Wed., 3/4 Thurs., 3/5 Fri., 3/6	**19.6-.14** Capacitor & Resistor Circuits **Quiz** Ch 18, **19.15-17,19** Meters and RC Circuits **Lab 7** RC Circuits **(*focus on experimental uncertainty*)**	**RE18**, **Exp** 31-34 **Exp** 35-37
Mon., 3/9 Tues., 3/10	**20.1,3-4** Magnetic Force	**RE19** **HW19:**RQ.42, 49, 52; P.61, 66, 69

Next Friday: Sr. Thesis Presentation @ 4pm, and dinner

Signup

Handouts:

- Quiz on Ch. 18 prep
- Lab

Just us again.

Chapter 19 introduces capacitors into our circuits, and goes on to make the transition from a microscopic view to a more macroscopic view of circuits.

Review from a long time ago

Review:

Recall that back in the second chapter, Ch 15, we spent some quality time with two, oppositely charged plates, area A, separation s, and charge Q. One reason we did this was that this charge configuration constitutes a "capacitor", so named for its "capacity" to store charge. In Ch 15 we found that the electric field of a capacitor is:

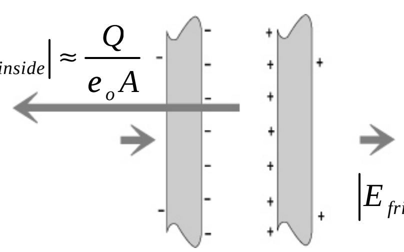
$$|E| \approx \frac{Q/A}{e_0} \quad \text{inside, not too close to edges ,}$$

In terms of the voltage difference across the gap, which we discussed in Ch. 16,

$$|\Delta V| = |E|s \approx \frac{Q/A}{e_0}s = \frac{Qs}{e_0 A}$$

$$|E_{\text{fringe}}| \approx \frac{Q/A}{2e_0}\left(\frac{s}{R}\right) \quad \text{just outside the plates .}$$

$$|E_{inside}| \approx \frac{Q}{e_o A}$$



$$|E_{fringe}| \approx \frac{Q}{2e_o A}\left(\frac{s}{R}\right)$$

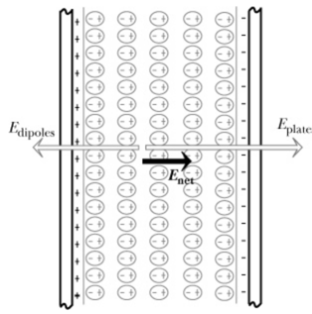
$$|\Delta V| = |E|s \approx \frac{Qs}{e_0 A}$$

Imagine for a moment that you have a capacitor wired up across a battery. In steady state, considering the voltage drops across the leads to be negligible, the voltage drop across the capacitor must be equal to the voltage gain across the battery $|\Delta V| = |\Delta V_{batt}|$. Thus the charge on a plate must equal

$$Q \approx \frac{e_0 A}{s} |\Delta V_{batt}|$$

Dielectrics

Then in Ch 16 we introduced the idea of sandwiching a polarizable insulator between these plates. An insulator (dielectric) inside will reduce the electric field inside by a factor of *K*, the dielectric constant of the material.



$$\vec{E}_{insulator} = \frac{\vec{E}_{empty}}{K} \text{ where K = dielectric constant}$$

(varies from 1 for empty to 310 for strontium titanate)

