exam 2** (17-20) Magnetic Field and Moving Charges **20.9-10** Dipole's Potential Energy, Motors & Generators	**RE21** **HW20:**RQ.28, 34, 36; P.45, 56, **Lab 7** **RE22**, **Exp** 26

Equipment:

Parallel and anti-parallel wires

Hand crank generator

Magnet – wire swing

Cow magnet down a pipe.

Experiments:

Exp 20.23 – Switching the direction of the current or the magnetic field (flipping magnet) will reverse the force on the wire.

Exp 20.24 – (a) The magnet attracts the coil if their magnetic fields are in the same general direction, which occurs when the coil is perpendicular to the magnet. (b) The coil twists when it isn't perpendicular to the magnet.

Exp 20.25 – There is a larger current and deflection with two batteries in series, but no change when there are two in parallel.

Force on a Current-Carrying Wire:

The magnetic force on a single charge is:

$$\vec{F}_{\text{mag}} = q\vec{v} \times \vec{B}$$

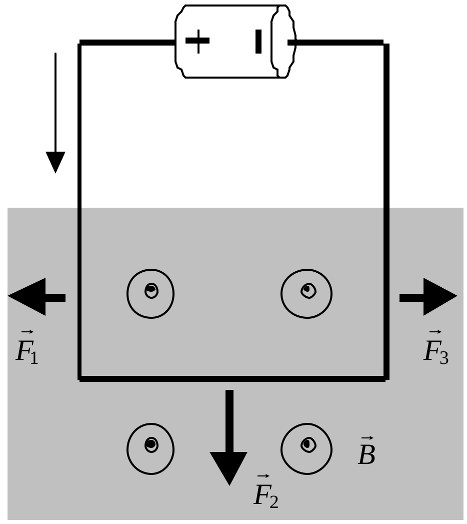
In a segment of wire of length $\Delta\ell$ and cross-sectional area A, there are $(nA\Delta\ell)$ charges moving with an average drift speed of $\bar{v}$. If the vector $\Delta\vec{\ell}$ is in the direction of the conventional current, then the force on the segment of wire is:

$$(nA\Delta\ell)q\vec{v} \times \vec{B} = (|q|nA\bar{v})\Delta\vec{\ell} \times \vec{B}$$

The conventional current is $I = |q|nA\bar{v}$, so:

$$\Delta\vec{F}_{\text{mag}} = I\,\Delta\vec{\ell} \times \vec{B}$$

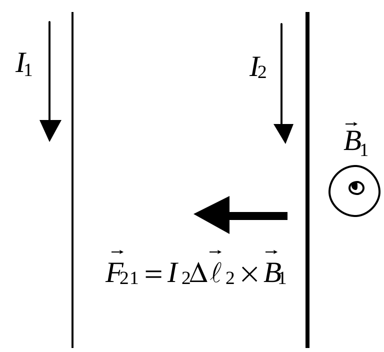
Example 1: loop of wire partially in a magnetic field



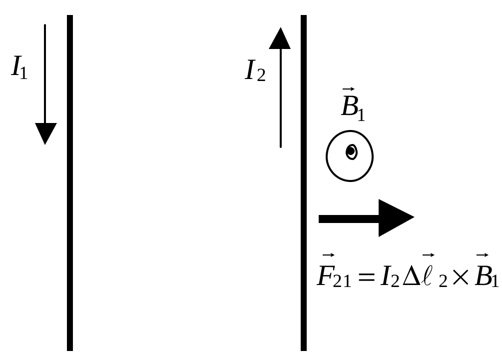
The force on each side points outward from the loop, so the net force IBL is downward.

<u>Example 2</u>: two parallel wires

(1) currents in same direction



(2) currents in opposite directions



The magnetic force acts on individual moving charges, so why does it cause a wire to move? The electrons are bound to the wire, so they drag the wire with them when they are pushed downward by the magnetic force.

The book notes that, in this case, the magnetic force of wire 1 on wire 2 is equal and opposite to that of wire 2 on wire 1; however, it parenthetically comments that this *needn't* always be the case! First look at that statement mathematically.

$$\vec{F}_{mag_{2\leftarrow 1}} = \frac{m_o}{4p}\frac{q_2\vec{v}_2 \times \left(q_1\vec{v}_1 \times \hat{r}_{2\leftarrow 1}\right)}{\left|r_{2\leftarrow 1}\right|^2} \text{ while } \vec{F}_{mag_{1\leftarrow 2}} = \frac{m_o}{4p}\frac{q_1\vec{v}_1 \times \left(q_2\vec{v}_2 \times \hat{r}_{1\leftarrow 2}\right)}{\left|r_{1\leftarrow 2}\right|^2}$$

Now, the denominator, thanks to the squaring, is the same in both cases, as is the factor out front, but the unit vector is flipped, and so are the roles of v_1 and v_2 in the cross-products. Flipping the unit vector in the second relation (to get back to the one in the first relation) introduces a negative sign.

