Fri., 3/6	**19.15-17,19** Meters and RC Circuits	
Mon., 3/9 Tues., 3/10 Wed., 3/11 Thurs., 3/12 Fri., 3/ 13	**20.1,3-4** Magnetic Force **20.2,5** Current and Motional Emf **Quiz** Ch 19, **Lab 8** Cyclotron & Electron Mass Lab **20.6-8** Reference Frames and Relativity, Torque **Bonus: Phys. Sr. Thesis Presentations @ 4pm**	**RE19** **HW19:**RQ.42, 49, 52; P.61, 66, 69 **RE20**, **Exp** 23-25 new **RE21**

Look for exercises

Announcements: Anyone else planning on attending the Sr. Thesis Presnetations?

The thrust of Wednesday's reading & discussion was moving us from a microscopic understanding of *why* circuit elements behave as they do to a macroscopic characterization of *how* they behave, which is one step closer to putting their behavior to use. "Using" moves us from theoretical toward experimental, and that means actually *measuring* the circuit element's behavior. You've done so in semi-qualitative ways, using a compass and perhaps a watch. Here's a little bit about how it is done more quantitatively.

Review: Julie's Slides

.73 Capacitance of a spherical capacitor

<u>Power</u>:

Suppose the change in electric potential energy as a charge Δq moves across a component is ΔU_e.The power is the work done in the time Δt that it takes for the charge to move:

$$\text{Power}=\frac{\Delta U_e}{\Delta t}.$$

The potential difference is the change in potential energy per charge, so:

$$\text{Power}=\frac{\Delta U_e}{\Delta t}=\frac{\Delta q\cdot\Delta V}{\Delta t}=I\Delta V.$$

The relationship above is true for any kind of component. It is literally the rate at which the charged particles loose electric potential energy. If they flowed un-resisted, that would equal the rate at which they *gained* kinetic energy, and they'd accelerate.

However, in resistors, they are opposed and they don't accelerate. So they must simultaneously be imparting the kinetic energy they gain to their environment (via all those collisions). For a resistor, $I=\frac{|\Delta V|}{R}$, so:

$$\text{Power}= I\Delta V=I^2R=\frac{(\Delta V)^2}{R}$$

Interestingly, if that environment, a filament say, is at a constant temperature, it in turn must be imparting energy to *its* environment at this rate as well. So this is the rate at which a light bulb radiates energy as light.

Meanwhile, for a Capacitor, as it's charging up, current and voltage change:

$Q = C|\Delta V|$ the rate of energy build up is $\text{Power} = \frac{dU_e}{dt} = \frac{d(q \cdot \Delta V)}{dt} = \frac{dC(\Delta V)^2}{dt} = C\Delta V \frac{d\Delta V}{dt}$ or the energy invested at a particular time would be the integral of this:

$$U = \int \frac{dU_e}{dt} dt = \int dU_e = \int C\Delta V d\Delta V = \tfrac{1}{2} C(\Delta V)^2$$

RC Circuits: resistor and capacitor in series. You've experimentally charged up and discharged a capacitor in different configurations with a resistor / light bulb. Now that we've built up some easy to use relations for how capacitors and resistors behave in circuits, we're ready to get more quantitative in our theoretical treatment of charging and discharging.

The relations we developed last time are

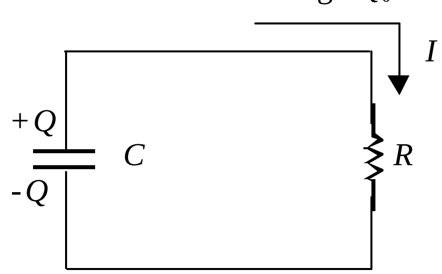
$$\Delta V_R = -I_R R \text{ and } Q_C = C\Delta V_C,$$

where, of course, $I_R = \frac{dQ_R}{dt}$ running through an element.

(1) Discharging

This is easier to treat because there's one less circuit element to consider (no battery) than in charging, so we'll do it first.