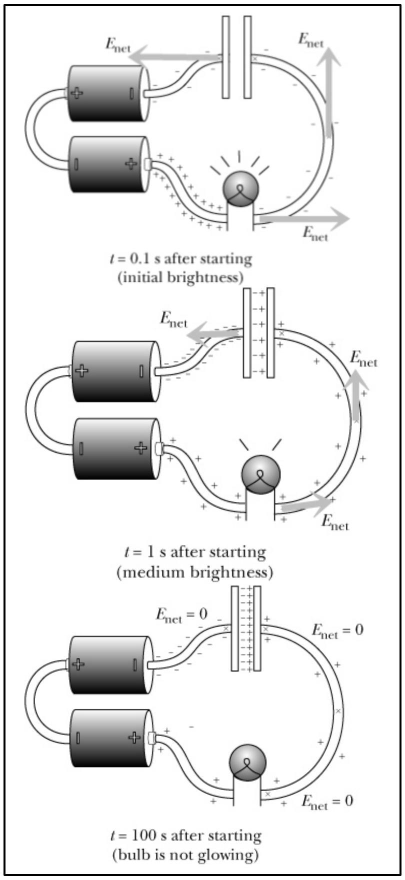
$$|E| \approx \frac{Q/A}{e_0 K} \quad \text{inside, not too close to edges}$$

$$\text{Similarly, } \Delta V_{insulator} = \frac{\Delta V_{empty}}{K}$$

$$|\Delta V| \approx \frac{Qs}{e_0 KA}$$

Last Time

- **Introduced Capacitors**
  - **Charged** – with a lightbulb in series, initially bright, then dimmed
  - **Discharged** – remove battery, lightbulb initially bright, then dimmed.
  - **Why?**
    - **Charging.**



- Initially, charges driven out of the battery's negative terminal and some end up on the left plate, while they can't go anywhere, they can repel charges away from the right plate, and the domino effect continues…charges move through the bulb.
  - $\left|E_{\text{fringe}}\right| \approx \frac{Q/A}{2e_0}\left(\frac{s}{R}\right)$ just outside the plates
- Subsequent charges feel the repulsive "fringe field" due to those already on the capacitor, so they don't flood onto the left plate or off the right plate quite so quickly – current slows.
- Eventually, the charge built up on the capacitor generates a strong enough fringe field that it stops current all together.

Discharging.

- Remove the batteries, now the fringe field feels no opposition – the charges rush off the left plate, through the wire, to the right plate.
- This reduces the fringe field, so the next round of charge coming off isn't driven as hard – lower flow.
- Eventually, there's no more fringe field and no more current.

With this picture in mind Consider two scenarios.

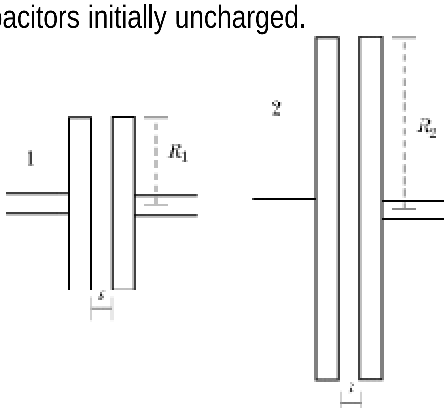
EXERCISES: What effects do the following changes have on the charging of a capacitor? Hint: Think about what happens in the first fraction of a second. Explain your reasoning in detail.

$$\left|E_{\text{fringe}}\right| \approx \frac{Q/A}{2e_0}\left(\frac{s}{R}\right) \quad \text{just outside the plates}$$

1. Replace the original capacitor with one that is larger.

(see Ch19 clicker – capacitor.doc question 5)

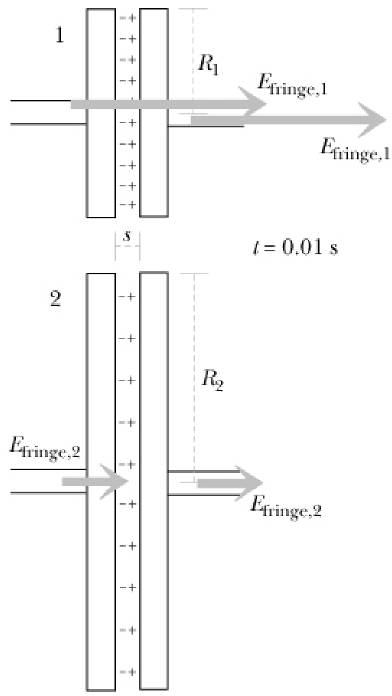
Q19.5a

<table>
<tr>
<td>Capacitors initially uncharged.