$\vec{F}_{mag_{1\leftarrow 2}} = -\frac{m_o}{4p}\frac{q_1\vec{v}_1 \times \left(q_2\vec{v}_2 \times \hat{r}_{2\leftarrow 1}\right)}{\left|r_{2\leftarrow 1}\right|^2}$. Now, *if* v1 and v2 are parallel, then it's intuitive that the order of the cross-products doesn't matter, but what if they *aren't* parallel?

Well, generally, $A\times(B\times C) = B\times(A\times C) + C\times(B\times A)$

More specifically,

$$\vec{F}_{mag_{1\leftarrow 2}} = -\frac{m_o}{4p}\frac{q_1\vec{v}_1 \times \left(q_2\vec{v}_2 \times \hat{r}_{2\leftarrow 1}\right)}{\left|r_{2\leftarrow 1}\right|^2} = -\frac{m_o}{4p}\frac{q_2\vec{v}_2 \times \left(q_1\vec{v}_1 \times \hat{r}_{2\leftarrow 1}\right) - \hat{r}_{2\leftarrow 1} \times \left(q_2\vec{v}_2 \times q_1\vec{v}_1\right)}{\left|r_{2\leftarrow 1}\right|^2}$$

$$\vec{F}_{mag_{1\leftarrow 2}} = -\vec{F}_{mag_{2\leftarrow 1}} + \frac{m_o}{4p}\frac{\hat{r}_{2\leftarrow 1} \times \left(q_2\vec{v}_2 \times q_1\vec{v}_1\right)}{\left|r_{2\leftarrow 1}\right|^2}$$

Or,

$$\vec{F}_{mag_{1\leftarrow 2}} + \vec{F}_{mag_{2\leftarrow 1}} = \frac{m_o}{4p}\frac{\hat{r}_{2\leftarrow 1} \times \left(q_2\vec{v}_2 \times q_1\vec{v}_1\right)}{\left|r_{2\leftarrow 1}\right|^2}$$

This last term goes away only if the two velocities are parallel or anti-parallel.

One might hope that, if we included the electric interaction (which is the proper thing to do), using the corrected form of Coulomb's law (corrected for the fact that the sources are moving), we'd then find and equal and opposite excess of force that will cancel off. The book claims that this is not the case. I've worked out the case with constant velocities (say, steady-state in a wire), and one aspect of the result is that there *is* also an 'extra' electric force, but it points along r_{21} (which isn't intuitive since the particles are moving), while the extra magnetic force must be perpendicular to r_{21} (obvious in the math since the force results from crossing something into r_{21} and must therefore be perpendicular to r_{21}. So the extra magnetic and electric forces are perpendicular to each other – there's no way they could cancel. I've looked at the equations when the particles are free to accelerate, not enough to discount the possibility of cancelation, but enough to see that it isn't obviously there. At any rate, if we believe the authors, the moral is that if we treat this interaction as simply a two-party thing, then Newton's 3rd Law is violated, and, by extension, conservation of momentum is violated. That's very, very bad. The solution is that we *don't* treat it as a two-party interaction. Recall that we've previously found that conservation of energy fails unless we consider the change in the *field's* energy. Similarly, we need to consider the change in the magnetic (and electric) *field's* momentum. This adds extra weight to our notion of a field / its necessity in our structure of physics. You may feel more comfortable with this weight if we phrase it in terms of photons. You've already met the idea of photons – these little packets of light that transfer energy *and* momentum from one charged object to another. Well, at any given time, some of the energy and momentum of this interaction are invested in the photons that mediate it.

Students aren't prepared for the following, but it proves that the electric forces don't make up the difference in the magnetic forces (both measured in the lab frame, not in the particles' frames.)