Suppose the capacitor starts out with an initial charge Q_0 at time $t = 0$.



The voltage difference across the capacitor (from negative to positive plate) is $\Delta V_{\text{capacitor}} = Q/C$. The loop rule for a clockwise loop gives:

$$\begin{aligned} \Delta V_c + \Delta V_R &= 0 \\ Q_c/C - IR &= 0 \end{aligned}.$$

Here's a slightly subtle sign issue: $I_R = \frac{dQ_R}{dt}$, where dQ_R is the current flowing into the resistor per time interval. In the same time, the charge on the capacitor must change by an amount $dQ_C = -dQ_R$, the sign difference indicates that, while we're counting a *positive* amount of charge passing through the resistor, the charge is *reducing* on the capacitor. So, $I_R = \frac{dQ_R}{dt} = -\frac{dQ_C}{dt}$.

$$\frac{Q_C}{RC} + \frac{dQ_C}{dt} = 0$$

Re arranging this, and making explicit the charge's time dependence, it might look a little more familiar:

$$\frac{d}{dt} Q_C(t) = \left(-\tfrac{1}{RC}\right) Q_C(t)$$

Q: There are two approaches to determining the functional form of $Q_c(t)$. One is to deal directly with this differential equation in its current form and make a guess, apply some things you know to refine your guess until it works. The other is to rewrite this equation in integral form and then integrate. That's perhaps the more familiar technique to you at this stage in your mathematical education, so let's do that.

$$\frac{dQ_C(t)}{dt} = \left(-\tfrac{1}{RC}\right) Q_C(t)$$

So, getting the *dQ* and *Q* on one side of the equals sign and the *dt* on the other, I have

$$\frac{1}{Q_C(t)} dQ_C(t) = \left(-\tfrac{1}{RC}\right) dt$$

Then, integrate both sides

$$\int_{Q_c(0)}^{Q_c(t)} \frac{1}{Q_C(t)} dQ_C(t) = \left(-\tfrac{1}{RC}\right) \int_0^t dt$$

$$\ln(Q_C(t)) - \ln(Q_C(0)) = \left(-\tfrac{1}{RC}\right) t$$

$$\ln\left(\frac{Q_C(t)}{Q_C(0)}\right) = \left(-\tfrac{1}{RC}\right) t$$

Exponentiate both sides to get

$$\frac{Q_C(t)}{Q_C(0)} = e^{-\frac{t}{RC}}$$

$$Q_C(t) = Q_C(0) e^{-\frac{t}{RC}}$$

Or using the short hand $Q_o = Q_c(0)$

$$Q_C(t) = Q_o e^{-\frac{t}{RC}}$$

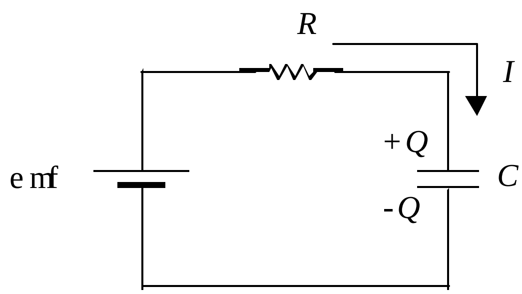
and the current is then:

$$I = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC}.$$

Both the charge and current drop exponentially to zero.

(2) Charging

Suppose the capacitor is initially uncharge ($Q = 0$ at time $t = 0$).



The loop rule for a clockwise loop gives:

$$\Delta V_{batt} + \Delta V_c + \Delta V_R = 0$$
$$emf - Q_c/C - I_R R = 0$$

The change in amount of charge on the capacitor is now positive, so in time *dt*, $dQ_C = dQ_R = I_R\, dt$, so $I = +dQ_C/dt$ (the positive here is because charge of the capacitor increases when current flows). This gives:

$$\text{emf} - \frac{Q_c}{C} - \frac{dQ_c}{dt} R = 0.$$

Rearranging that, the equation is quite similar to the one we just solved for discharging, though we've got an additional constant term:

$$\frac{d}{dt} Q_c(t) = \left(\tfrac{emf}{R}\right) - \tfrac{1}{RC} Q_c(t)$$

For kicks, we'll now take the *other* approach to solving this differential equation: guessing the form of the solution, and then refining the guess until it works. Reason being, this equation looks a heck of a lot like the one we just solved for charging. So I'd be prepared to guess that the solution has a similar form. The only difference is that there's a constant emf/R in the equation, so I'll guess that the solution is the same, but with an additional constant. How about if our function has the form

$$Q_c(t) = A + Be^{-t/RC}$$

First, at **t = 0**, there should be no charge, where as this guess gives

$$Q_c(0) = 0$$
$$Q_c(0) = A + B$$

Okay, that's satisfied if B = -A. So we refine our guess to be

$$Q_c(t) = A\left(1 - e^{-t/RC}\right)$$

Let's plug it into the equation and see what it gives

$$\frac{d}{dt}A\left(1 - e^{-t/RC}\right) = \left(\tfrac{emf}{R}\right) - \tfrac{1}{RC}A\left(1 - e^{-t/RC}\right)$$
$$\tfrac{A}{RC}e^{-t/RC} = \left(\tfrac{emf}{R}\right) - \tfrac{1}{RC}A\left(1 - e^{-t/RC}\right)$$

If this is a solution, it's a solution for *all t.* So, a nice convenient time to look at is t = 0..

$$\tfrac{A}{RC} = \left(\tfrac{emf}{R}\right) - \tfrac{1}{RC}A(1-1) \Rightarrow A = emfC$$

There we have it:

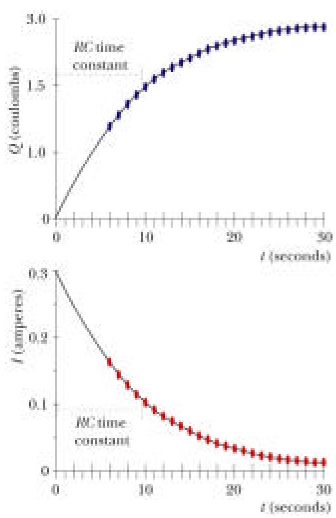
$$Q_c(t) = emfC\left(1 - e^{t/RC}\right)$$

The charge starts at zero and approaches $Q = C(\text{emf})$. The final static equilibrium makes sense because I is zero (see the first equation).

The current is:

$$I = +\frac{dQ}{dt} = \left(\frac{\text{emf}}{R}\right)e^{-t/RC},$$

which starts at a maximum and drop exponentially to zero.



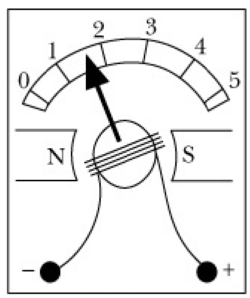
In both cases, the size of the potential difference across the capacitor (what you measure) is proportional to the charge on it: $\Delta V_{\text{capacitor}} = Q/C$.

RC Time Constant:

The product of resistance and capacitance has units of time. We call *RC* the *time constant* for discharging or charging. In that amount of time, the exponential drops from its original value ($e^0 = 1$) to $e^{-1} = 0.37$, which is a 63% (about 2/3) decrease. For both discharging and charging, the current drops by that factor ever interval of time *RC*.

Ammeters:

You have used a compass as a simple ammeter. The needle was deflected from North by the magnetic field generated by the current, and you could, given the geometry, deduce the magnitude of the current.