$E_{fringe} = \frac{Q/A}{\varepsilon_0}\left(\frac{s}{2R}\right)$</td>
<td>After 0.01 s of charging:

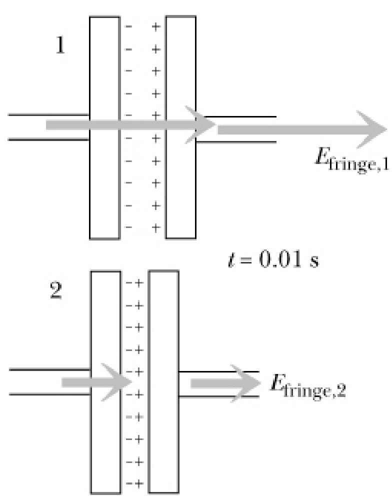
1) The fringe field of each capacitor is the same
2) The smaller capacitor (#1) has a larger fringe field
3) The larger capacitor (#2) has a larger fringe field</td>
</tr>
</table>

Initially, the capacitor is uncharged so the electric field in the wires is due to the battery. About the same charge would end up on each capacitor in the first time interval. The area of the larger capacitor is larger, so *Q*/(*AR*) is smaller and the fringe electric field it generates is smaller. So, in the process of putting charge Q on the plates, the net field in the wire has reduced less, thus the current has reduced less. So, the current dies off more slowly with the big capacitor than the small capacitor.



Replace the original capacitor with one that has a smaller gap.

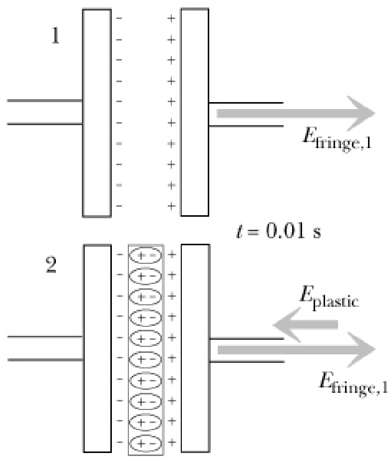
Initially, the capacitor is uncharged so the electric field in the wires is due to the battery. About the same charge would end up on each capacitor in the first time interval. The gap of the second capacitor is smaller, so its fringe electric field is smaller. That means that there is a larger net field and the current does <u>not</u> decrease as quickly.



$$E_{fringe} = \frac{Q/A}{\varepsilon_0}\left(\frac{s}{2R}\right)$$

2. **Replace the original capacitor with one that has an insulating layer between the plates**.

   Initially, the capacitor is uncharged so the electric field in the wires is due to the battery. About the same charge would end up on each capacitor in the first time interval. The electric field of the polarized insulator is opposite in direction to the fringe field, i.e., in the same direction as the field due to the charge gradient on the wires. That means that there is a larger net field ($\vec{E}_{wire} + \vec{E}_{plate} + \vec{E}_{insulator}$) and the current does <u>not</u> decrease as quickly.



$$E_{fringe\text{-}plastic} = \frac{Q/A}{e_0 K}\left(\frac{s}{2R}\right)$$

In all cases, more charge flows onto the capacitor. Tomorrow, we will define the *capacitance* as the ability to store charge (for a given potential difference.)