Our text doesn't really facilitate answering this question in terms of the position vector we're already using, so I'll borrow from Griffiths:

$$\vec{F}_{E_{2\leftarrow1}} = \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{\hat{r}_{2\leftarrow1}}{\left|r_{2\leftarrow1}\right|^2}\left[\frac{1-\left(\frac{v_1}{c}\right)^2}{\left(1-\left(\frac{v_1}{c}\right)^2\sin^2\theta_{r-v_1}\right)^{3/2}}\right] \text{ and similarly } \vec{F}_{E_{1\leftarrow2}} = -\frac{q_1 q_2}{4\pi\varepsilon_o}\frac{\hat{r}_{2\leftarrow1}}{\left|r_{2\leftarrow1}\right|^2}\left[\frac{1-\left(\frac{v_2}{c}\right)^2}{\left(1-\left(\frac{v_2}{c}\right)^2\sin^2\theta_{r-v_2}\right)^{3/2}}\right]$$

So the sum of these two is

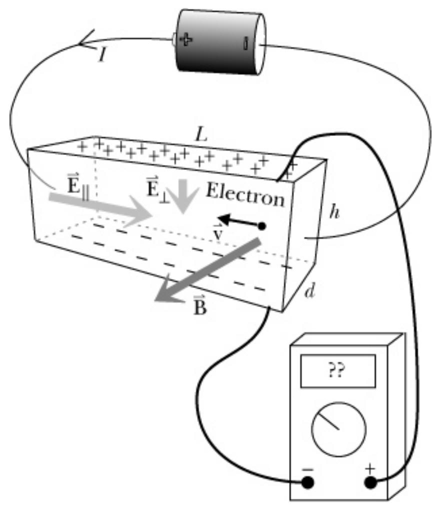
$$\vec{F}_{E_{2\leftarrow1}} + \vec{F}_{E_{1\leftarrow2}} = \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{\hat{r}_{2\leftarrow1}}{\left|r_{2\leftarrow1}\right|^2}\left[\frac{1-\left(\frac{v_1}{c}\right)^2}{\left(1-\left(\frac{v_1}{c}\right)^2\sin^2\theta_{r-v_1}\right)^{3/2}} - \frac{1-\left(\frac{v_2}{c}\right)^2}{\left(1-\left(\frac{v_2}{c}\right)^2\sin^2\theta_{r-v_2}\right)^{3/2}}\right]$$

$$\vec{F}_{E_{2\leftarrow1}} + \vec{F}_{E_{1\leftarrow2}} = \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{\hat{r}_{2\leftarrow1}}{\left|r_{2\leftarrow1}\right|^2}\left[\frac{\left(\frac{v_1}{c}\right)^2\left(1-\left(\frac{v_2}{c}\right)^2\sin^2\theta_{r-v_2}\right)^{3/2} - \left(\frac{v_2}{c}\right)^2\left(1-\left(\frac{v_1}{c}\right)^2\sin^2\theta_{r-v_1}\right)^{3/2}}{\left(\left(1-\left(\frac{v_1}{c}\right)^2\sin^2\theta_{r-v_1} - \left(\frac{v_2}{c}\right)^2\sin^2\theta_{r-v_2} + \left(\frac{v_1 v_2}{c^2}\sin\theta_{r-v_1}\sin\theta_{r-v_2}\right)^2\right)\right)^{3/2}}\right]$$

There's no way for this to be equal and opposite to the excess magnetic force. For one thing, the two forces don't even point in the same direction (the electric force, rather remarkably, is along r (in spite of the particles' motions) while the magnetic force is perpendicular to it).

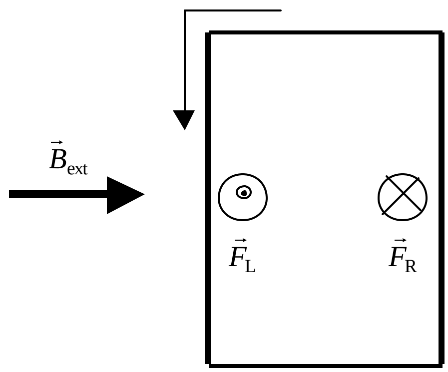
Of course, allowing that the particles, when interacting with each other thus (and magnetically) will be accelerating, the result must change and there will be a component *not* along r_{21}. It's hard to guess what the result will look like, but it could be found by using Griffith's 10.67 and incorporating the accelerations (presumably assuming non-relativistic is good enough) due to the interactions into the equations.

What we did is add up the (average) forces on all of the moving charges. As we discussed for the Hall effect, the magnetic force causes opposite surface charges on opposite sides of the wire.

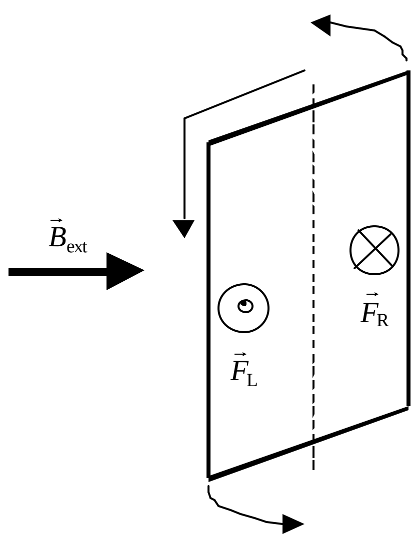


The surface charges in the diagram above produce a downward electric field, which exerts a downward force on the positive atomic cores.