Nondigital ammeter

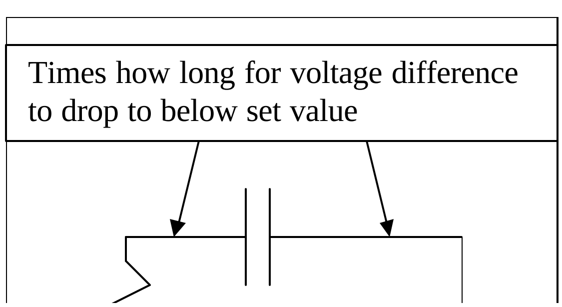
Traditional ammeters, known as "galvanometers" use a similar idea. The galvanometer contains a permanent magnet and the current you're measuring is routed through a coil mounted on a spring (to resist rotation). The coil is like an electromagnet – the more current runs through it, the stronger a magnet it is – and the stronger it tries to rotate to get its North /South pole near the permanent magnet's South/North pole. But the spring resist the rotation – so for a given electro magnet strength / current strength, the coil will deflect to a given angle – thus the current strength can be read from the angle of deflection.

Having an idea how this works helps us to appreciate some constraints on its use.

First: In Series. The current you want to measure has got to be routed *through* the meter, so the circuit actually has to be broken and the meter inserted into it. So if you want to know the current running through a resistor, you need to attach the ammeter in *series* with the resistor, so it catches all that same current.

Second: Small Internal Resistance. For the current we measure to be equal (enough) to the current that would run through the circuit if the meter weren't there, the meter can't significantly impede the current – thus it must have negligible internal resistance.

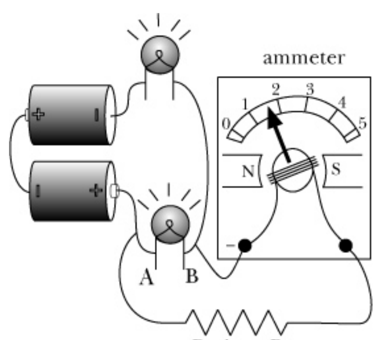
Third: Breaks in parallel! putting these two together, if the ammeter is applied, say in *parallel* with the resistor of interest, then, not only will it not measure the current going through the resistor, it will likely pass a very large current, perhaps large enough to damage the meter.



Modern Ammeter. Timing. In more modern, digital ammeters, the current you want to measure is routed across a very small and well characterized resistor (small so that it doesn't significantly diminish the current that you're trying to measure), and the voltage drop across that resistor is used to charge up a well characterized capacitor. Then the capacitor is allowed to discharge, and the time that takes is related to the voltage to which it was charged, which is related to the current through the known resistor.

<u>Voltmeters</u>:

A voltmeter measures the potential difference. A simple voltmeter can be made by placing a large, known resistance *R* in series with an ammeter. The combination is connected between the points that you want to know the potential difference between (see figure below).



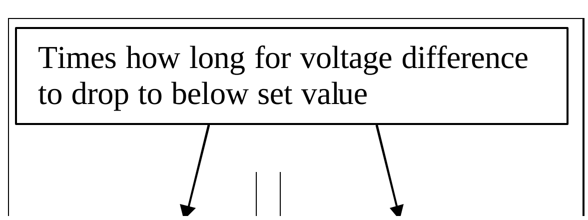
If a small current *I* is measured, the potential difference between points A and B is $|\Delta V| = RI$.

The resistance of the voltmeter should be large so that not much current flows through it, instead of what it is hooked up to.

Note the voltmeter must be attached in *parallel* to the element of interest.

Modern Voltmeter

Digital Voltmeters work much as digital ammeters, except they now *don't* use an internal resistor.



<u>Ohmmeters</u>:

An ohmmeter measures resistance. A small voltage source in series with an ammeter is a simple ohmmeter. The current that flows is proportional to the resistance. Since an ohmmeter is an "active" device (with a voltage source), it should not be used in a circuit. Resistors should be measured individually.

Complex Circuits

19.RQ.57 p.690

.58 Capacitor circuts with different bulb arrangements

.65 Remove plastic slab from capacitor

.70 Multiple loops

Current in Metals

The acceleration experiment

Thermal gradients would have a similar effect. (where it's hot, the more mobile parties disperse more than the less mobile ones, and with them, goes the charge – a reduced negative charge density at the hot end and increased negative charge density at the cold end. If positive charges were the mobile ones, the hot end would be left more negative and the cold end would be more positive.)