Overview

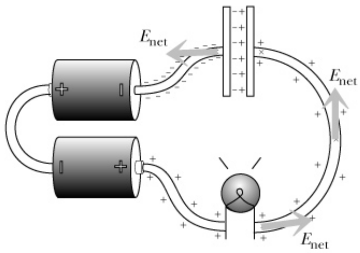
- **19.6 Energy Considerations**
- **19.7 Current Node Rule with a Capacitor**

  Macroscopic Analysis of Circuits
- **19.8 Capacitance**
- **19.11 Resistance**
  - **Ohmic, non-ohmic (semiconductors, batteries)**
- **19.12 Series and Parallel Resistance**
- **19.13 Work and Power in Circuirs**
- **19.14 Real Batteries and Internal Resistance**

Any particular questions, things you'd like us to go over?

19.6 Energy Considerations

- This subsection was essentially two in-line exercises. So let's pause and consider them.
    - Read 19.X.6 (p. 666)



- t = 1 s after starting
(medium brightness)

- Read 19.X.7 (p.666)

19.7 Current Node Rule with a Capacitor

- The Node Rule was argued from the necessity that, *in steady state,* charge isn't building up or depleting anywhere – thus the same amount of charge must flow into a spot as flows out again.
- Well, when a capacitor is charging or discharging, charge build up and depletion is what it's all about! So the node rule doesn't hold for an capacitor plate. *However,* it does still hold for the capacitor as a whole – for each electron added to the – side, an electron is bumped off the + side, so the current *is* the same on either side of the capacitor.

Do exercise on effects of changing properties of capacitors (in Wed. notes) – tie conclusions into the definition of capacitance…

Introduction

Thus far with circuits, we've been thinking a lot about the microscopic scale – what the charges are doing, what the fields are that they create, etc. That gives us a good understanding of the *fundamentals* of how circuits work; however, there's some quite complex circuitry out there, and it would be quite burdensome to always be thinking down on the fundamental level. Today we make the transition from microscopic to more macroscopic terms. Part of that is mentally separating out the constants that get set in the factory when a circuit element is built from the variables that are adjusted by the user. We'll define **Capacitance** and **Resistance** as (approximate) factory set constants that characterize how individual capacitors and 'resistors' behave. Then we'll start seeing how our fundamental physical relationships are rephrased in terms of these constants and the variables that are most easily measured: **Voltage differences** and **Conventional Currents**.

Definition of Capacitance:

Since we were focused on the capacitor yesterday, we'll tackle "capacitance" first.

Qualitatively, a particular capacitor's "capacitance" characterizes how easy it is for the capacitor to store charge. Let's see what 'factory-set' parameters determine this.

Recall from last time, and from Ch 15,

$$|\Delta V| = |E|s \approx \frac{Q/A}{e_0} s = \frac{Qs}{e_0 A}$$

For an 'empty' capacitor, or, more generally allowing for some kind of dielectric,

$$|\Delta V| \approx \frac{Qs}{e_0 KA}$$

Given our qualitative definition, 'how easy it is for the capacitor to store charge', it'll be handy to isolate the 'stored charge' in this equation.

$$Q = \frac{e_0 KA}{s} |\Delta V|$$

Q: once a capacitor's been built, which of these 'variables' are essentially set constant?

The geometry's set: A and s, and the material of the dielectric is set, K, and of course ε_o is a 'fundamental' constant, so it's pretty much set too. So this whole combo is 'factory set.'

$$C \equiv \left(\frac{e_0 KA}{s} \right)$$

This has the right qualitative behavior: if C is bigger, then it's easier (takes less of a voltage difference) for the capacitor to store the same amount of charge, or, for the same voltage difference, the capacitor can hold more charge.

Having thought about the fundamental picture of capacitors, we understand why each factor, K, A, and s, influence the storage capacity as they do. But once a capacitor's been built, these factors are set, and in building a circuit it isn't so important *why* a capacitor has a particular capacitance, but *what* that capacitance is. This is the key parameter by which capacitors are described. The units are "Farads", and you'll see "1 F" on the side of the capacitor in your kit, declaring it to have a capacitance of one Farad (which is quite large, as capacitances go).

$$\boxed{Q = C|\Delta V|,}$$

Experiments:

Exp 19.31 – The initial current (when the capacitors are uncharged) is unaffected, so the light bulb starts out equally bright with one or two capacitors. There are two places for the charge to be stored, so the current must flow longer and the bulb stays lit longer.