Example 3: a rigid current loop in an external magnetic field



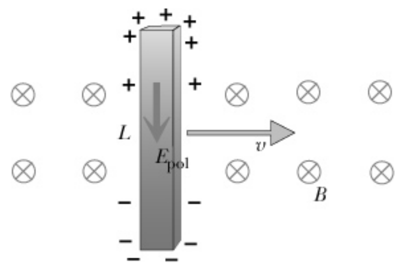
The forces cause the loop to twist.



The loop will be in equilibrium (no net force or torque) when it is perpendicular to the external magnetic field. If there is any friction, the loop will end up in an orientation so that $\vec{B}_{\mathrm{loop}} \,||\, \vec{B}_{\mathrm{ext}}$. Remember that the loop acts like a magnet (magnetic dipole).

Motional emf:

Suppose a metal bar of length L is moved through a magnetic field with a magnitude B into the page at a speed v.



There is a magnetic force on each electrons in the wire:

$$\vec{F}_{\text{mag}} = (-e)\vec{v} \times \vec{B}$$

$$F_{\text{mag}} = evB \qquad \text{downward}$$

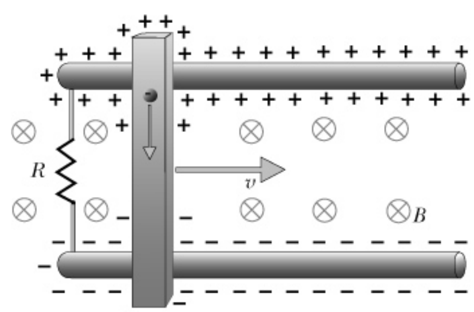
There is also a force on each proton, but they are not free to move. Electrons will move from the top of the bar to its bottom, polarizing the bar. This will produce a downward electric field and upward force $\vec{F}_{\text{e}} = -e\vec{E}$ on each electron in the bar. Equilibrium will be reached when the magnetic and electric forces are the same size:

$$evB = eE$$

The potential difference between the ends of the bar is:

$$|\Delta V| = EL = vBL$$

The bar acts like a battery because a charge separation is maintained by a non-Coulomb force (in this case the magnetic force). Suppose the bar is pulled along two frictionless metal rails which are connected by a resistor (as shown below).



The moving bar will cause the rails to become charged and cause a current through the resistor. Note that the electrons move the "wrong" way (opposite direction of the electric force) through the bar. This is the result of the non-Coulomb force, which does a work:

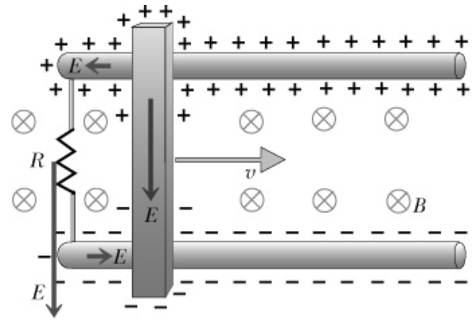
$$W = (evB)L$$

so the non-Coulomb work per charge or "motional emf" is:

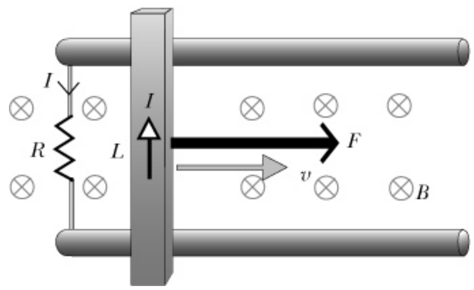
$$\text{emf} = vBL$$

The motional emf is usually relatively small and not sustained, so it can be hard to detect.

Look at the electric field to see that $\Delta V = -\oint \vec{E} \cdot d\vec{\ell}$ for a round trip is zero.



As the bar moves at a constant speed and a steady current flow, the resistor dissipates energy to its surroundings. Where does this energy come from? Work must be done on the bar to keep it moving.



If there is a current I flowing upward through the bar, the magnetic force on it is ILB to the left on it. Someone or something must apply the same size force to the right on the bar to keep it moving at a constant speed. The power input is work per time:

$$\text{Power} = F\Delta x/\Delta t = Fv = (ILB)v$$

The power dissipated by the resistor is:

$$\text{Power} = I(\text{emf}) = I(LBv)$$

These agree as they should, since no power is being stored up in the system.

Monday: Torque and Energy for magnetic dipoles (only <u>skim</u> Sect. 20.6)