Exp 19.32 – The light bulb will still light because initially it's as if the capacitors are <u>not</u> there. Current flows until the size of the potential difference across the two capacitors is the same as the emf of the battery.

Exp 19.33 – If a capacitor is charged with one battery through a light bulb and reversed, the bulb will initially glow as it were connected to <u>two</u> batteries. In static equilibrium (after the capacitor is fully charged), the size of the potential difference across the capacitor is the same as the potential difference across the battery.

Exp 19.34 – If the batteries are disconnected, current from the capacitor will keep the light glowing. In steady state, the potential difference across the capacitor is the same as the potential difference across the batteries.

<u>Resistance</u>:

Now for resistance. When you buy a stretch of wire – perhaps just for making a connection, or perhaps in the form of a light bulb filament, the "resistance" characterizes how hard it is to force charge flow through it. Again, it's the 'factory set' properties of the wire, filament, etc. Let's see what we know about passing currents

Fundamentally, conventional current is the amount of charge flowing through an element per time: $I = \frac{dQ}{dt}$ pointing in the direction of equivalent positive charge flow. This direction business gets rather awkwardly treated, but rather than going through the proper contortions (which the book does not) to treat it as a vector, we'll revisit this at the end of our argument.

This can be rephrased in terms of the charge of a carrier, the density of carriers, the cross-sectional area through which they're flowing, and their drift speed.

$$|I| = \left|\frac{dQ}{dt}\right|$$

$$|I| = \left|\frac{qdN}{dt}\right|$$

$$|I| = \left|\frac{qndVol}{dt}\right| = \left|\frac{qnAdL}{dt}\right| = |qnAv|$$

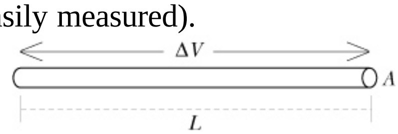
Which, in turn, can be rephrased in terms of the field.

$$|I| = |qnA(uE)|$$

Note: since current flow in a wire is a pretty one-dimensional thing, we often don't bother to apply the full formalities of vectors. Then again, not doing so can lead to confusion and error. This form lends itself very nicely to vector notation.

$$\vec{I} = qnA\vec{v} = |q|nAu\vec{E}.$$

And finally, we want to rephrase the field in terms of the corresponding voltage difference (since that's what is most easily measured).



$$\Delta V_{BA} = -\int_A^B \vec{E} \cdot d\vec{l} \ .$$

Now, through a particular circuit element, the field is essentially constant magnitude and the field points along it, following its every twist and turn. The only question is, does the path from A to B point *with* or *against* the field. So

$$|\Delta V_{BA}| = |EL_{BA}|,$$

And

$$|E| = \left|\frac{\Delta V_{BA}}{L_{BA}}\right|$$

Then, $|I| = \left|\dfrac{qnA(u\Delta V_{BA})}{L_{BA}}\right|$

Okay, thinking back on the qualitative definition of "Resistance" (how hard it is to make current flow), the collection of 'factory set' constants does that, except, that the *bigger* they are, the *bigger* current is – we want bigger resistance to mean smaller current. So:

$$R \equiv \left|\frac{L_{BA}}{qnAu}\right|$$

Measured in Ohms.

Then $\boxed{|I| = \dfrac{|\Delta V_{BA}|}{R}}$

Note: Bigger length L means smaller resistance and larger area A means smaller resistance.

Units for resistance are ohms (Ω) and $1 \text{ ohm} = 1 \text{ V/A}$.

Again, we have already reasoned out why the current should depend on each of these elements in this way, but once a resistive element is built, it's not so important *why* there's resistance as it is *how much* resistance there is.

Sign: Now to go back and get the sign right. Current flows "down hill" – in the direction of decreasing voltage. That means that if the voltage drop is positive from left to right, then the current flows in the opposite direction, from right to left. To keep that in mind, it's often handy to express this relation without the absolute value signs as

$$I = -\frac{\Delta V_{BA}}{R}.$$

Variability: The book makes a real point of the fact that treating R as a constant is just an approximation. We already know that mobility, u, is temperature dependent. Similarly, in some materials, the density of free charge carriers, n, is also temperature and voltage dependent. That said, "resistors" are designed so that, R remains fairly constant in most applications.

Resistivity, Current Density and Conductivity

It is sometimes handy to package the same information that we've just dealt with in different ways. For example, in our definition of Resistance, $R \equiv \left|\frac{L_{BA}}{qnAu}\right|$, there are some properties that are material specific, and others that are geometry specific. The material specific terms are often spun off by themselves and called either the

Resistivity: $r \equiv \left|\frac{1}{qnu}\right|$

Or, inversely, the Conductivity: $s \equiv |qnu|$.

There's no new physics here – just new packaging for convenience.

Similarly, it's sometimes convenient to separate out the geometry dependent factor in the relation between current and field:

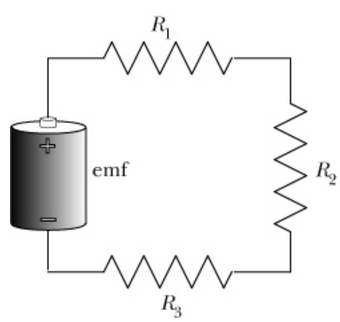
It is useful to define the current density (charge per cross-sectional area):

$$J = I/A = |q|nuE = sE,$$

where the conductivity is. The equation above could be generalized to a vector equation ($\vec{J} = s\vec{E}$), since the field and current density have directions.

<u>Resistors in Series</u>

Resistors are in series if they have the same current through them.



Use the loop rule in the clockwise direction with potential difference for a resistor given by $\Delta V = -RI$:

$$\text{emf} - R_1I - R_2I - R_3I = 0$$

$$\text{emf} = (R_1 + R_2 + R_3)I = R_{\text{series}}I$$

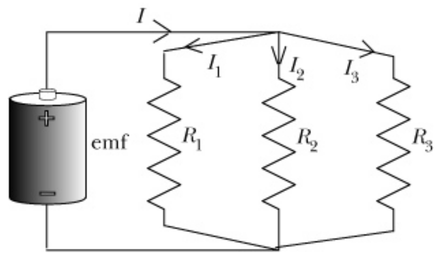
The resistors in series are "equivalent" to a single resistance of:

$$R_{\text{series}} = R_1 + R_2 + \ldots$$

A single resistance of this value connected to the same battery will result in the same current as in the circuit above.

Resistors in Parallel

Resistors are in parallel if they have the same potential difference across them.



The potential difference through each resistor is the emf of the battery, so the current through one is $I_i = \frac{\text{emf}}{R_i}$. The node rule gives:

$$I = I_1 + I_2 + I_3 = \frac{\text{emf}}{R_1} + \frac{\text{emf}}{R_2} + \frac{\text{emf}}{R_3} = \frac{\text{emf}}{R_{\text{parallel}}}$$

The resistors in parallel are "equivalent" to a single resistance of:

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

A single resistance of this value connected to the same battery will result in the same current from the battery as in the circuit above.

Be careful with fractions! Example: $R_1 = 6\ \Omega$, $R_2 = 3\ \Omega$

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{6\ \Omega} + \frac{1}{3\ \Omega} = \frac{3+6}{18\ \Omega} = \frac{1}{2\ \Omega} \quad \Rightarrow \quad R_{\text{parallel}} = 2\ \Omega$$

Power:

Suppose the change in electric potential energy as a charge Δq moves across a component is ΔU_e.The power is the work done in the time Δt that it takes for the charge to move:

$$\text{Power} = \frac{\Delta U_e}{\Delta t}.$$

The potential difference is the change in potential energy per charge, so:

$$\text{Power} = \frac{\Delta U_e}{\Delta t} = \frac{\Delta q \cdot \Delta V}{\Delta t} = I\Delta V.$$

The relationship above is true for any kind of component. It is literally the rate at which the charged particles loose electric potential energy. If they flowed un-resisted, that would equal the rate at which they *gained* kinetic energy, and they'd accelerate.

However, in resistors, they are opposed and they don't accelerate. So they must simultaneously be imparting the kinetic energy they gain to their environment (via all those collisions). For a resistor, $I = \frac{|\Delta V|}{R}$, so:

$$\text{Power} = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

Interestingly, if that environment, a filament say, is at a constant temperature, it in turn must be imparting energy to *its* environment at this rate as well. So this is the rate at which a light bulb radiates energy as light.

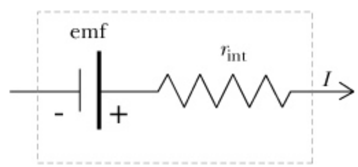
Meanwhile, for a Capacitor, as it's charging up, current and voltage change:

$Q = C|\Delta V|$ the rate of energy build up is $\text{Power} = \frac{dU_e}{dt} = \frac{d(q \cdot \Delta V)}{dt} = \frac{dC(\Delta V)^2}{dt} = C\Delta V \frac{d\Delta V}{dt}$ or the energy invested at a particular time would be the integral of this:

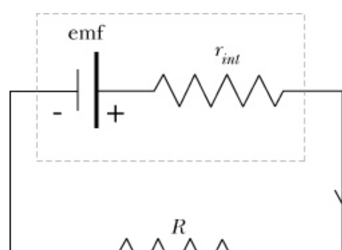
$$U = \int \frac{dU_e}{dt} dt = \int dU_e = \int C\Delta V d\Delta V = \tfrac{1}{2} C (\Delta V)^2$$

<u>Internal Resistance of a Battery</u>:

We will use the following to model the internal resistance of a battery, which is usually small compared to the other resistances in a circuit.



A battery connected to a single external resistor would be modeled as shown below.



Going clockwise, the loop rule for the circuit above gives:

$$+\text{emf} - r_{\text{int}} I - RI = 0$$

$$I = \frac{\text{emf}}{r_{\text{int}} + R}$$

For a D battery, the emf is about 1.5 volts and a typical internal resistance is 0.25 ohm. With those values, the actual currents for a few values of R compared with the ideal values (what they would be with no internal resistance are shown below).

R	I_{actual}	I_{ideal} ($r_{\text{int}} = 0$)
100 ohms	0.0149 A	0.015
10 ohms	0.146 A	0.15 A
1 ohm	1.2 A	1.5 A

The internal resistance can be ignored if the external resistor is much bigger than the internal resistance. If the external resistance is not much larger than the internal resistance, the current will be substantially smaller than expected for an "ideal" battery (no internal resistance).

Experiments: Have the students perform these, then discuss them.

Exp 19.35 – The bulb may be slightly brighter when one in parallel with it is unscrewed. With less current flowing, there is less of a potential difference across the internal resistance and more of a potential difference across the light bulb (the two have to add up to the emf). Also, the internal resistance is more important (see the table on p. 686) when the resistance in the circuit is smaller and two parallel bulbs have a smaller resistance than one bulb.

Exp 19.36 – The light bulb should stay lit about twice as long because the energy is being provided by two batteries instead of one.

Exp 19.38 – Two batteries in series should have twice the short-circuit current of one battery or two batteries in parallel.

Schedule changes:

Do Exp. 19.10 and 19.11 for tomorrow (Friday)

Friday: Meters and mathematical treatment of RC circuits

Monday: circuits lab